



Towards Robust Low-Thrust Trajectory Design under Missed Thrust Events

AMLAN SINHA

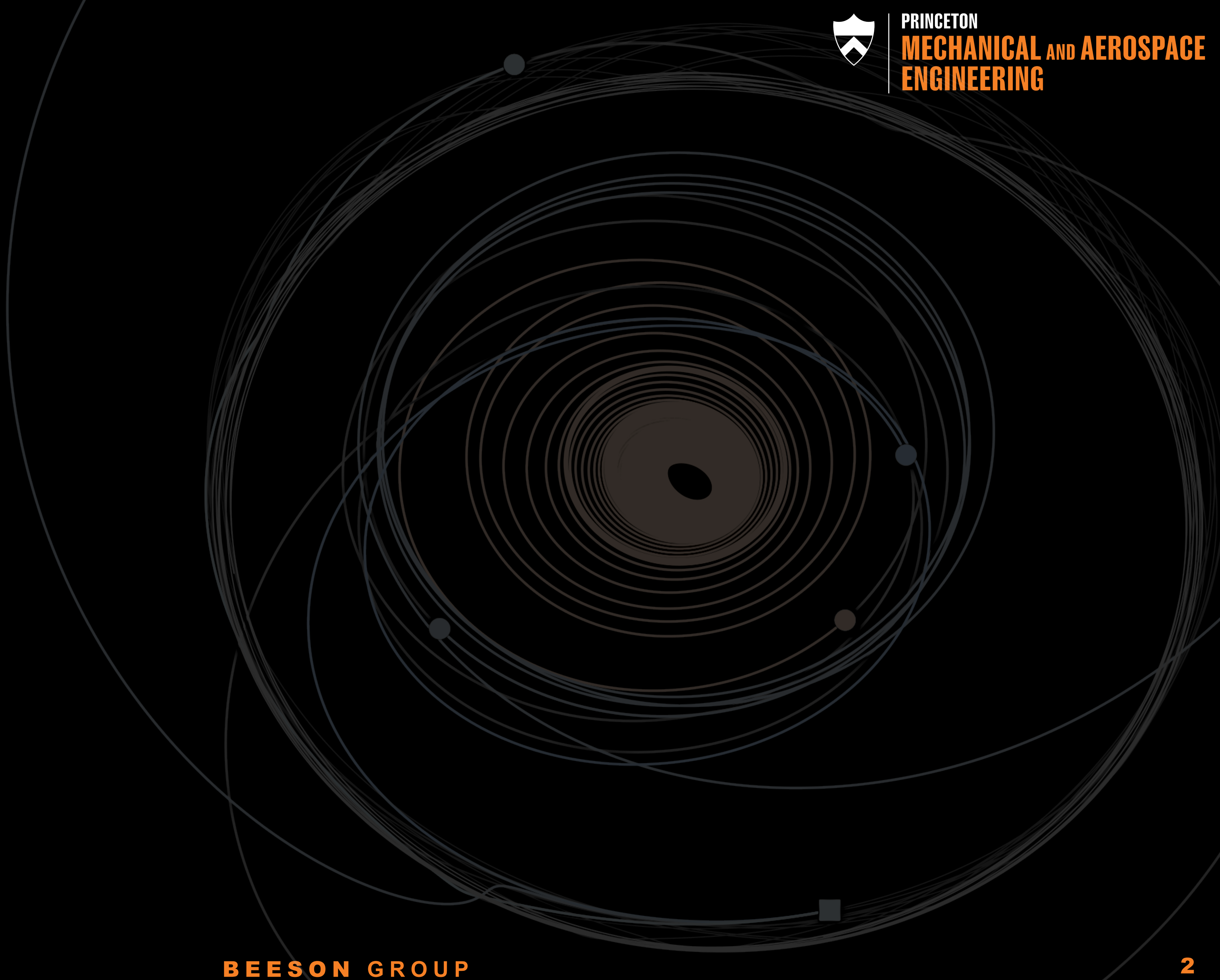
Adviser: Professor Ryne Beeson

Committee: Prof. Naomi Leonard, Prof. Robert Vanderbei

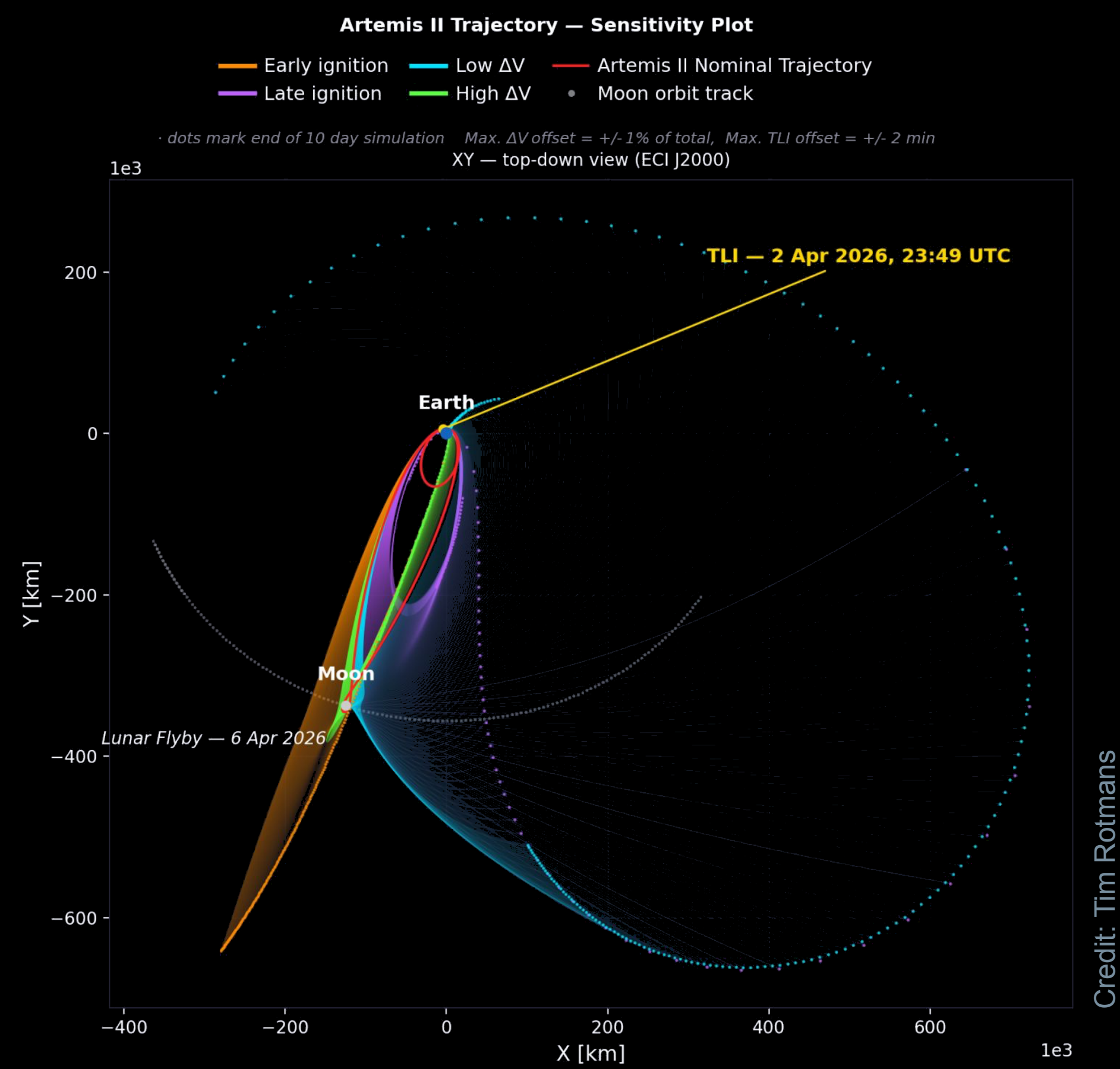
Examiners: Prof. Anirudha Majumdar, Prof. Robert Vanderbei



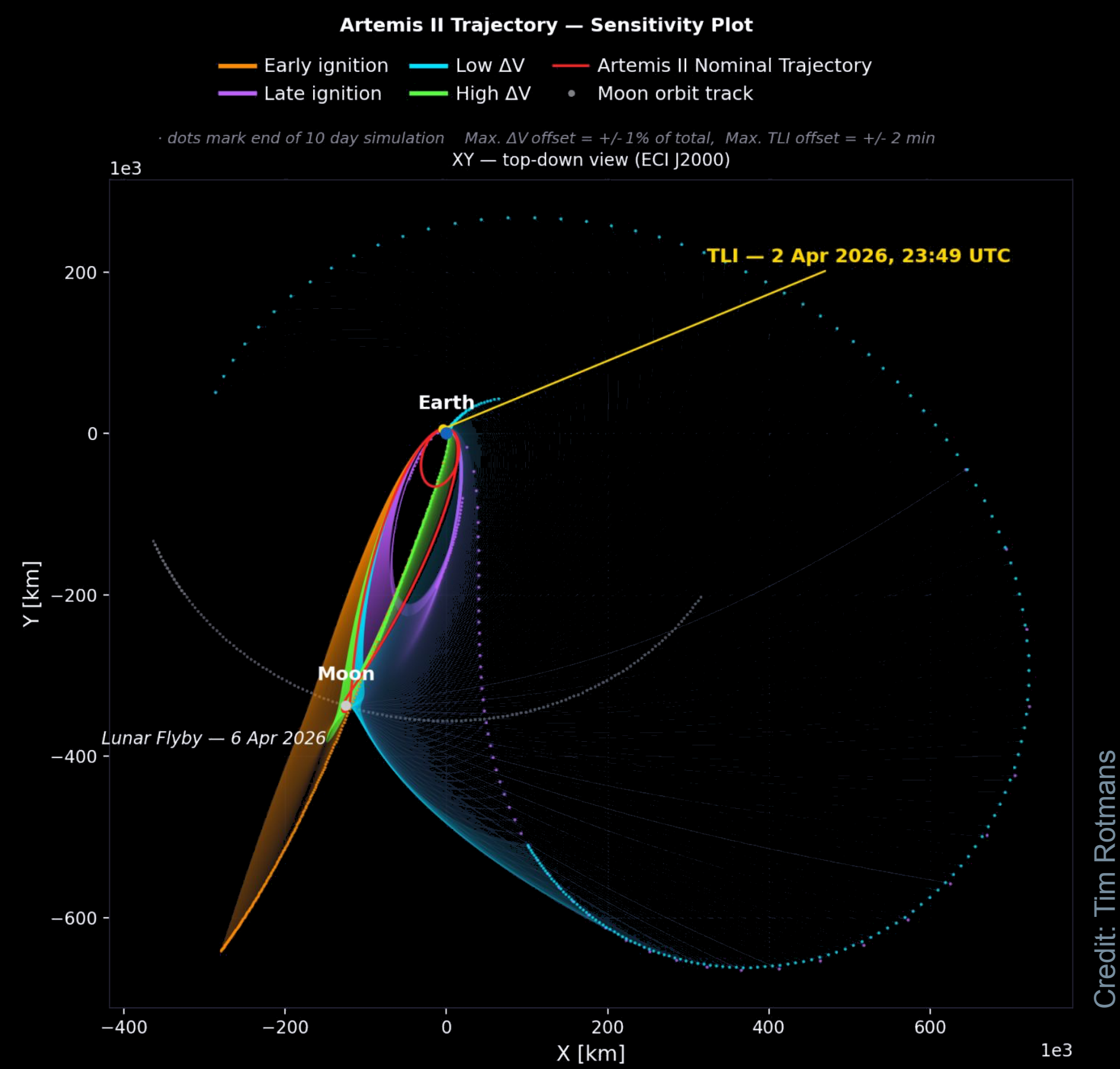
Motivation



Artemis II (2026)

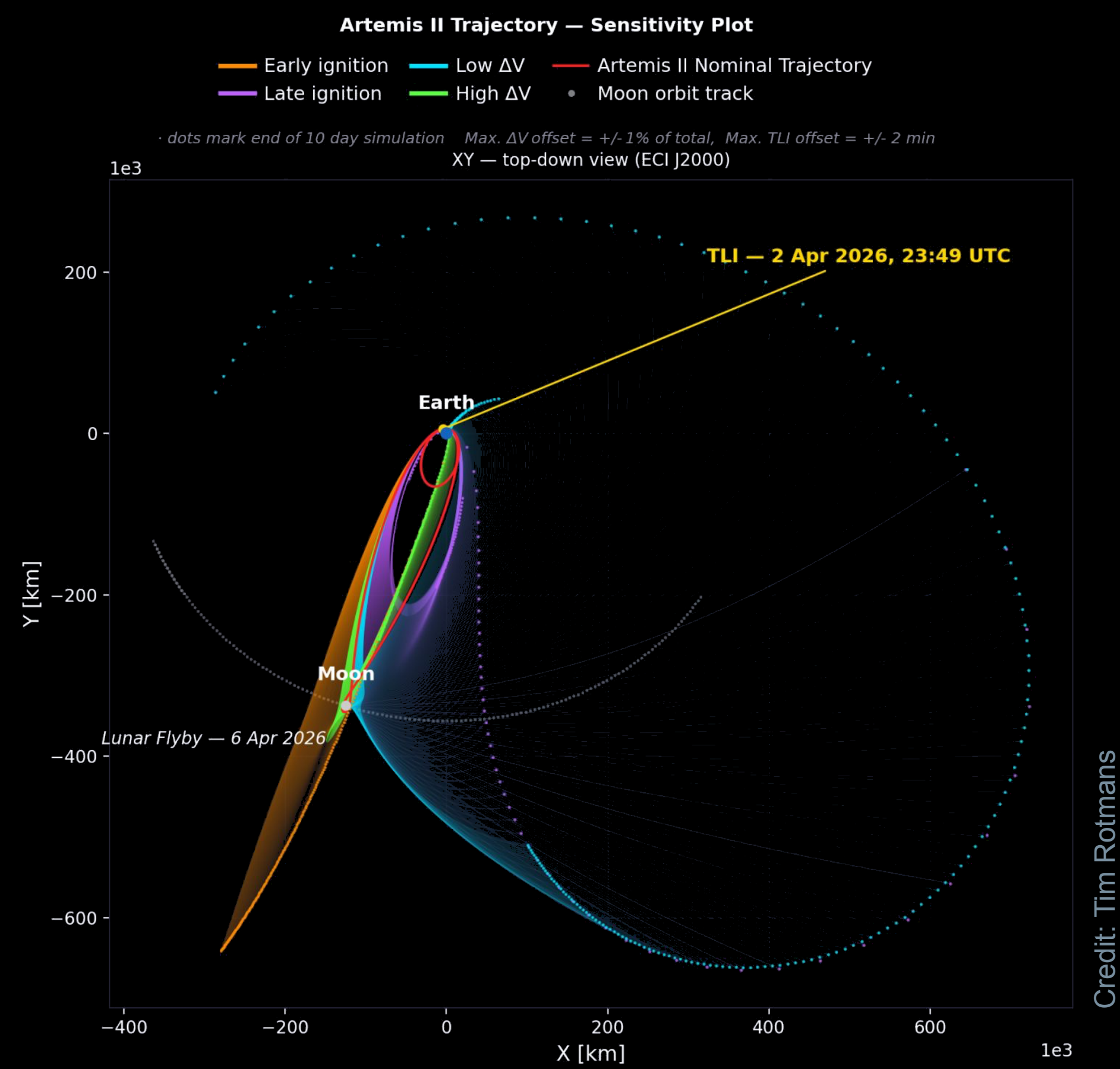


Artemis II (2026)



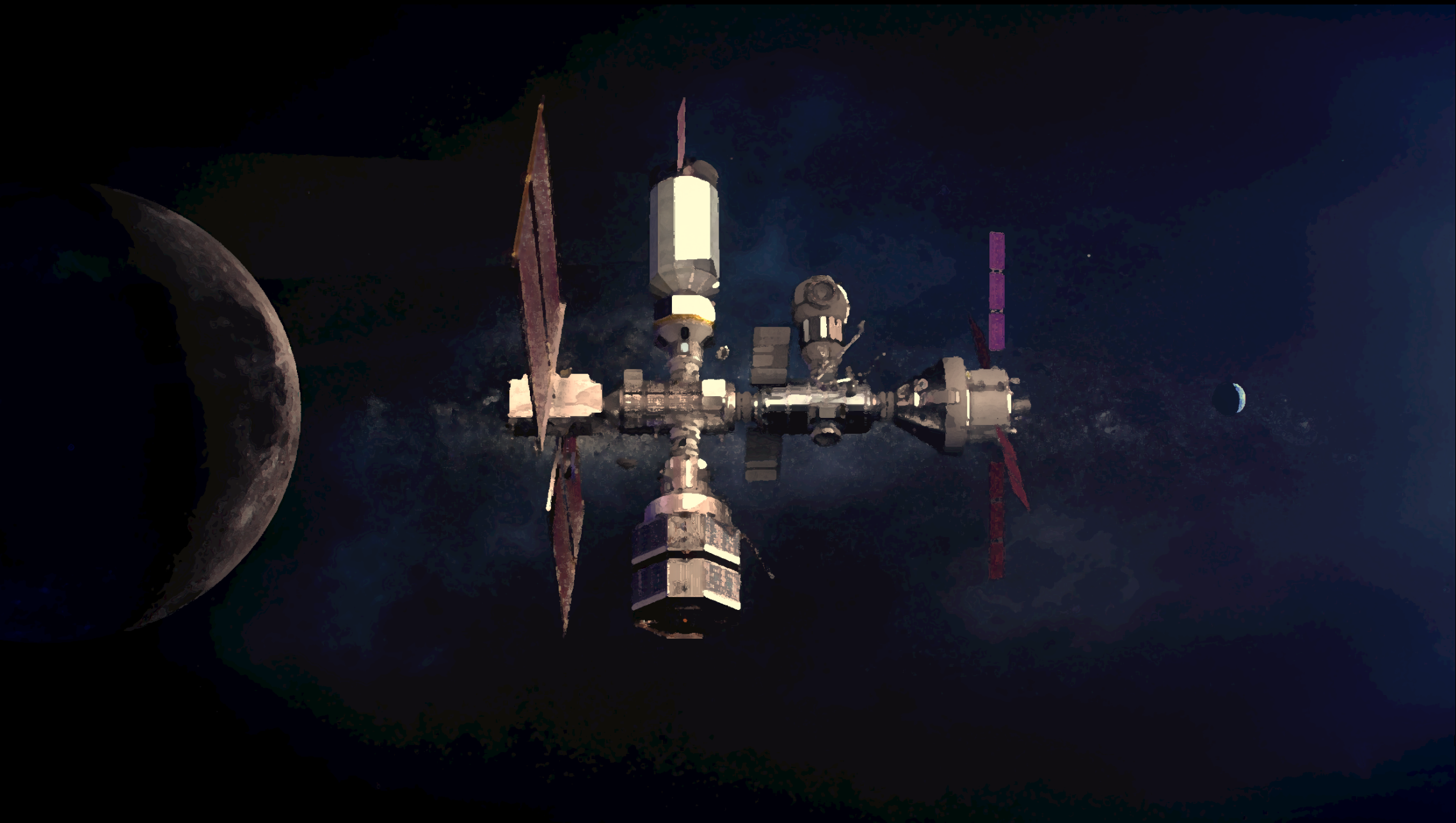
Small maneuver errors $O(1 \text{ m/s})$ or timing offsets $O(1 \text{ min})$ could have resulted in not reaching the Moon at all.

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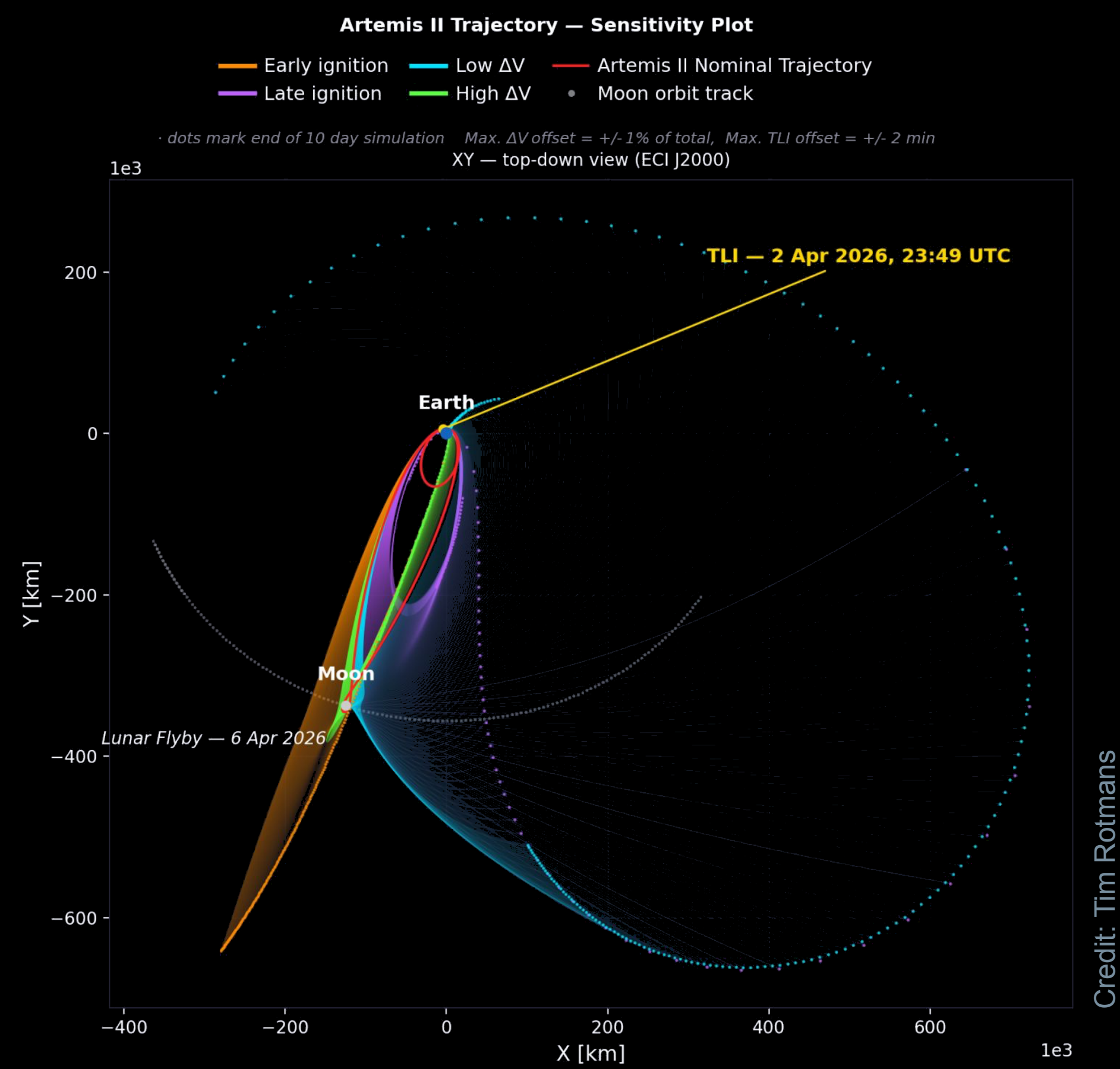


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Gateway (20XX)

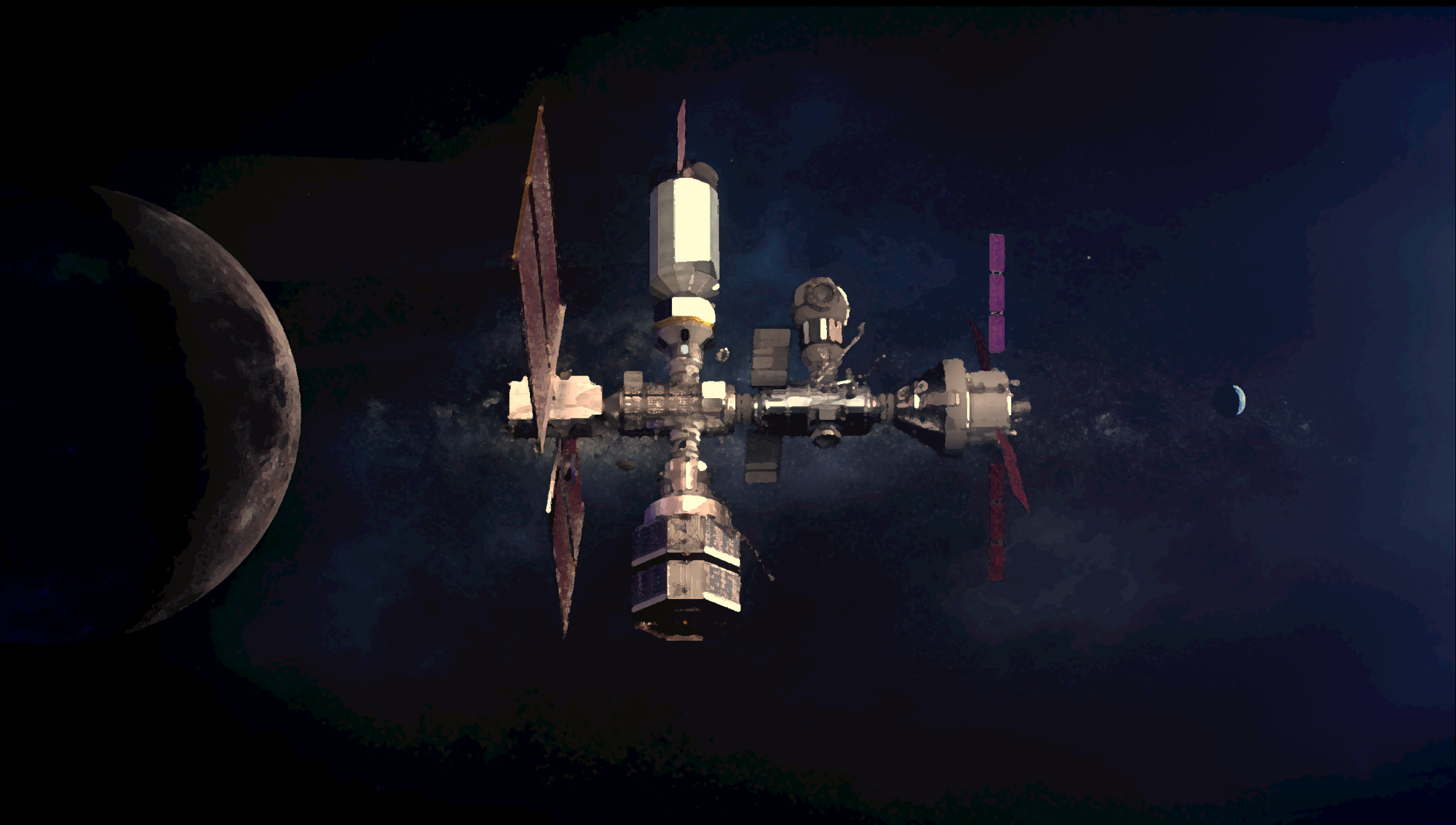


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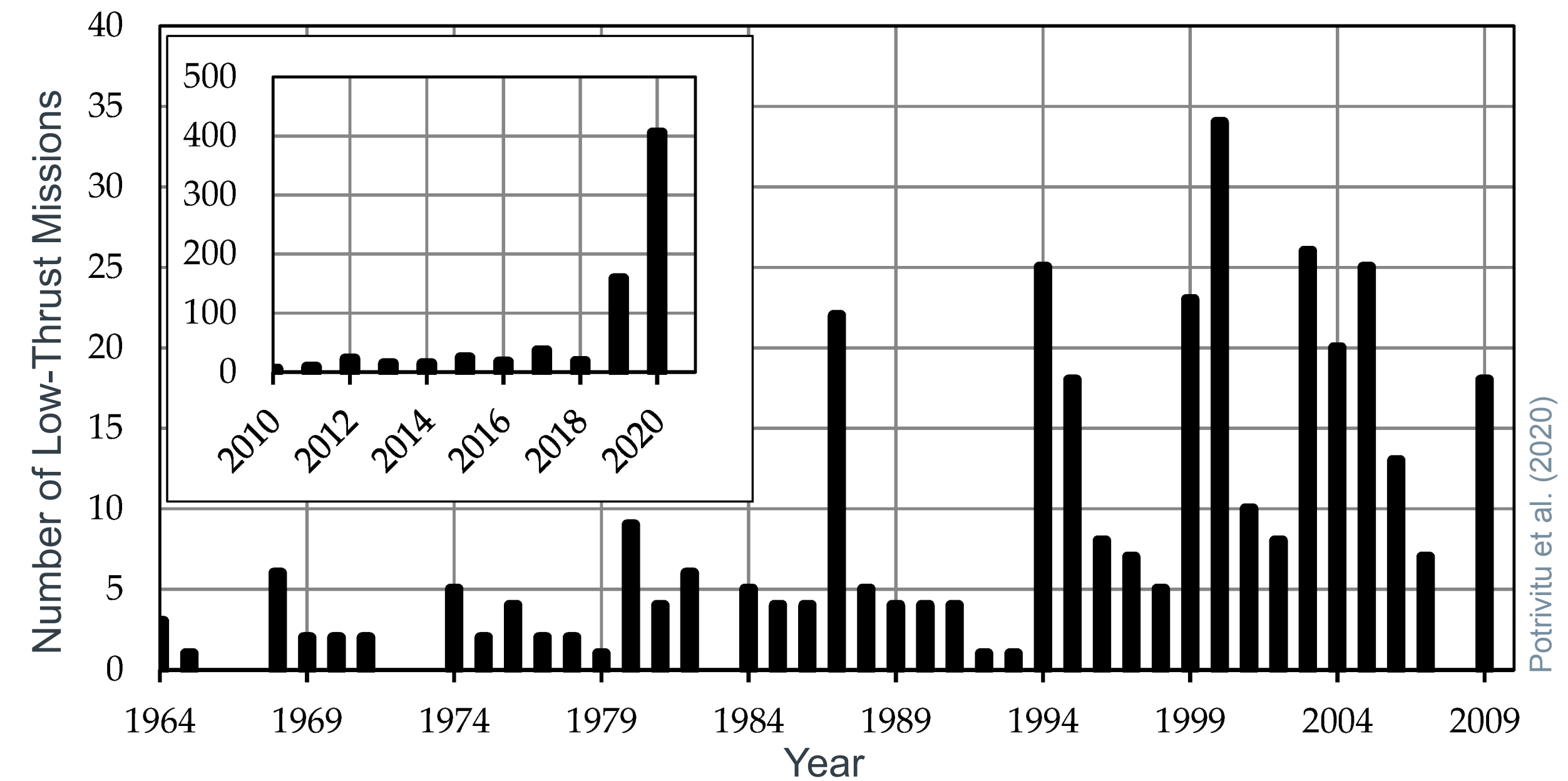


How do we recover from unplanned maneuver execution errors?

Low-thrust (LT) propulsion systems are becoming increasingly popular in space missions

LT offers better fuel efficiency compared to their HT counterparts

- Exponential growth in the last decade
- Attractive because:
 - ✓ High effective payload mass
 - ✓ Large cumulative Δv
 - ✓ Better rideshare compatibility
- But efficiency comes at the cost:
 - ✗ Low control authority
 - ✗ Long flight times



2014 - 2020
Technology Demonstration



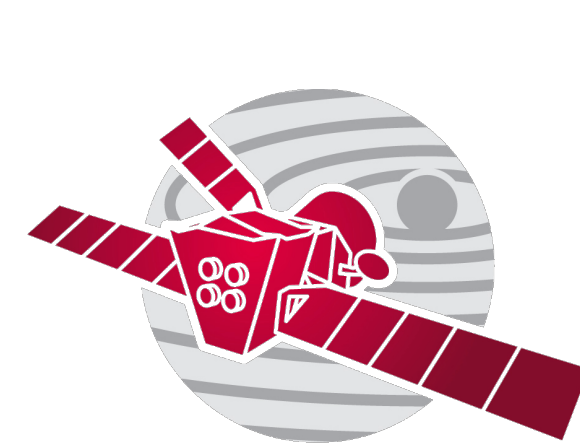
2014 - 2020
Asteroid sample return



2007 - 2018
Asteroid belt origins



2023 – present
Metal world science

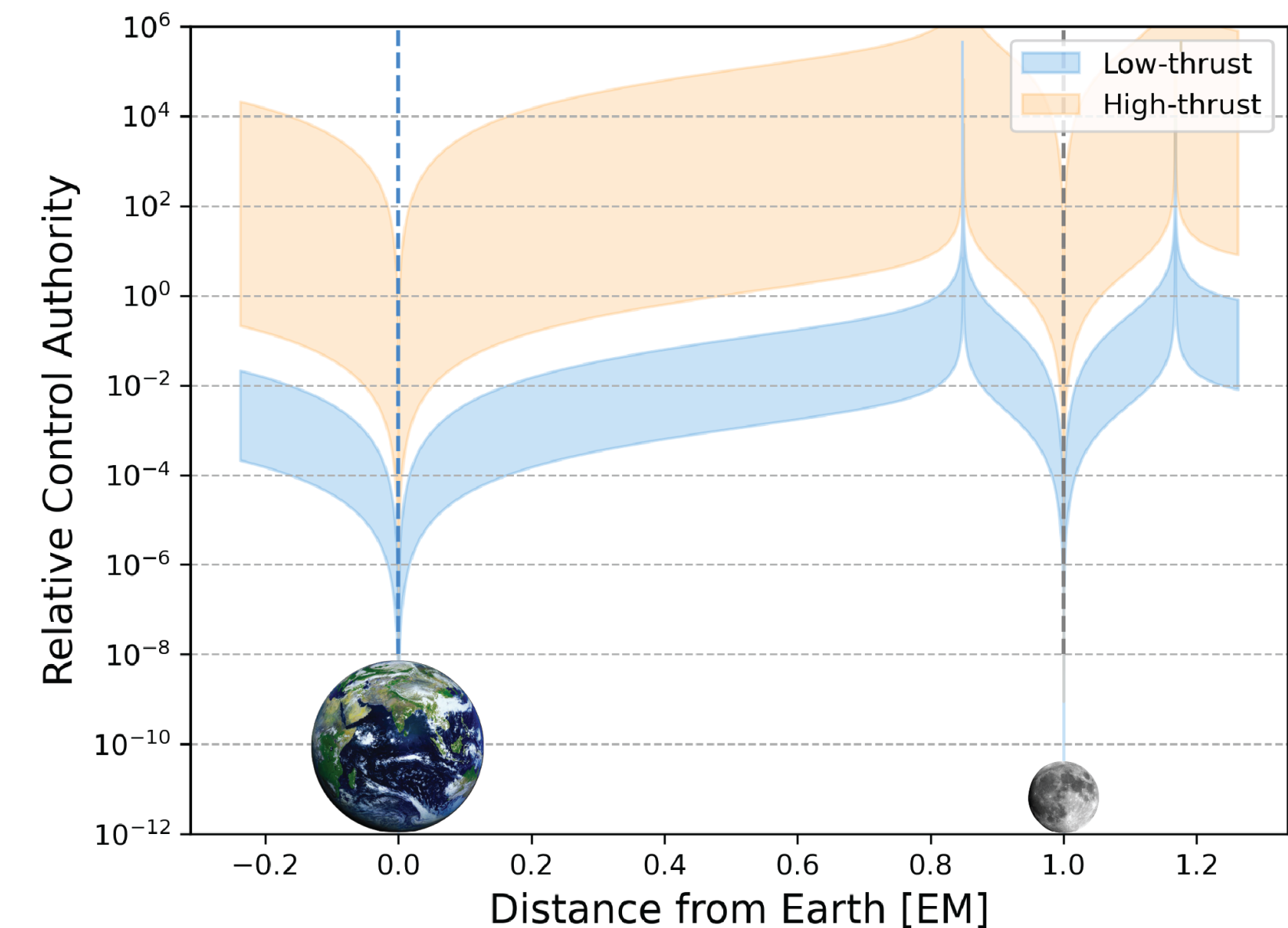


2023 – present
Mercury system science

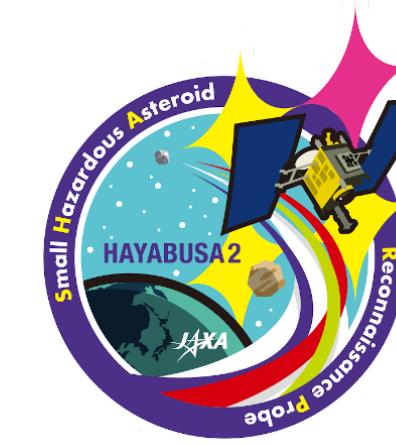
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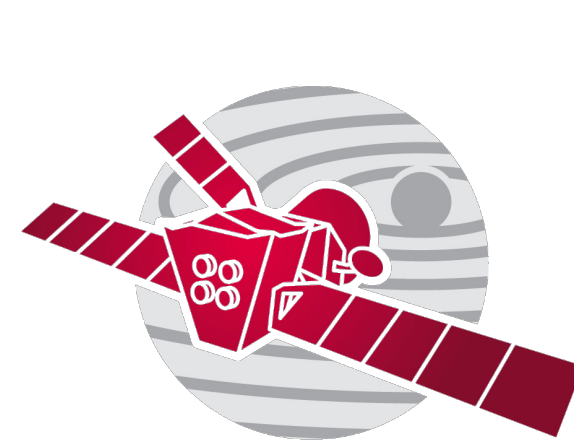
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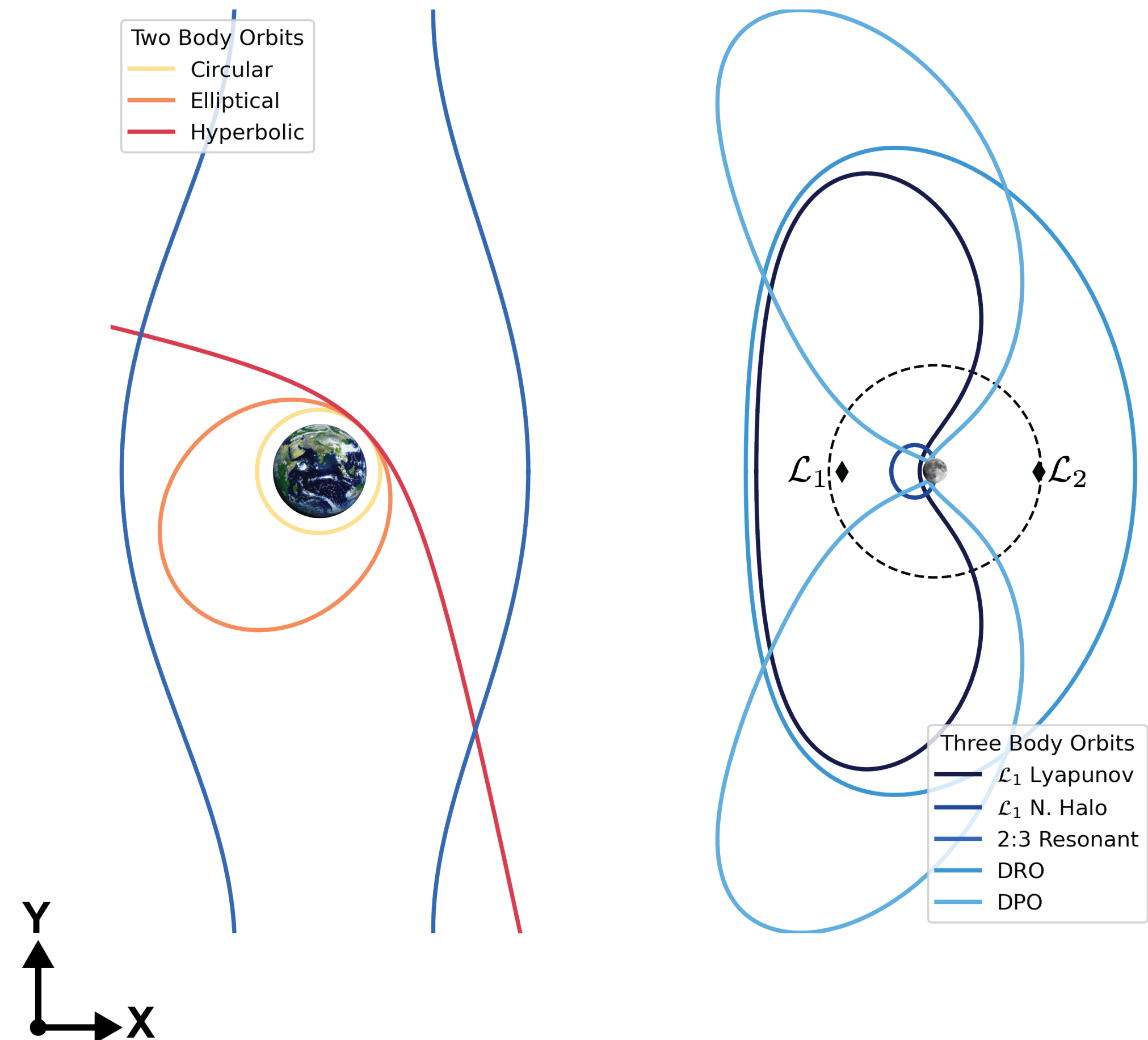


2023 – present
Mercury system science

The cislunar regime is fundamentally different than the near-Earth regime from a dynamical perspective

Cislunar dynamics is governed by complex gravitational influences of both Earth and Moon

- Cislunar dynamics is:
 - Chaotic
 - Non-integrable
 - Governed by invariant structures
- Cislunar mission design requires iterative exploration of a **high-dimensional solution space**



One major class of operational uncertainty is Missed thrust Events (MTEs)

Unless specifically accounted for, MTEs can critically jeopardize mission success

- Inability to execute a maneuver can result in **substantial deviations** from the nominal trajectory, especially within **chaotic multibody dynamics**
- MTEs are relatively common for **LT** space missions
 - Thrust arcs can last from weeks to months
 - 60% likelihood of encountering MTEs every 200 days

Imken et al (2018)

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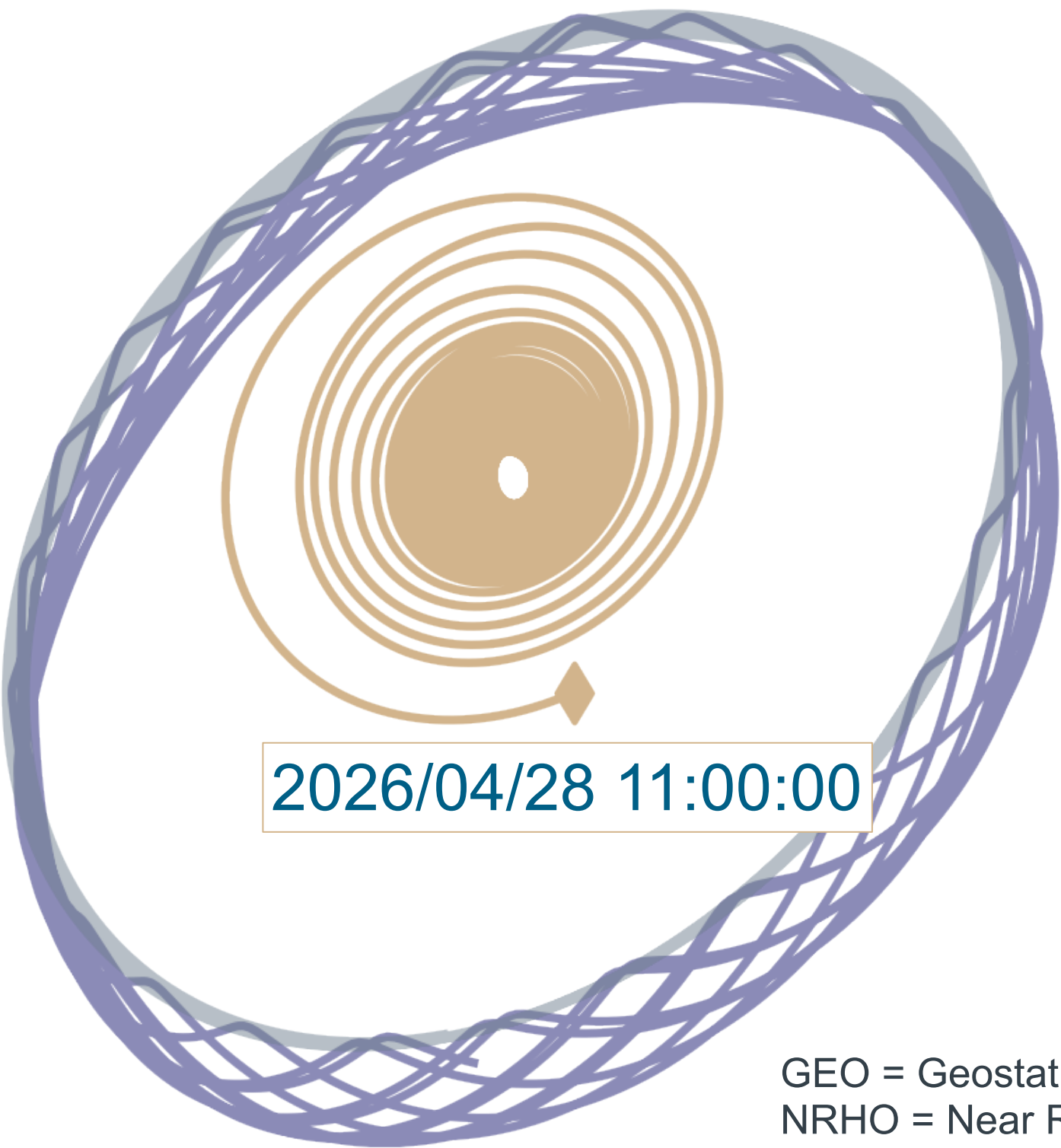


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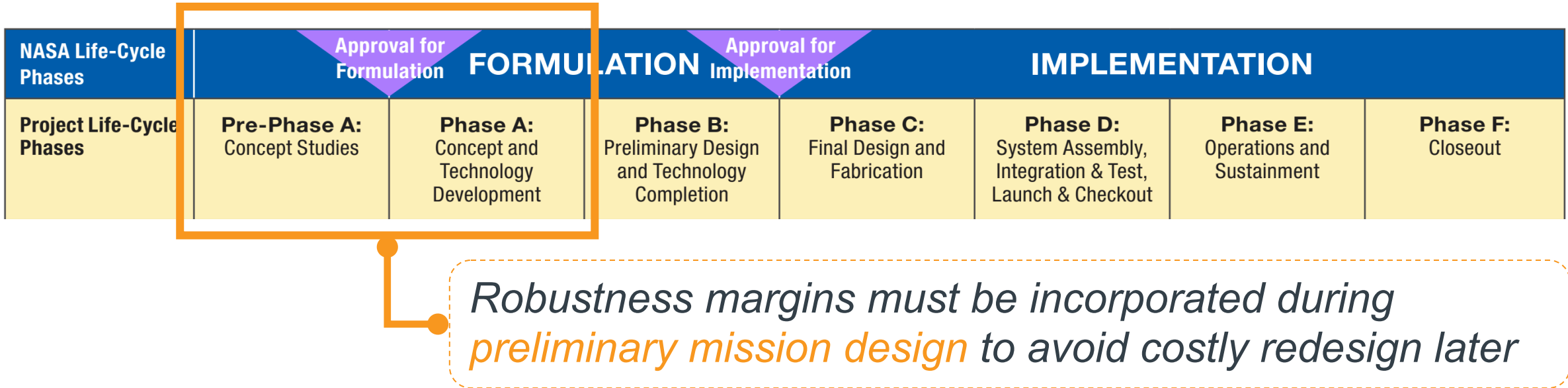
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Extent of impact on nominal solution depends on the **location** and **duration** of the MTE

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GEO = Geostationary Orbit
NRHO = Near Rectilinear Halo Orbit

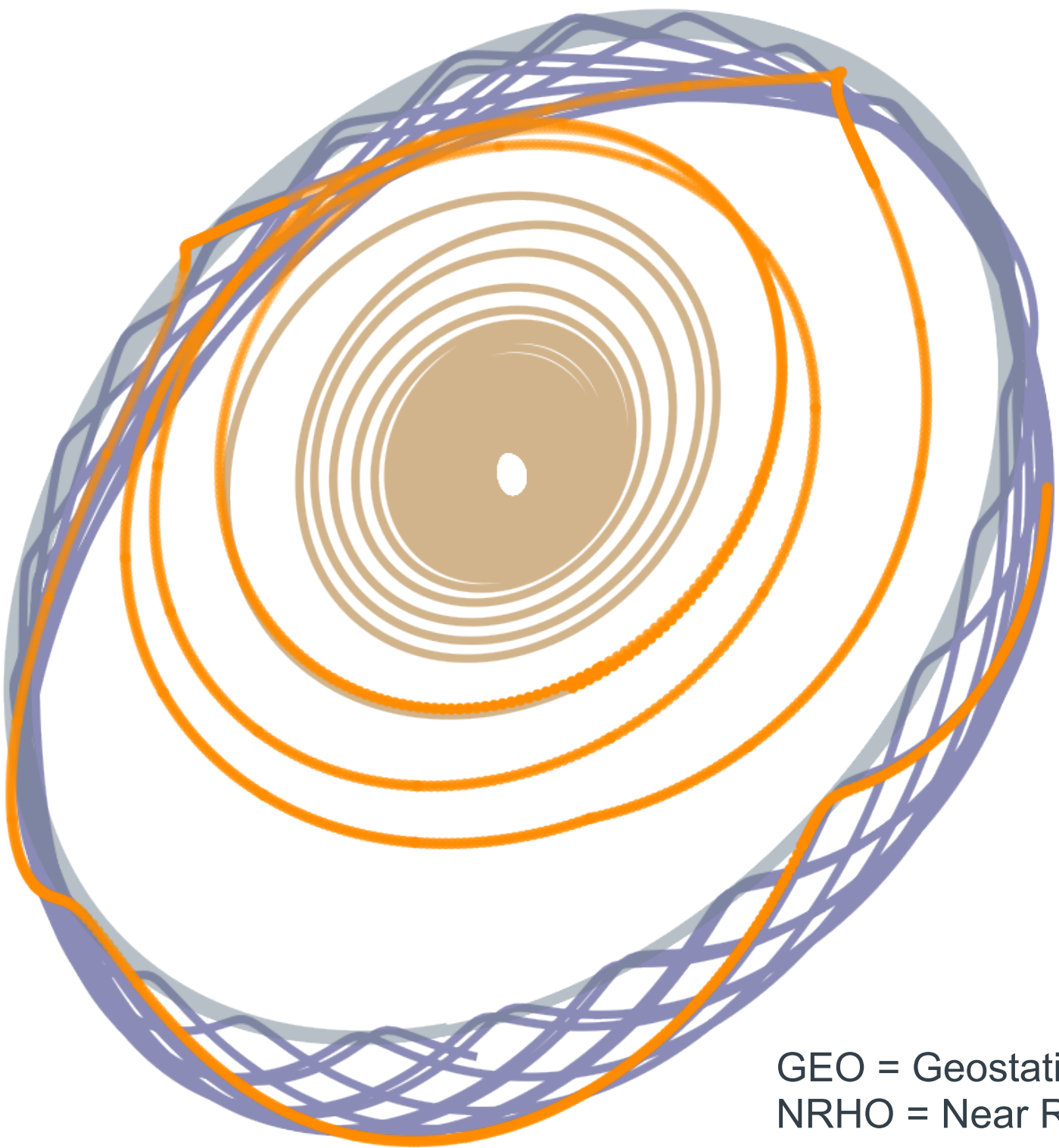


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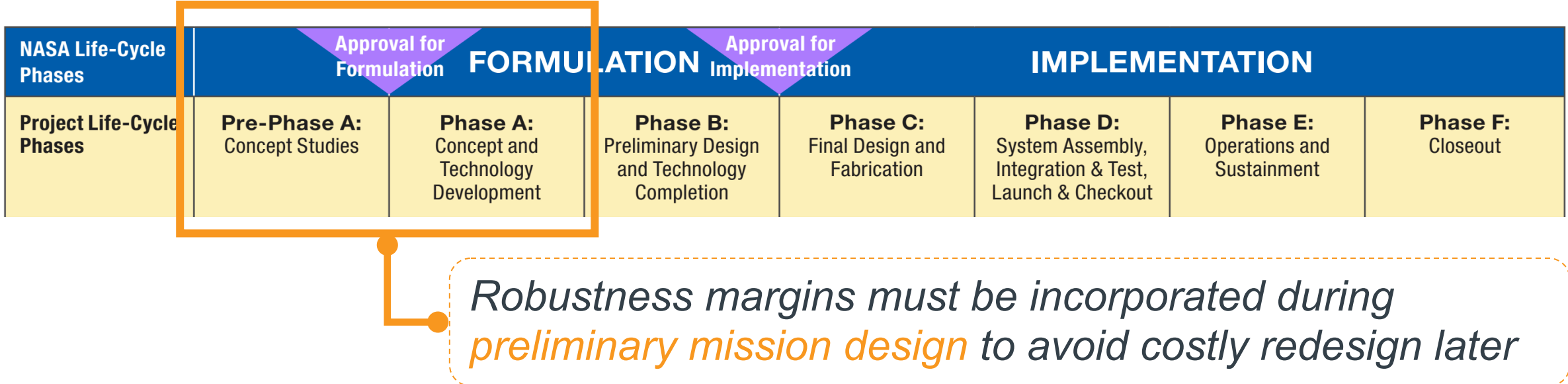
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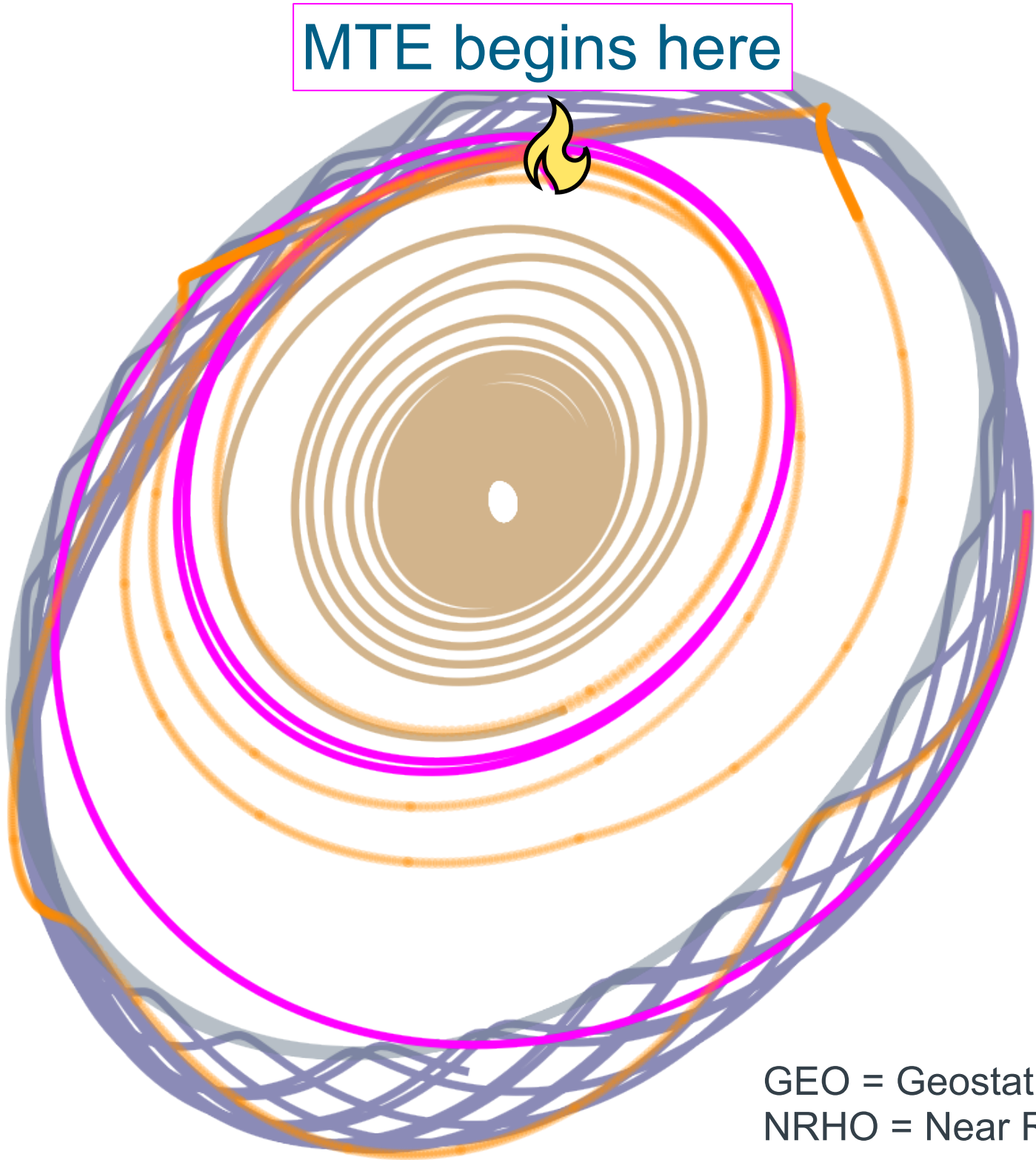
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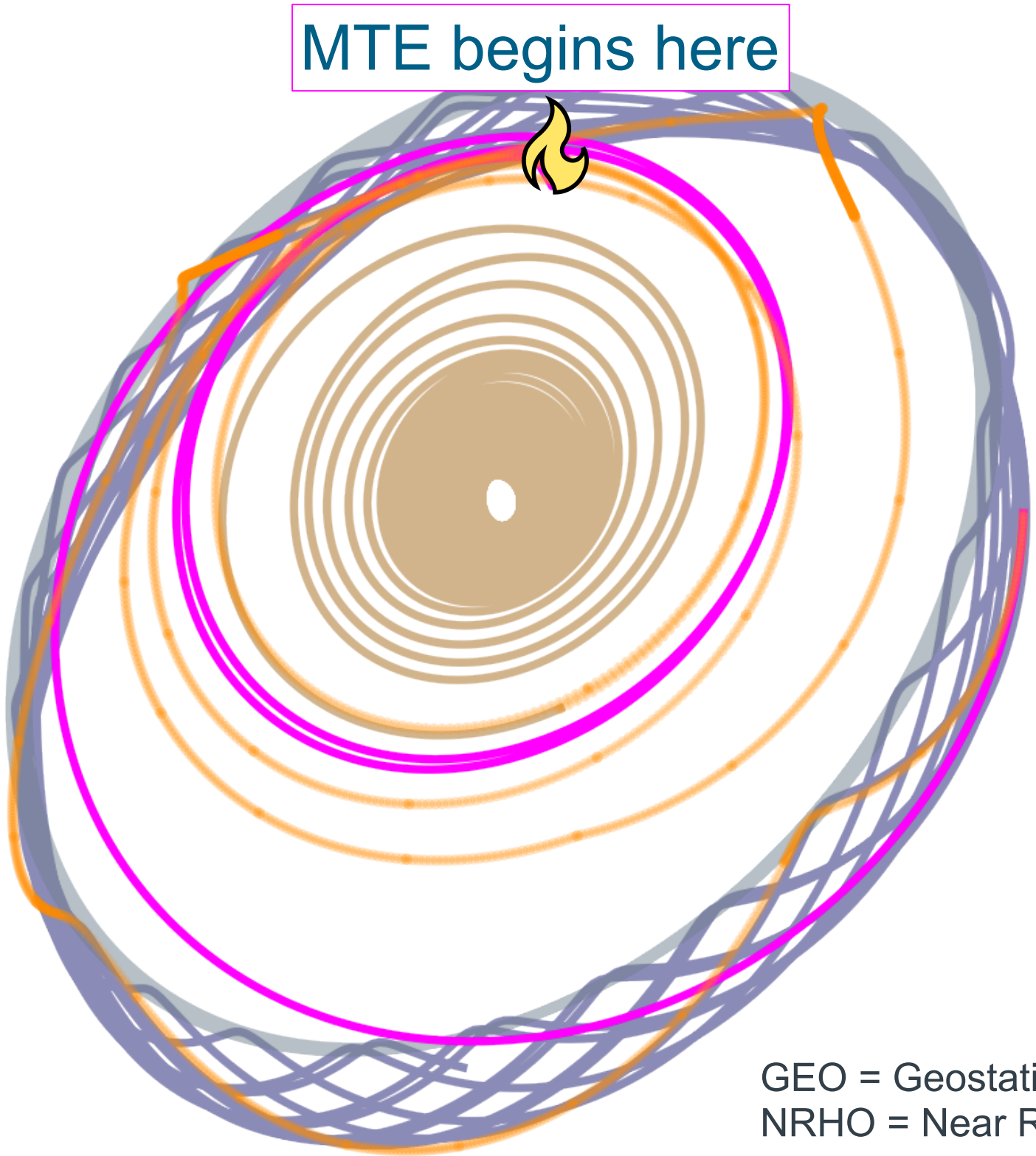
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NASA Life-Cycle Phases	Approval for Formulation		Approval for Implementation		IMPLEMENTATION		
Project Life-Cycle Phases	Pre-Phase A: Concept Studies	Phase A: Concept and Technology Development	Phase B: Preliminary Design and Technology Completion	Phase C: Final Design and Fabrication	Phase D: System Assembly, Integration & Test, Launch & Checkout	Phase E: Operations and Sustainment	Phase F: Closeout

Robustness margins must be incorporated during preliminary mission design to avoid costly redesign later



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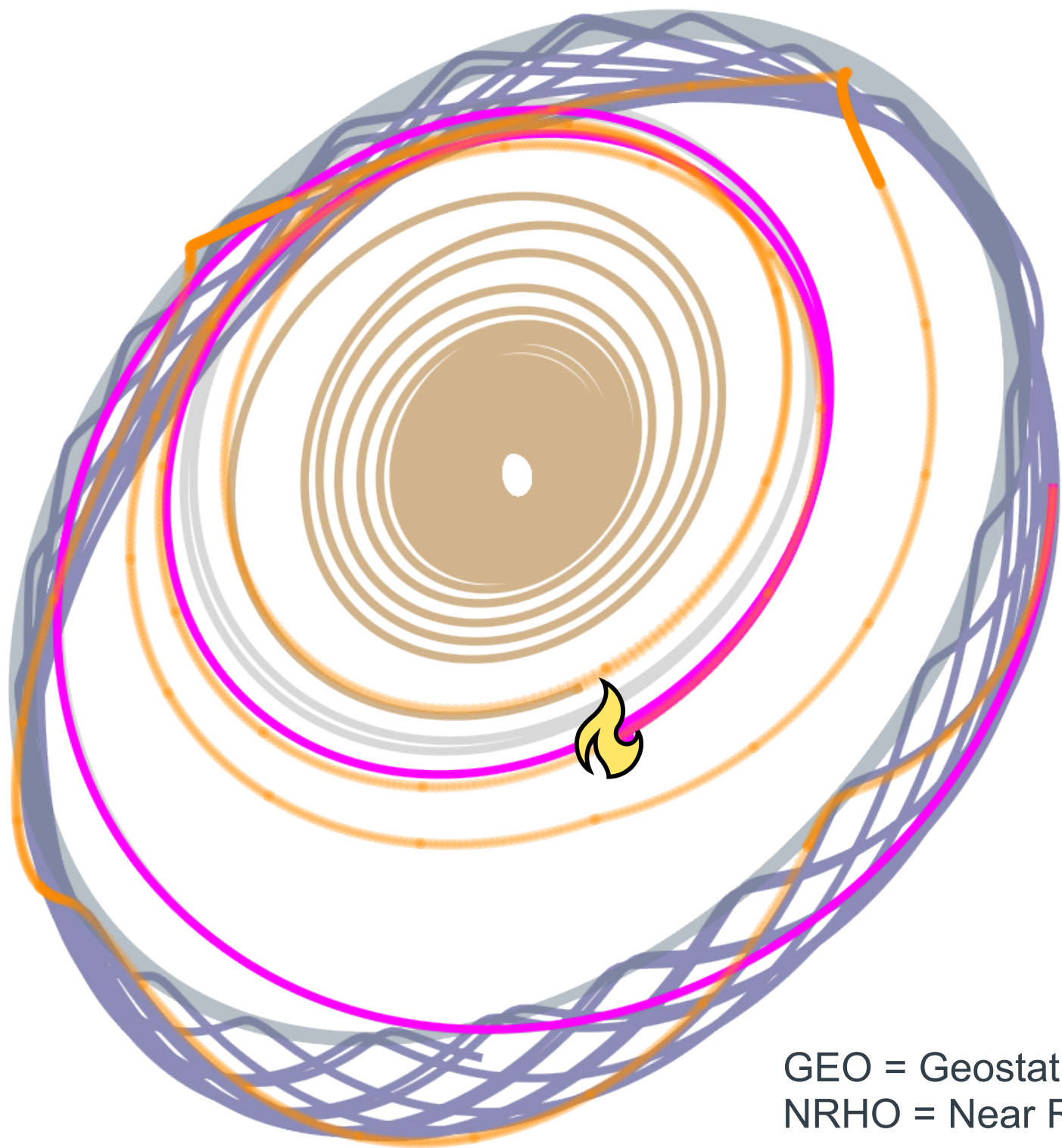
Fuel Consumption ↑ 9.19%

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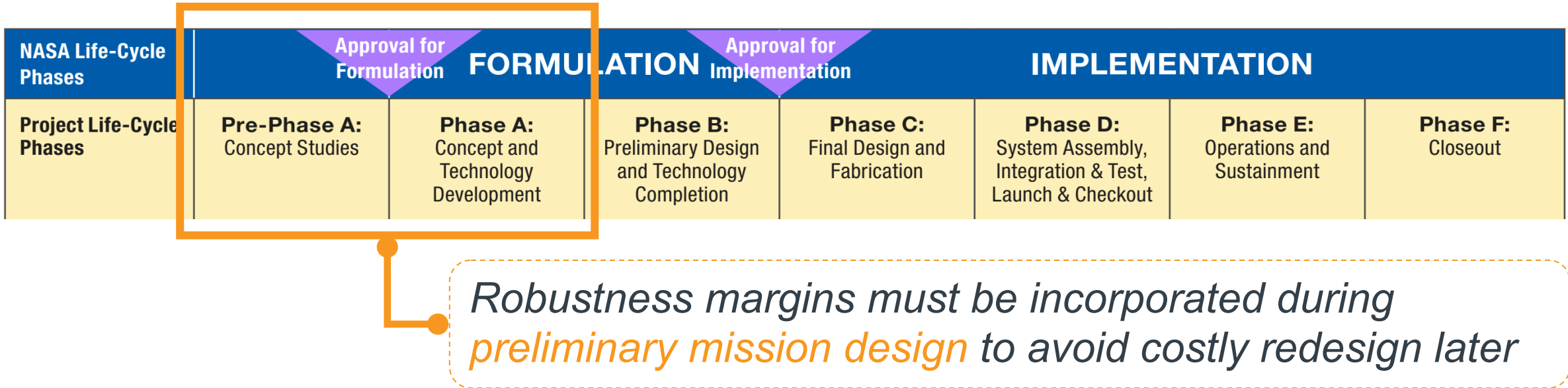
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Fuel Consumption ↑ 6.40%



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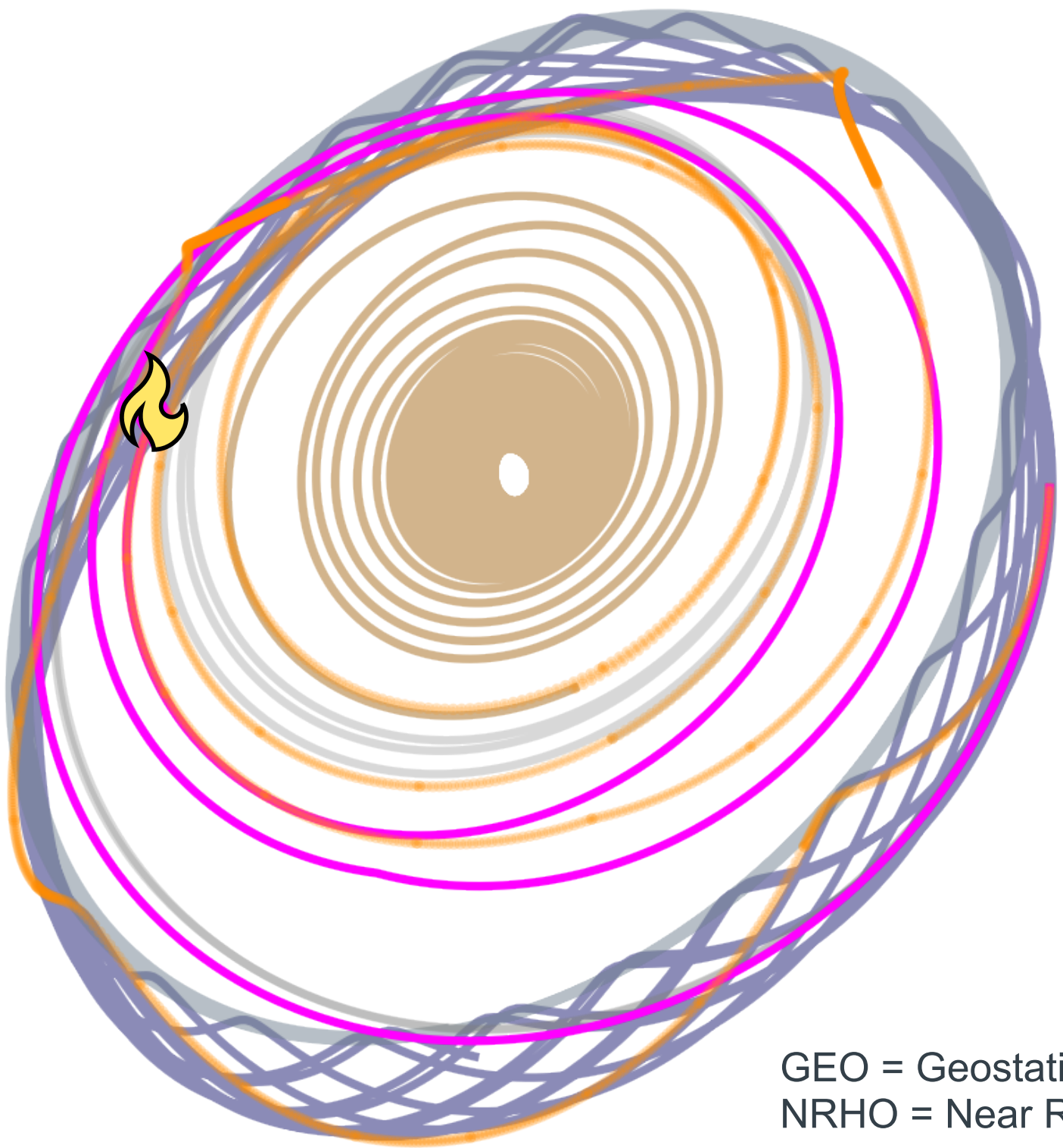
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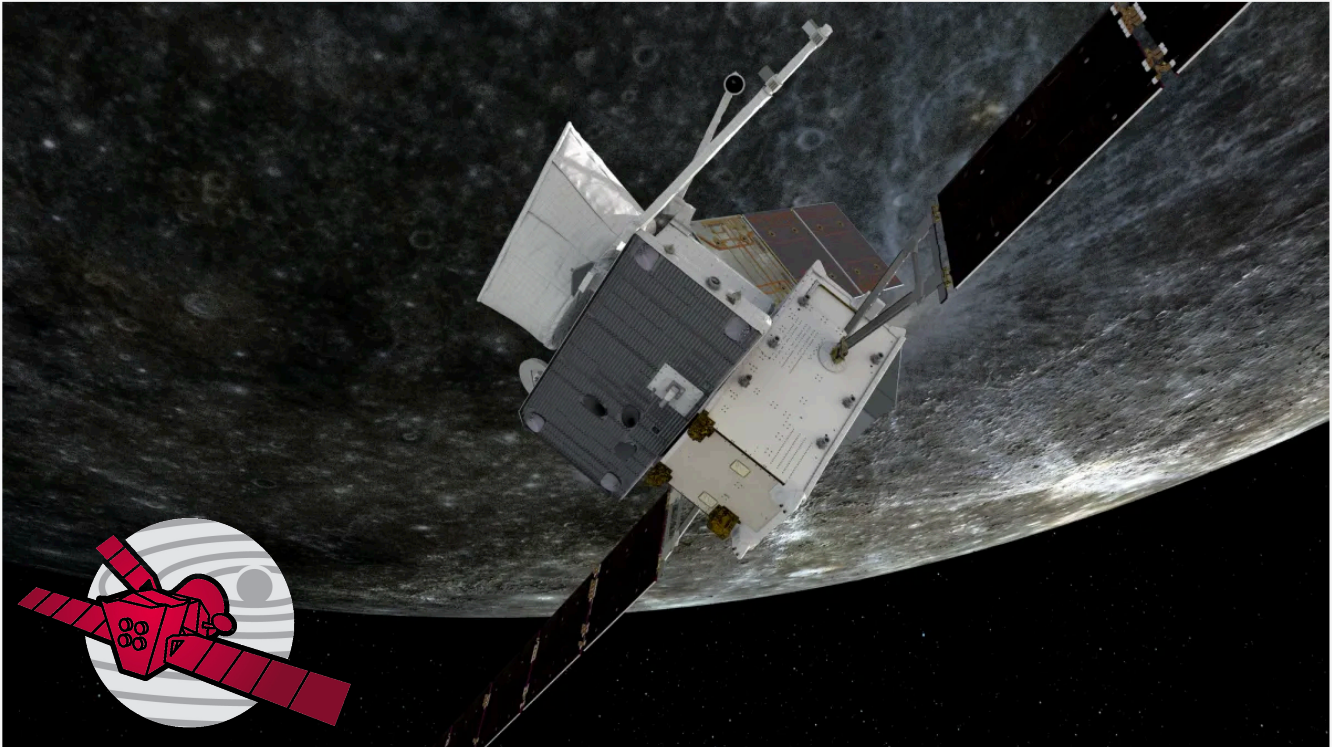
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Fuel Consumption ↑ 0.87%

MTEs can occur due to many different reasons...

ESA delays BepiColombo orbital insertion because of thruster problem

by Jeff Foust September 2, 2024



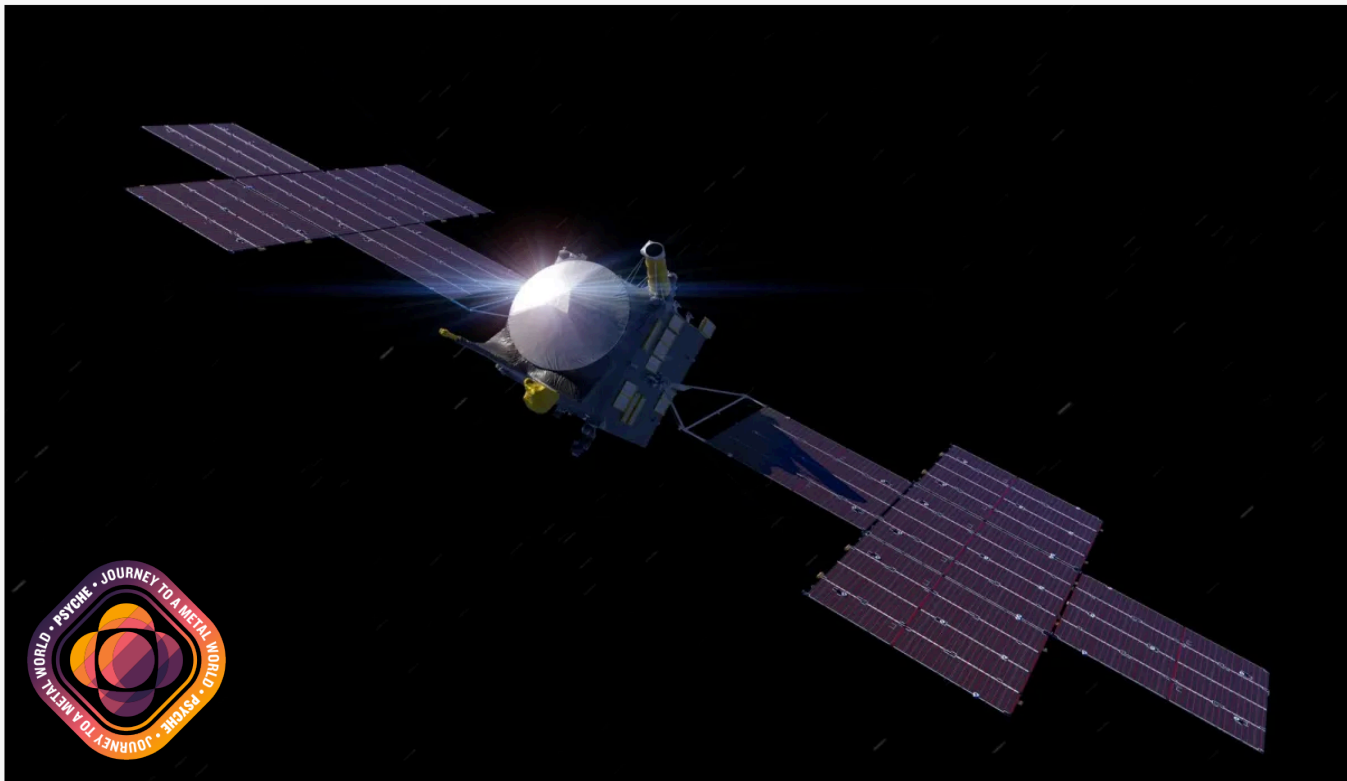
The ESA/JAXA BepiColombo spacecraft will take a revised trajectory to reach Mercury in response to problems with its electric propulsion system. Credit: ESA

“ The European Space Agency is adjusting the trajectory of its BepiColombo mission to Mercury, delaying its orbit insertion by nearly a year to compensate for *reduced performance from its electric thrusters*. ”

- SpaceNews (September, 2024)

NASA investigating problem with Psyche electric thrusters

by Jeff Foust April 30, 2025



NASA's Psyche mission is set to arrive at a main belt asteroid also named Psyche in 2029. Credit: NASA/JPL-Caltech/ASU

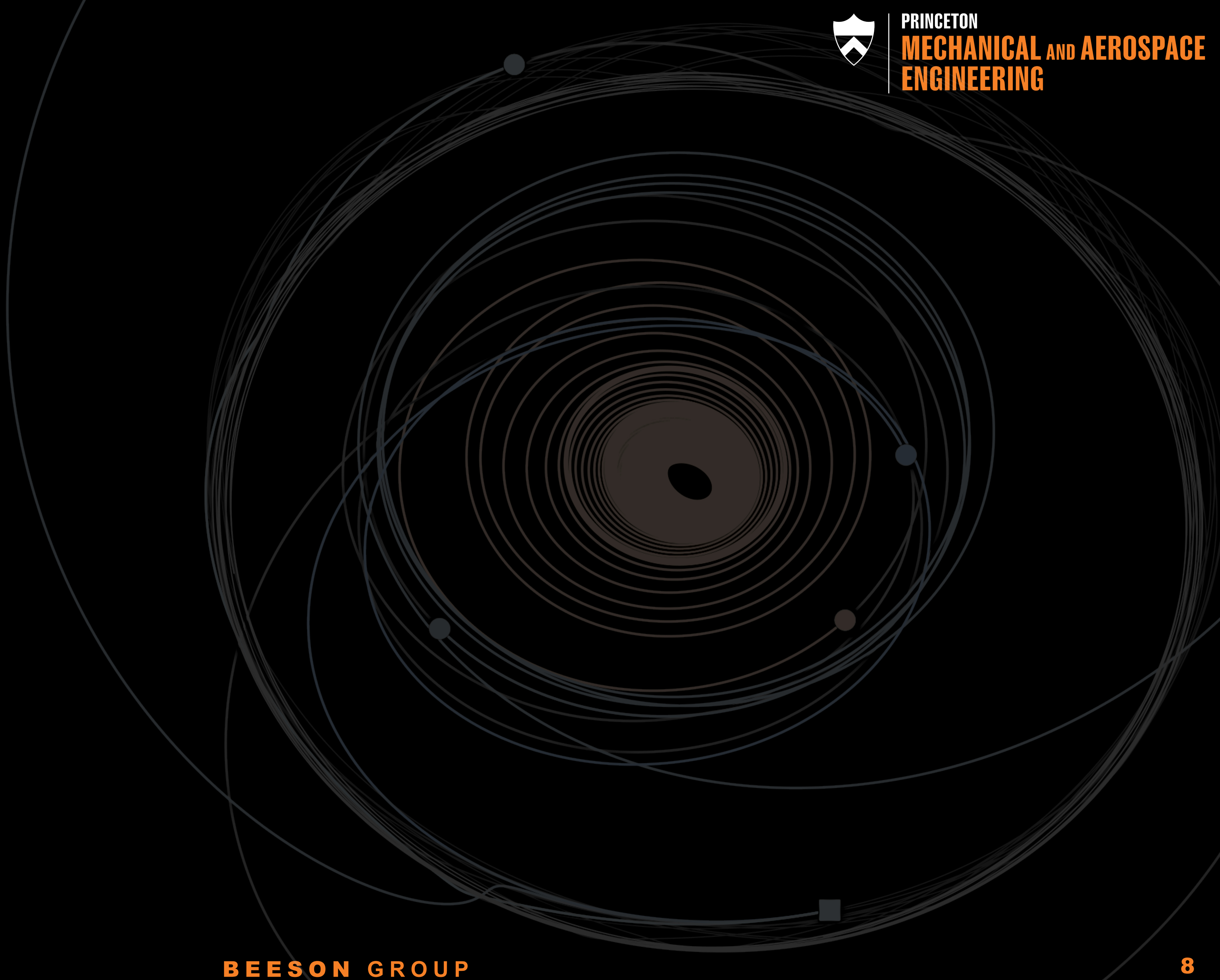
“ NASA is investigating a problem with the electric propulsion system on its Psyche asteroid spacecraft... due to a recent *decrease in fuel pressure in its propulsion system*. ”

- SpaceNews (April, 2025)

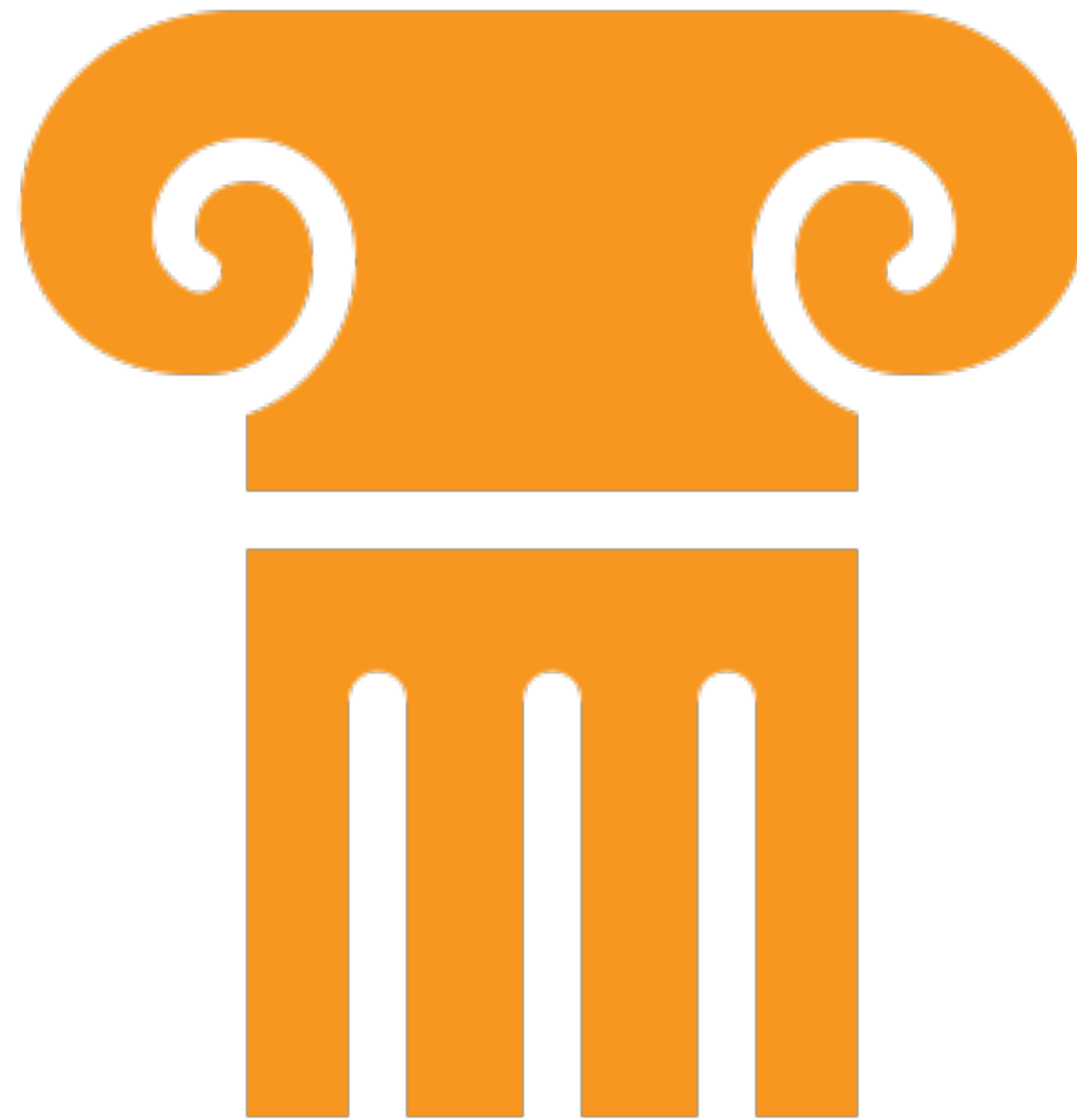
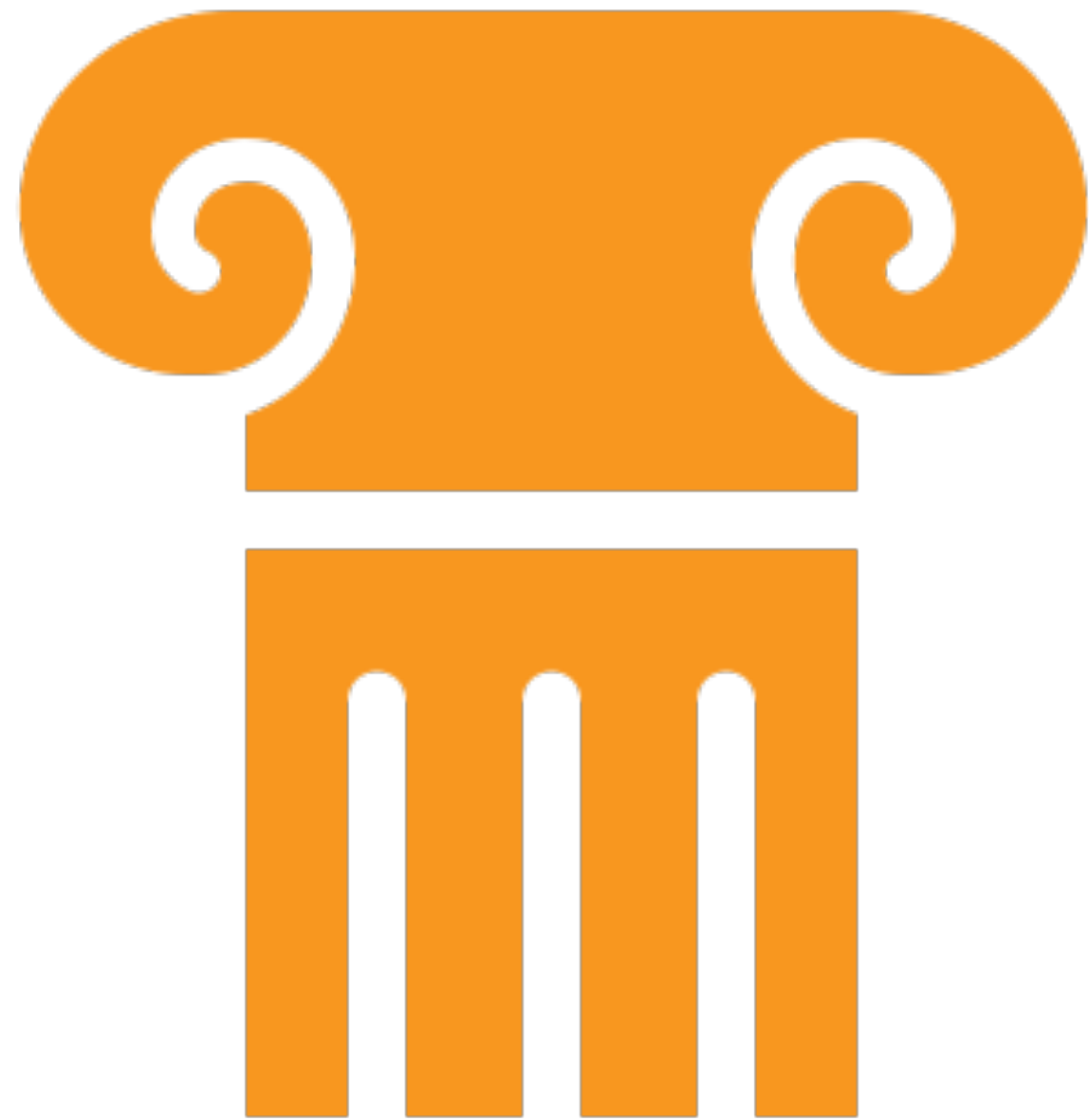




Outline

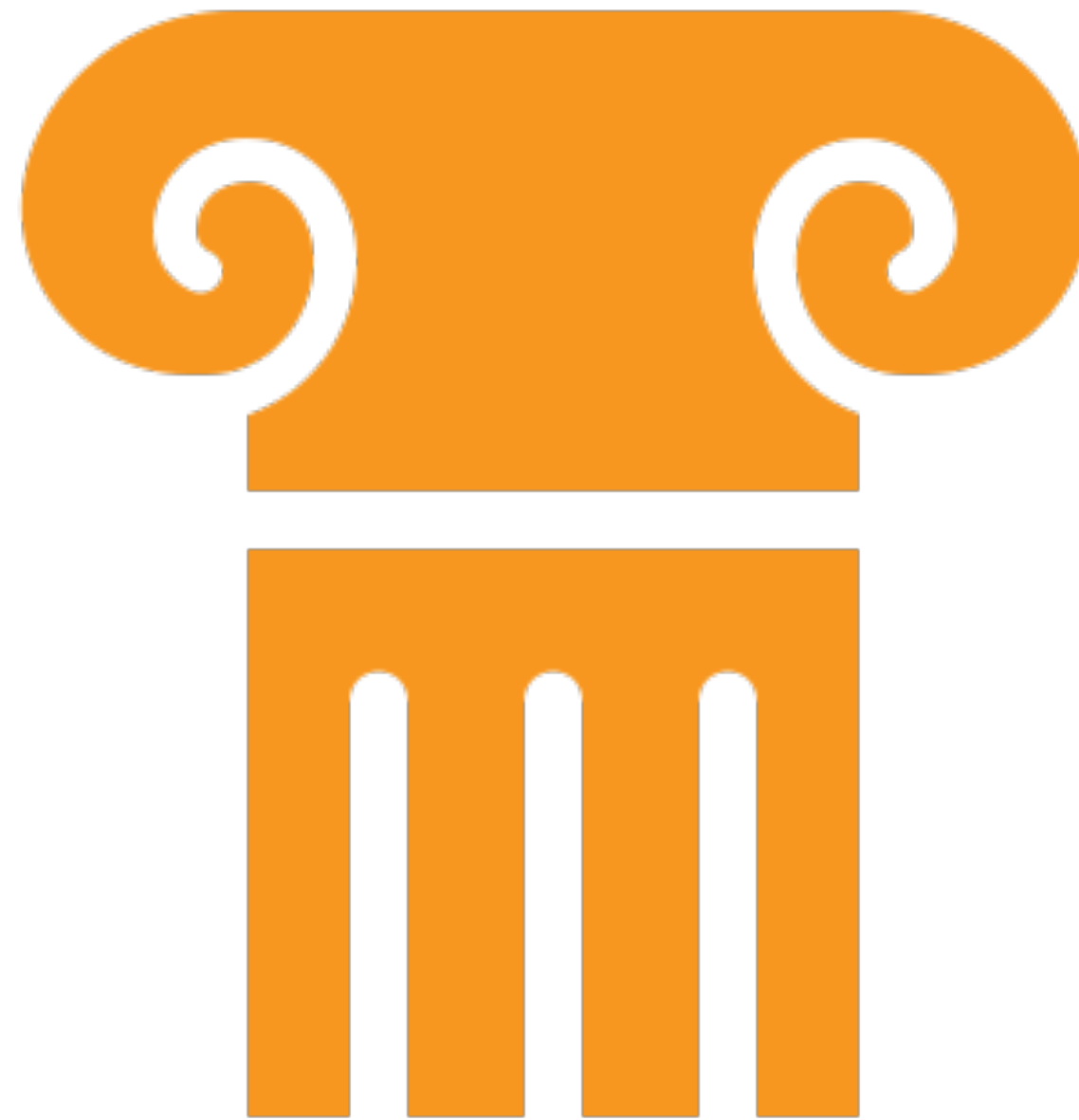
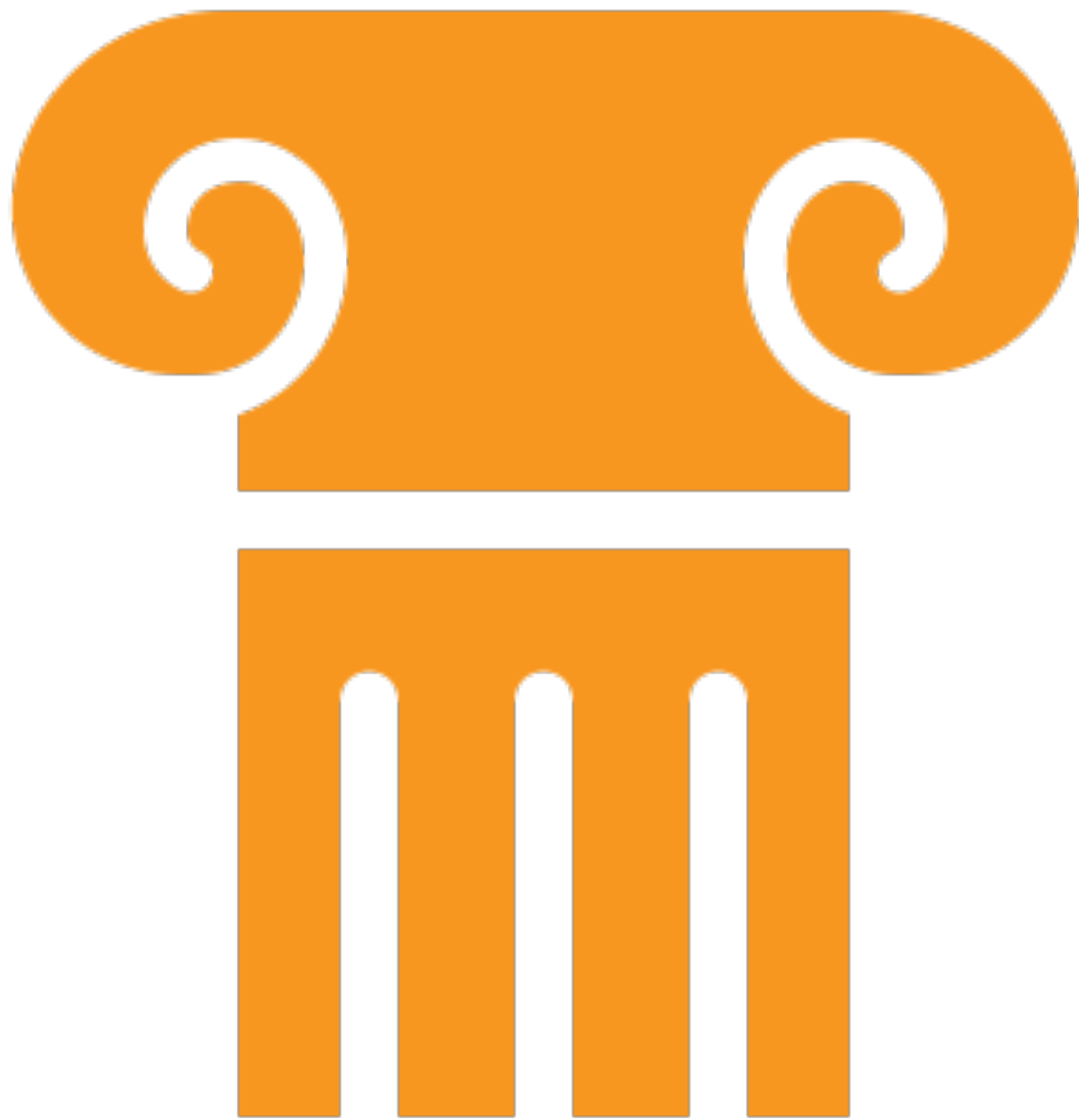
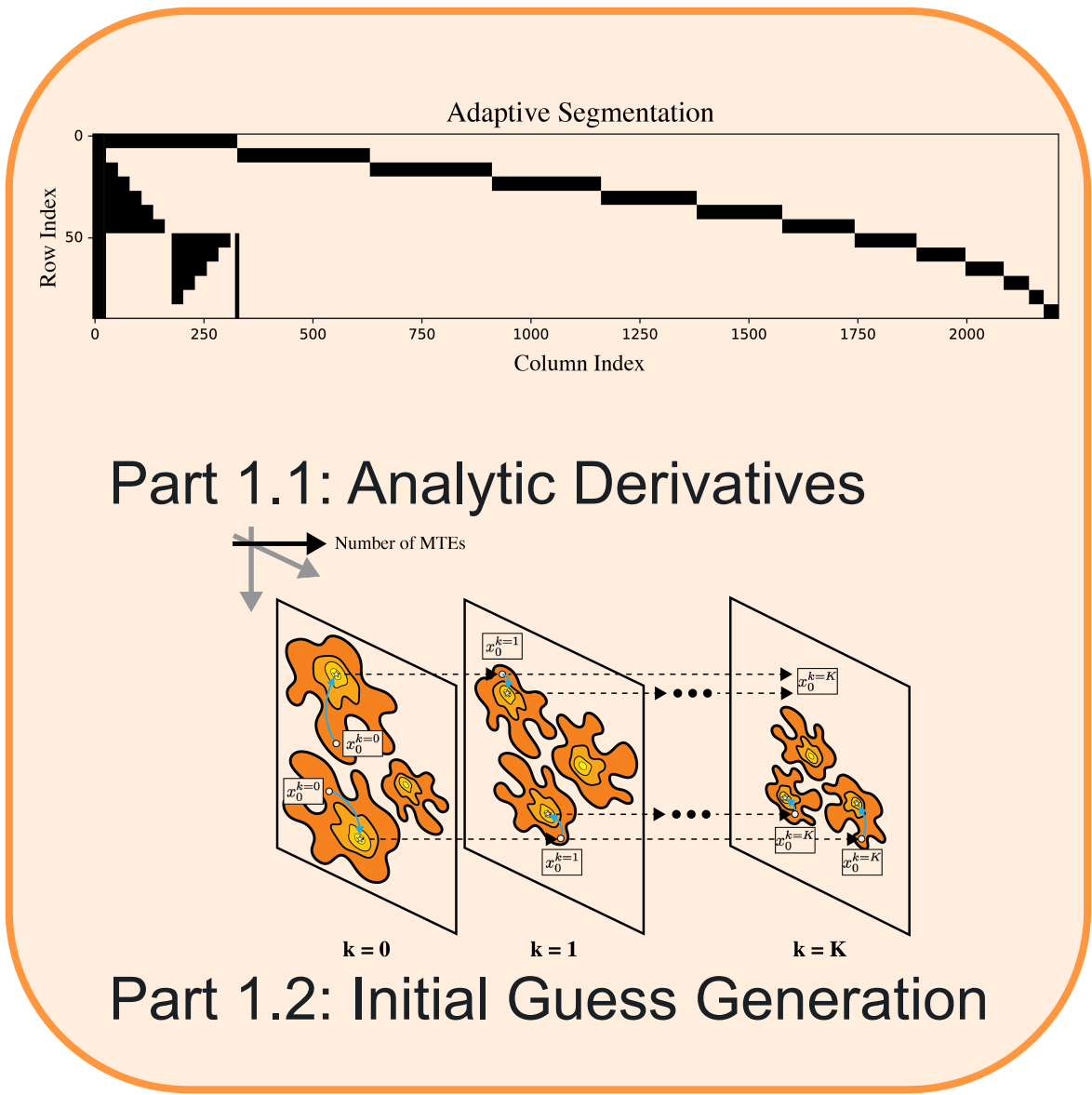


This dissertation builds on three foundational pillars...



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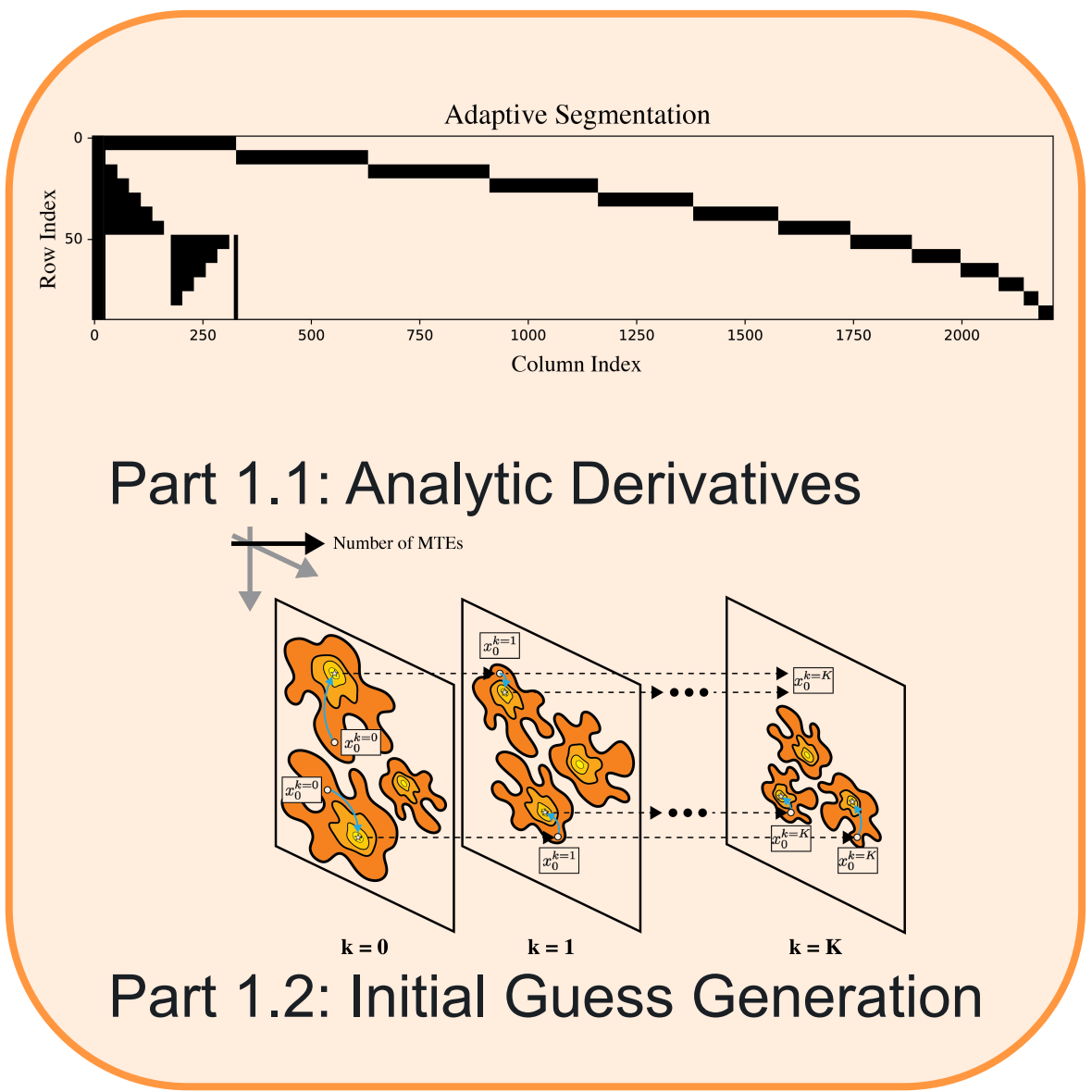
Part 1: Scalable Computational Methods



This dissertation builds on three foundational pillars...

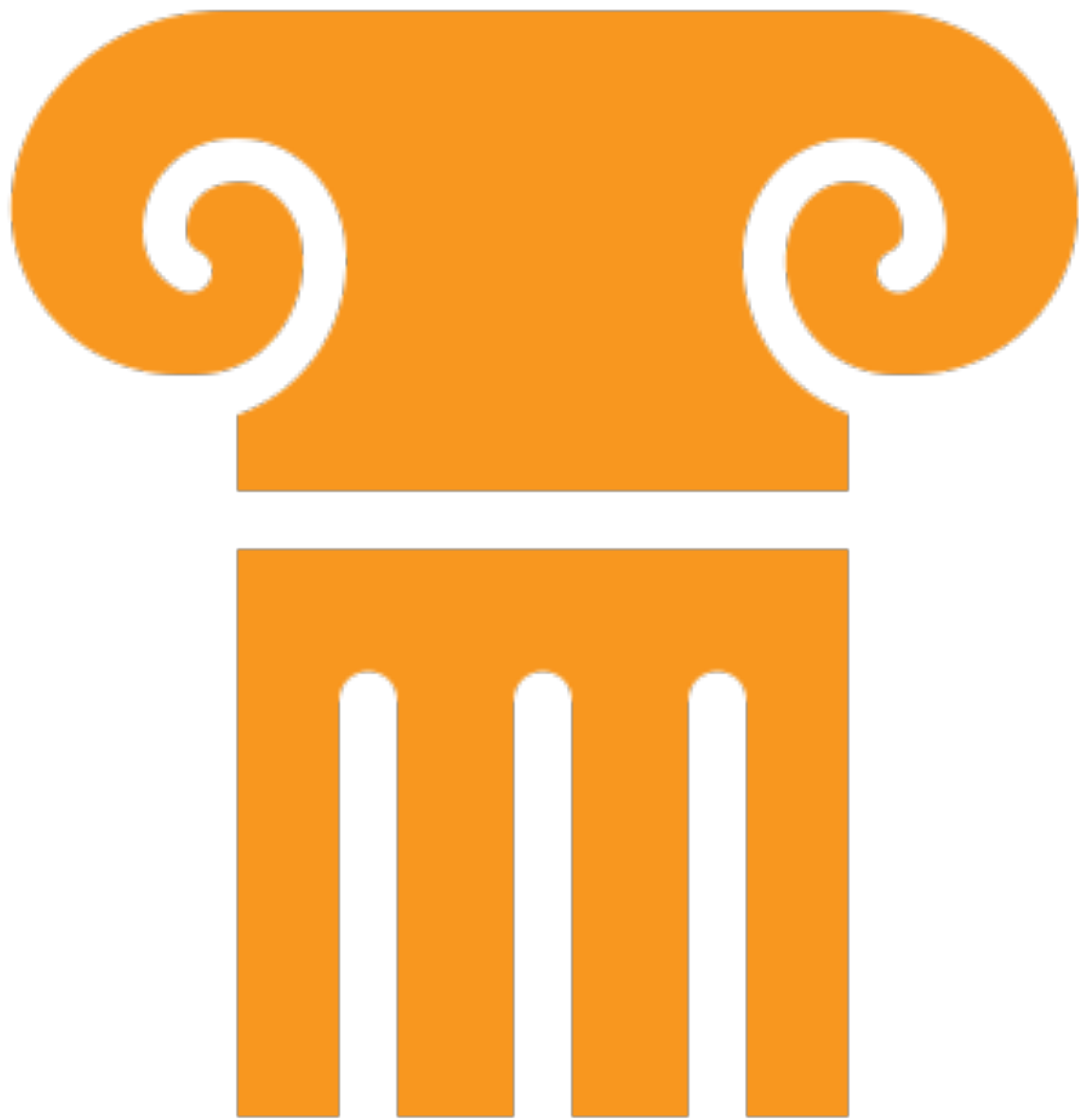
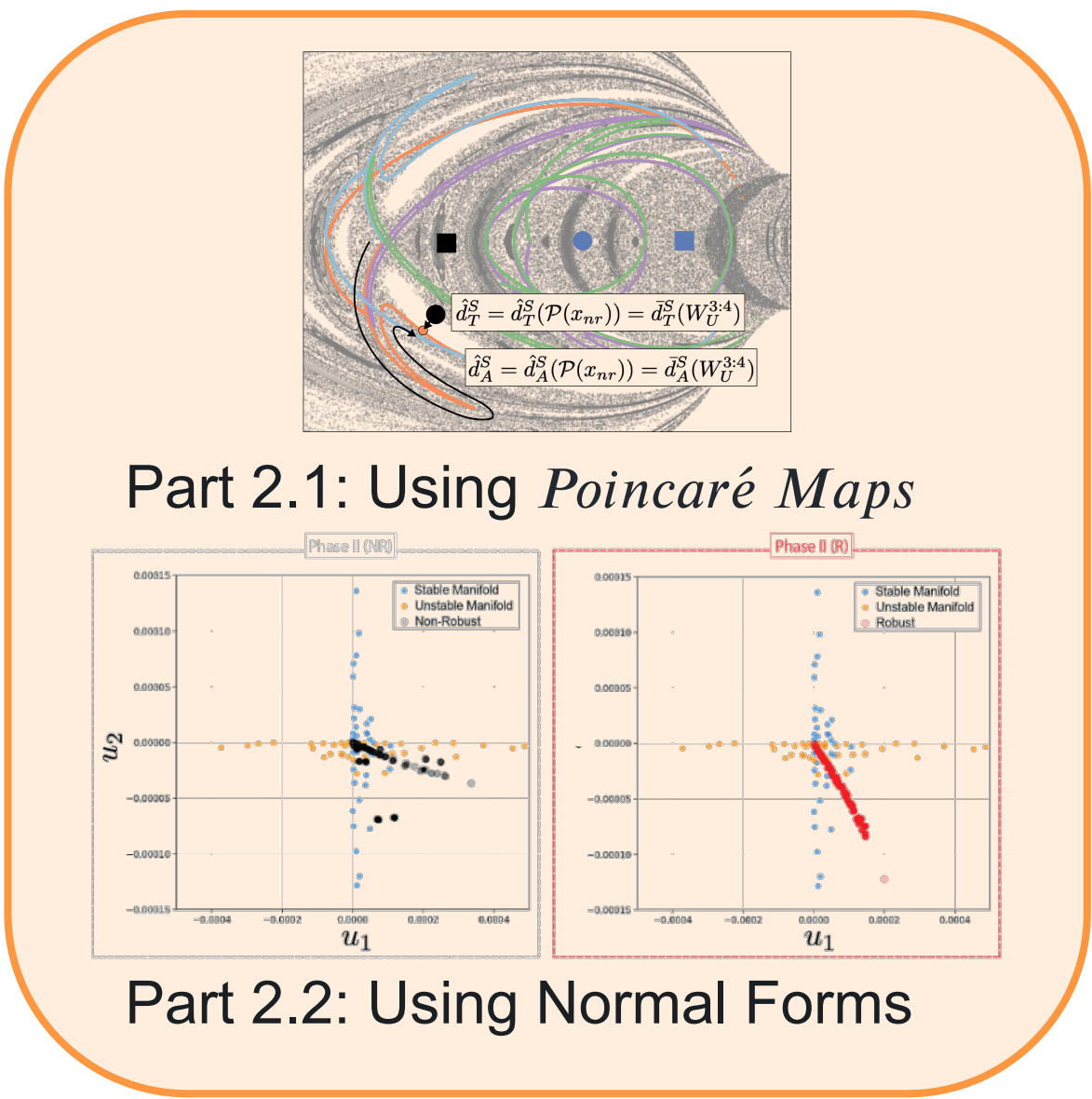
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Scalable Computational Methods



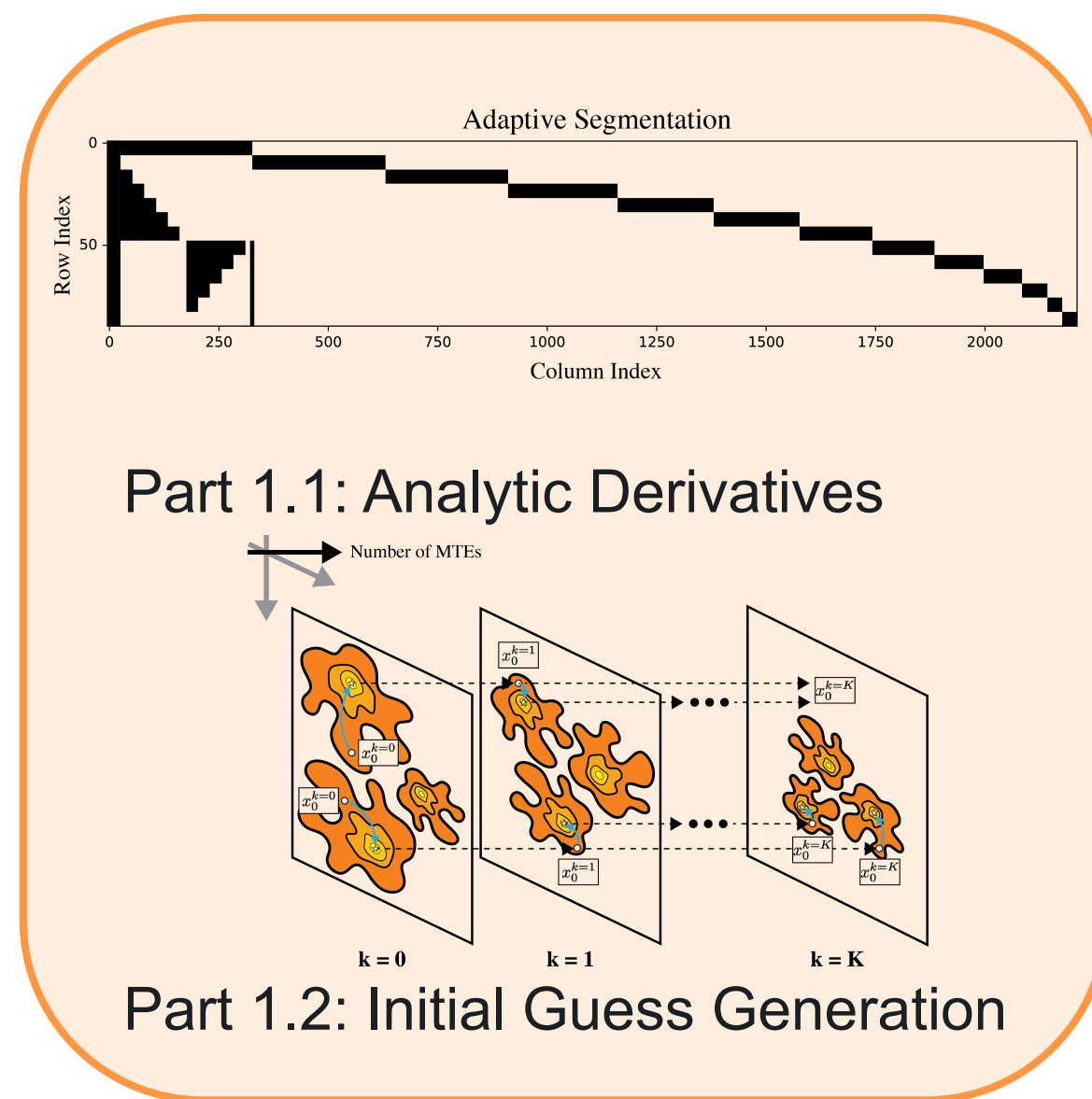
Part 2:

Dynamical Systems Analysis

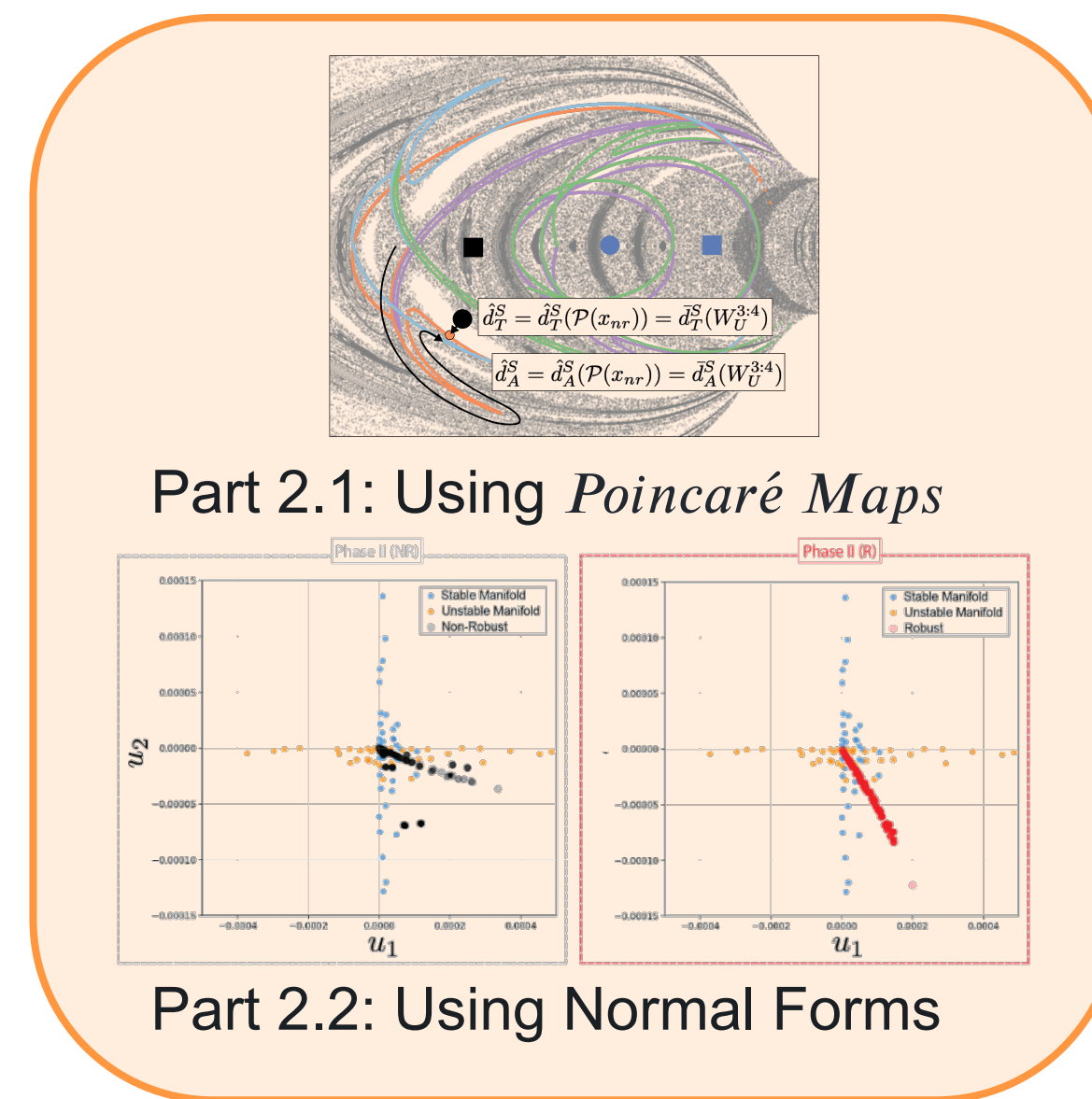


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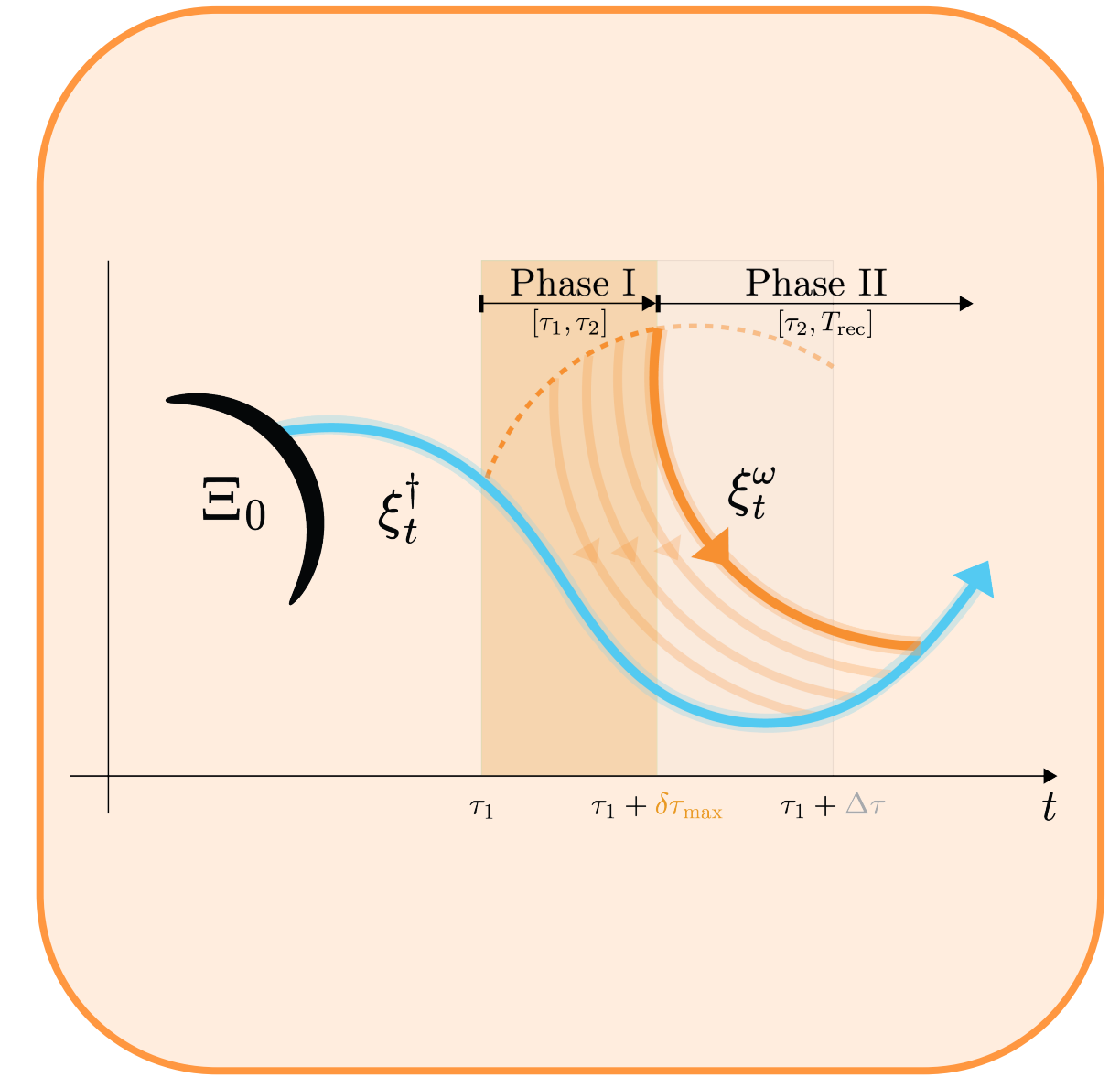
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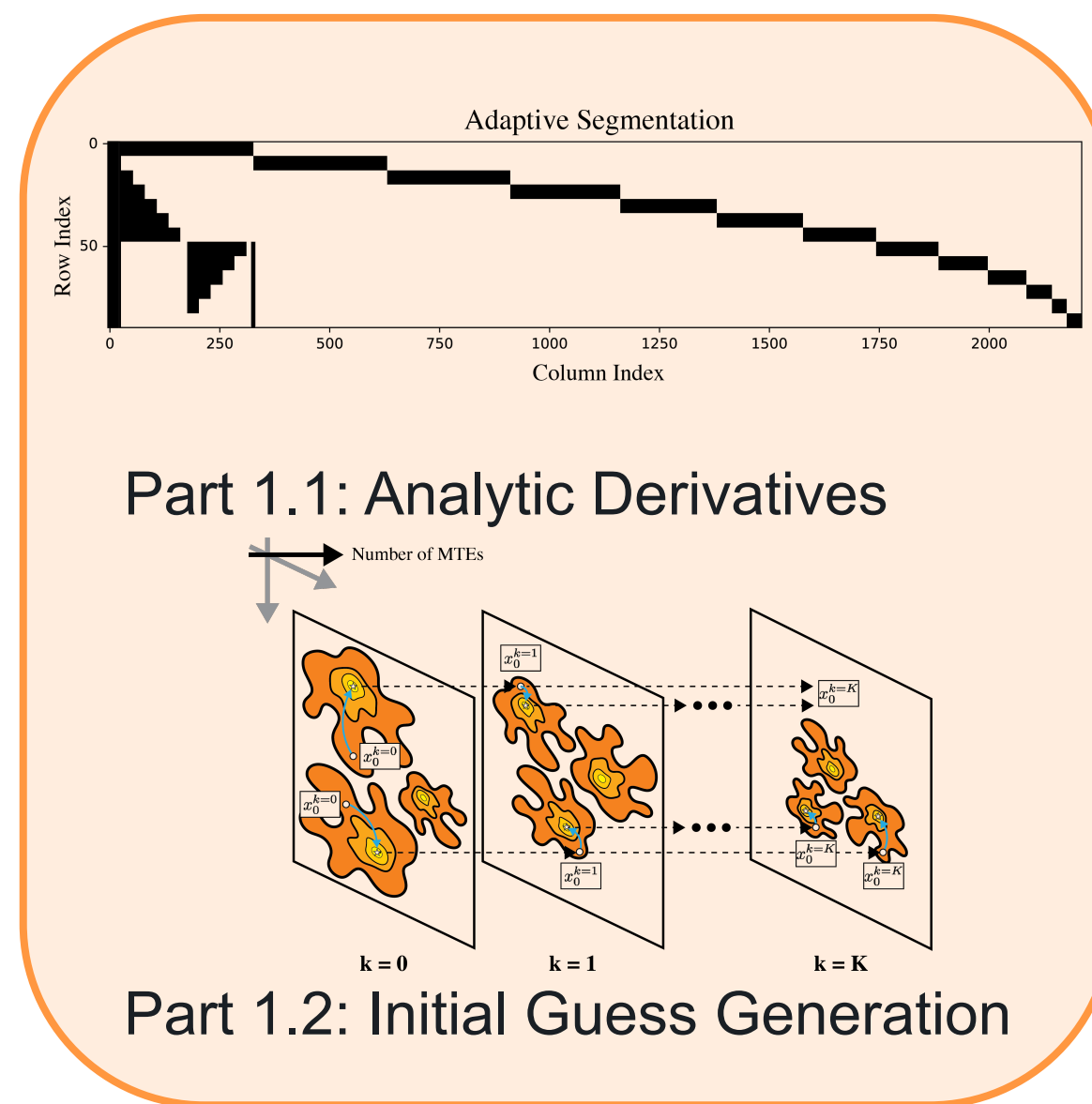


Part 3: Bilevel Optimal Control Framework

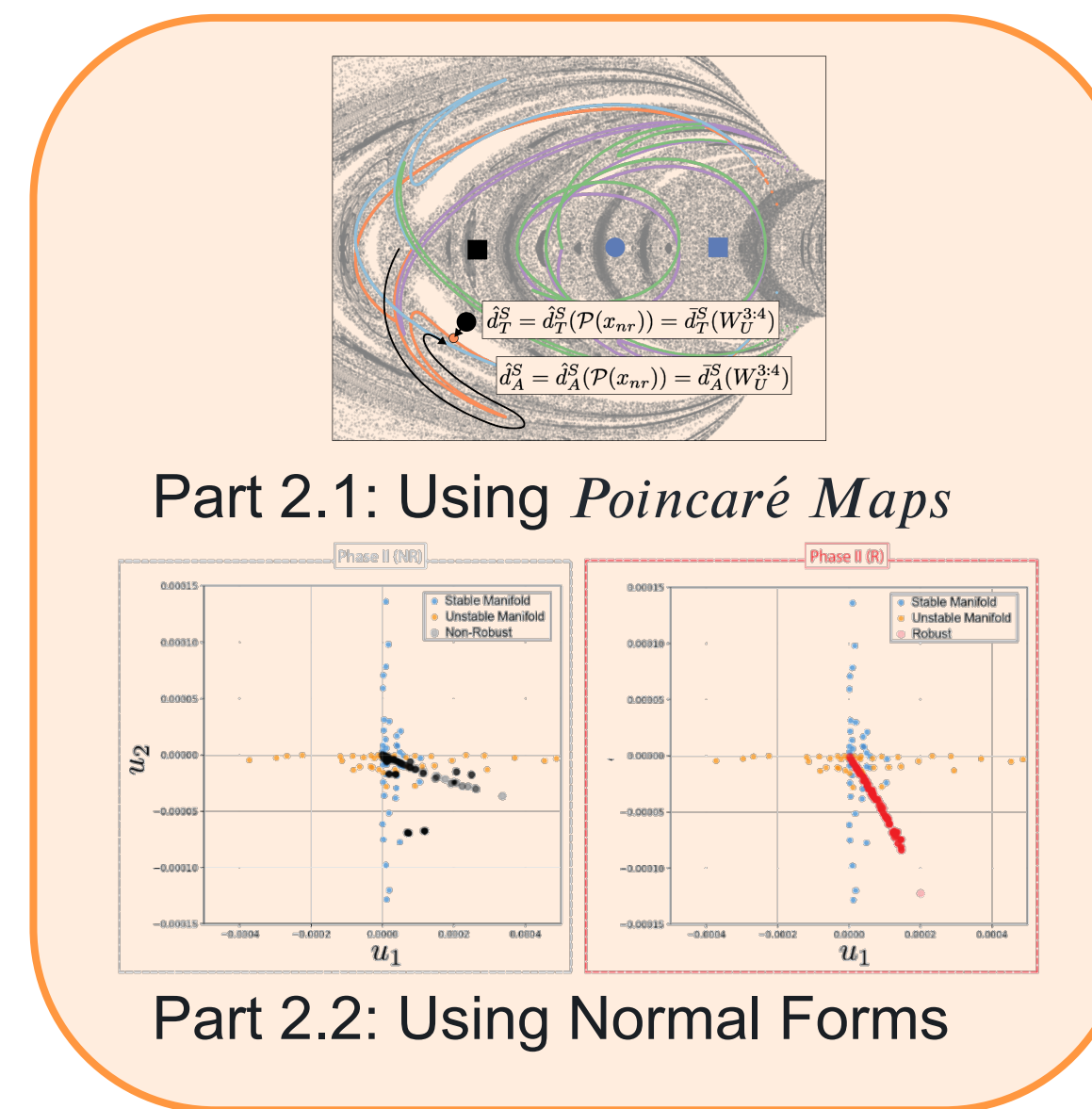


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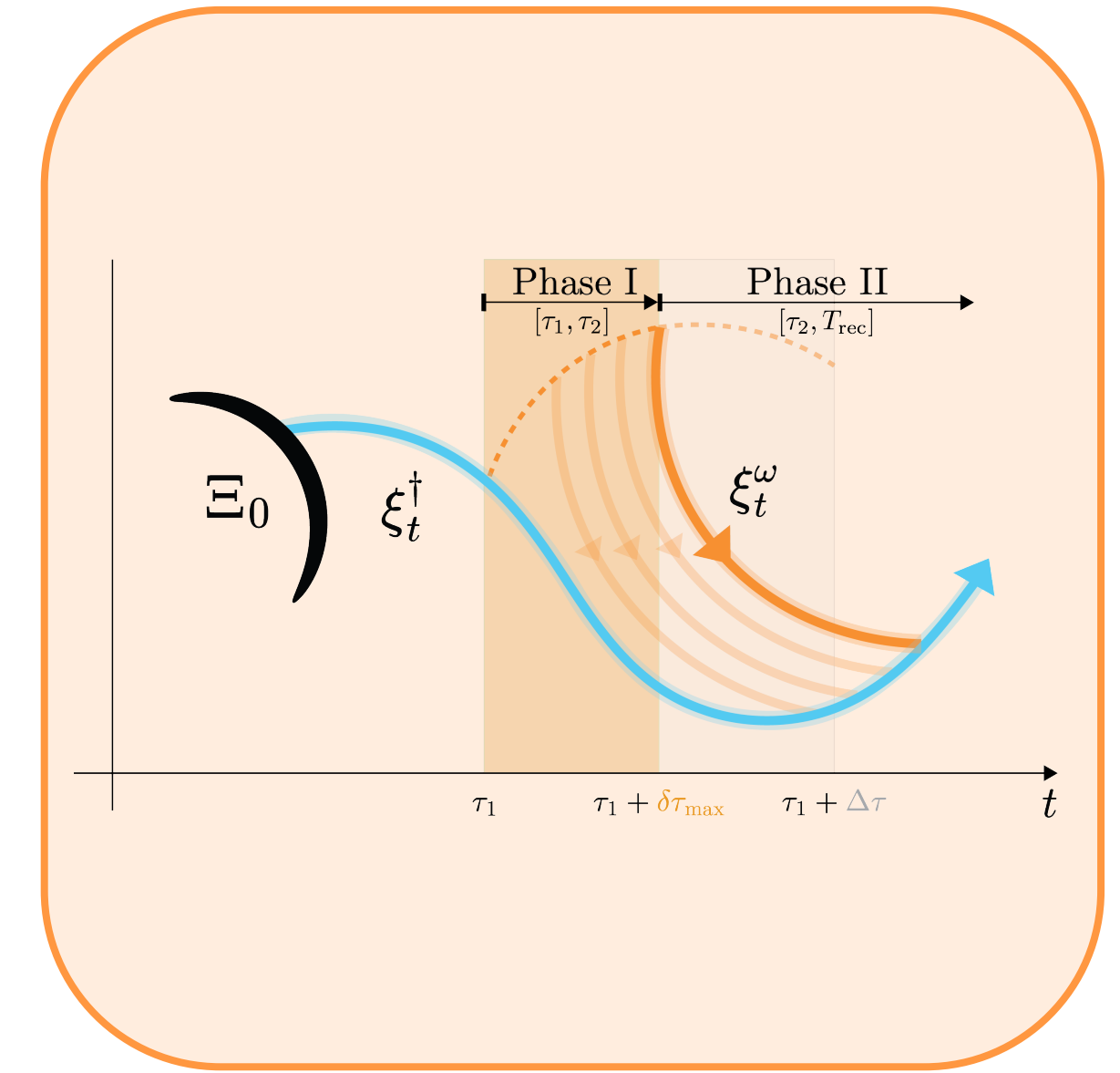
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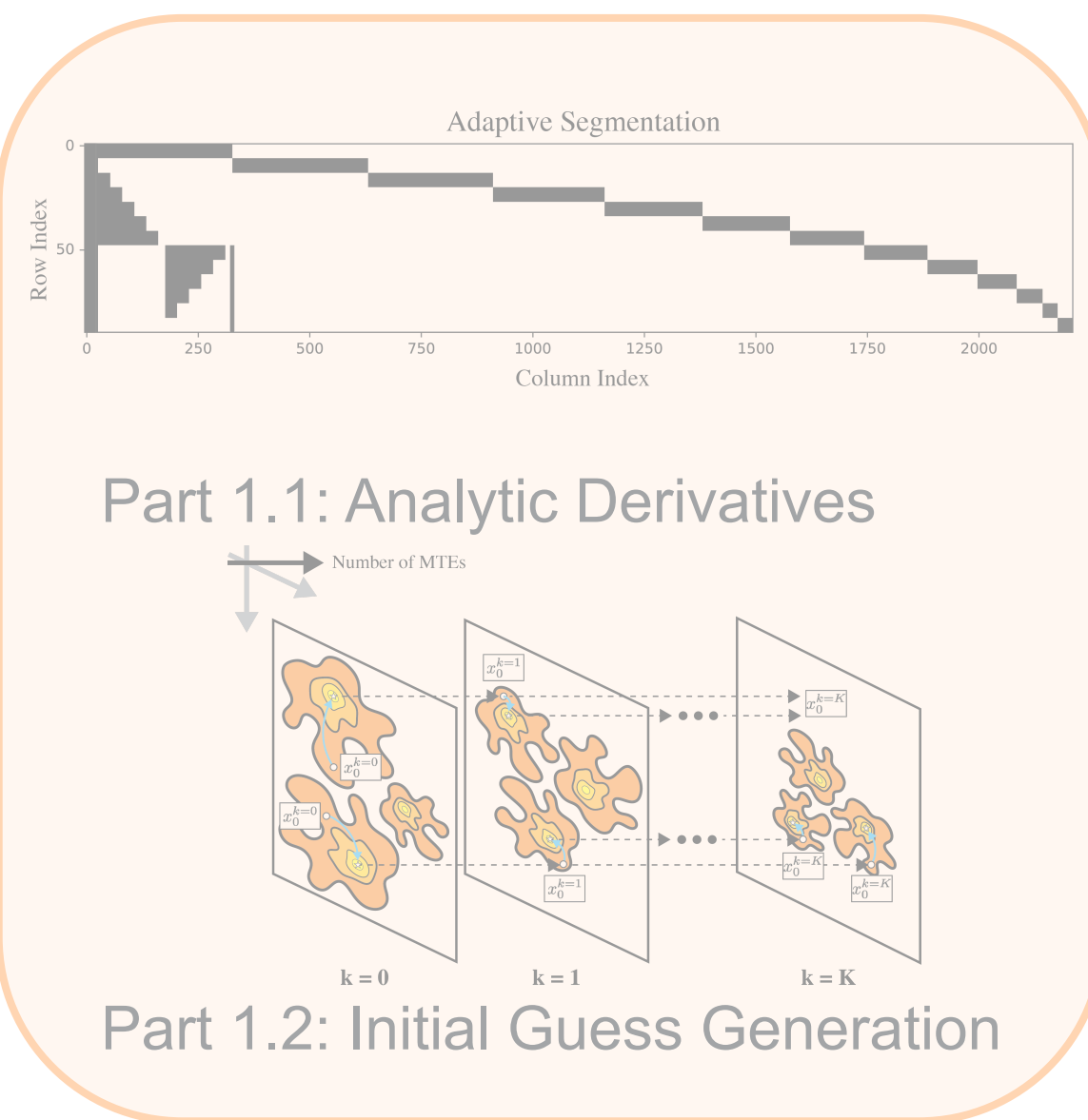


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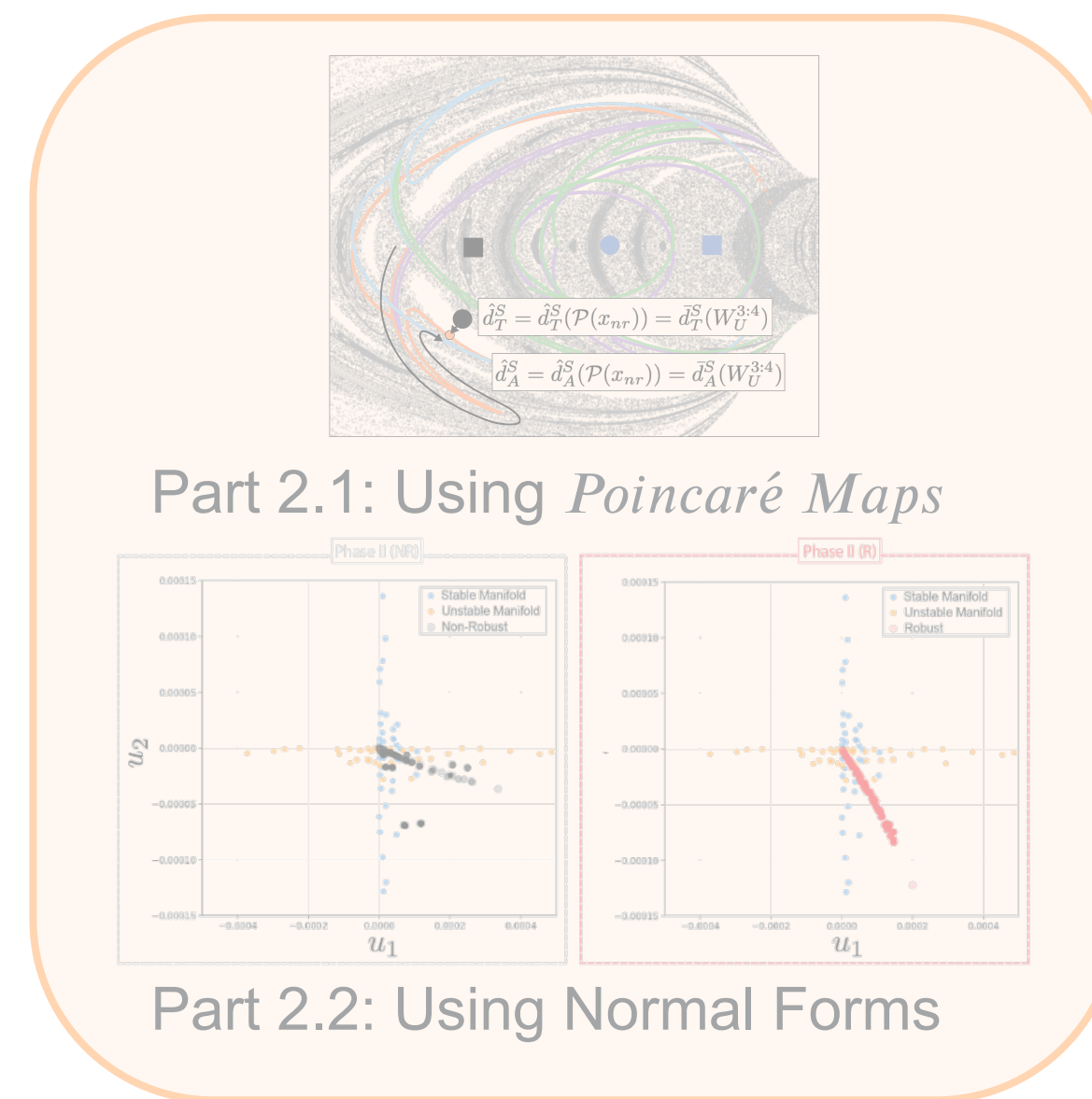


Part 0: Missed Thrust Design Problem Formulation

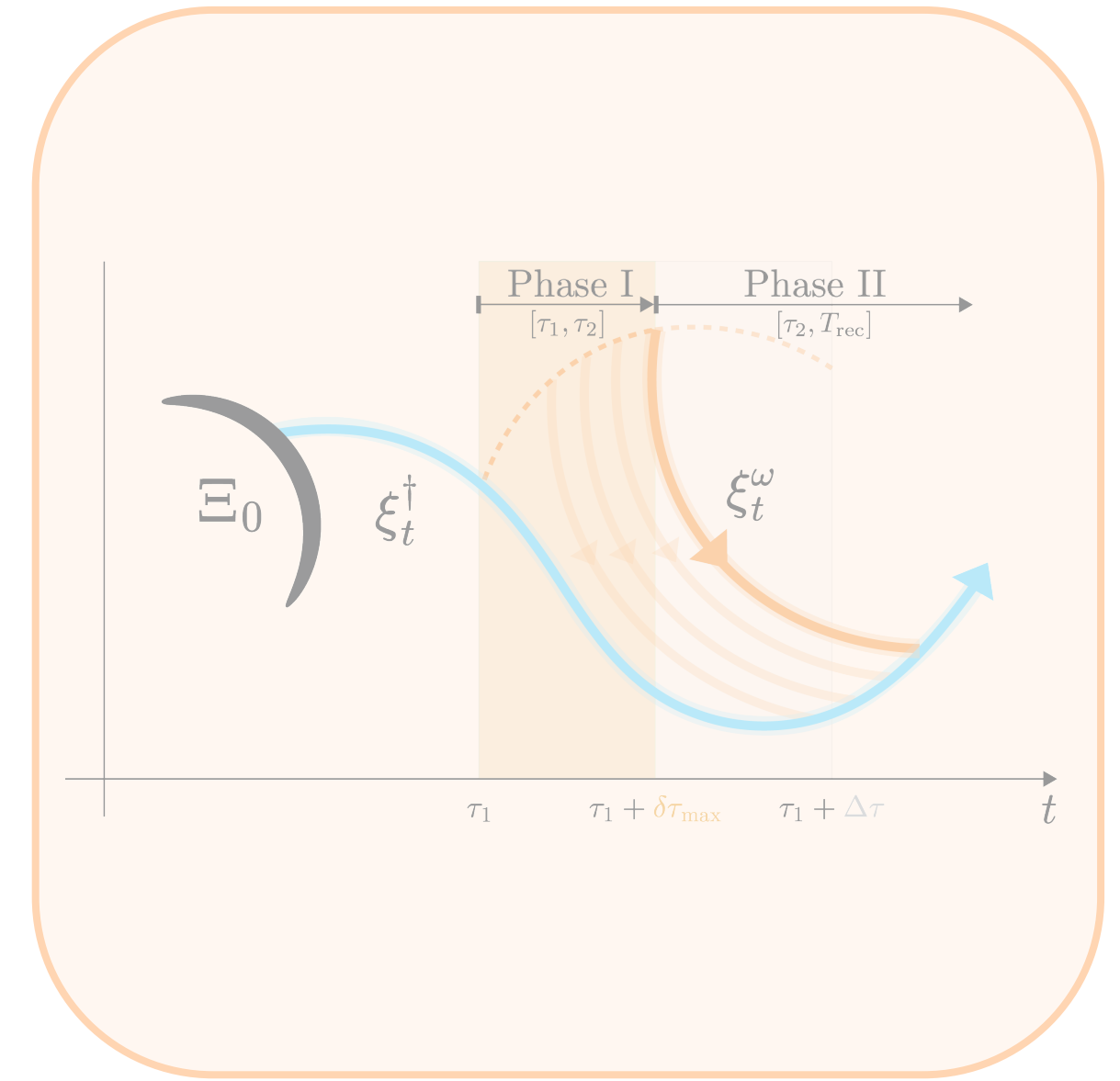
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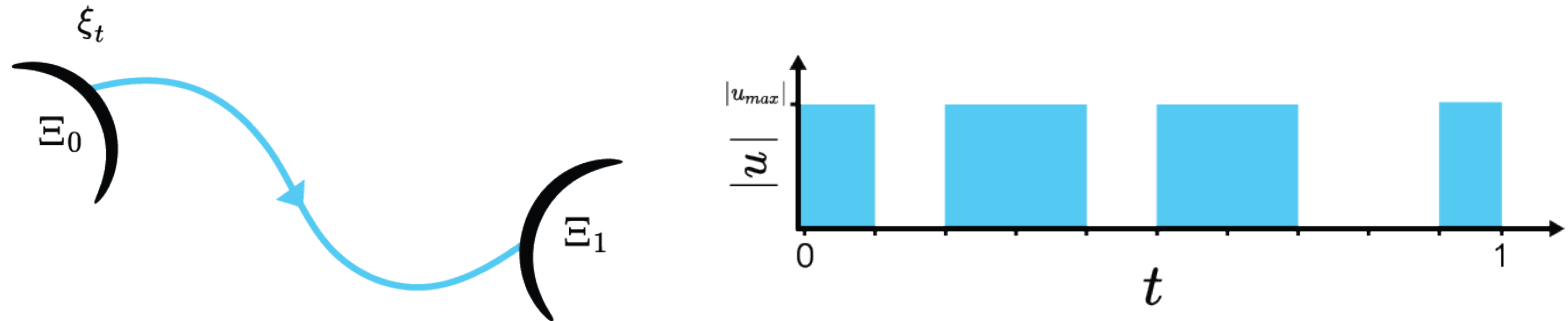


Part 0: Missed Thrust Design Problem Formulation



Part 0: Problem Formulation

Designing trajectories involves solving an infinite dimensional Optimal Control Problem (OCP)



Optimal Control Problem

$$\min_{u \in \mathcal{U}} J(u) := \phi(\xi_1) + \int_0^1 \mathcal{L}(s, \xi_s, u_s) ds$$

$J(u)$ may represent the propellant mass, flight time or a combination thereof

$$\text{s.t. } \xi(t, u) = \xi_0 + \int_0^t f(s, \xi_s) ds + \int_0^t g(s, \xi_s, u_s) ds \quad \forall t \in [0, 1],$$

f represents the natural dynamics, while g represents the forcing dynamics

$$\xi_0^\dagger \in \Xi_0, \quad \xi_1^\dagger \in \Xi_1$$

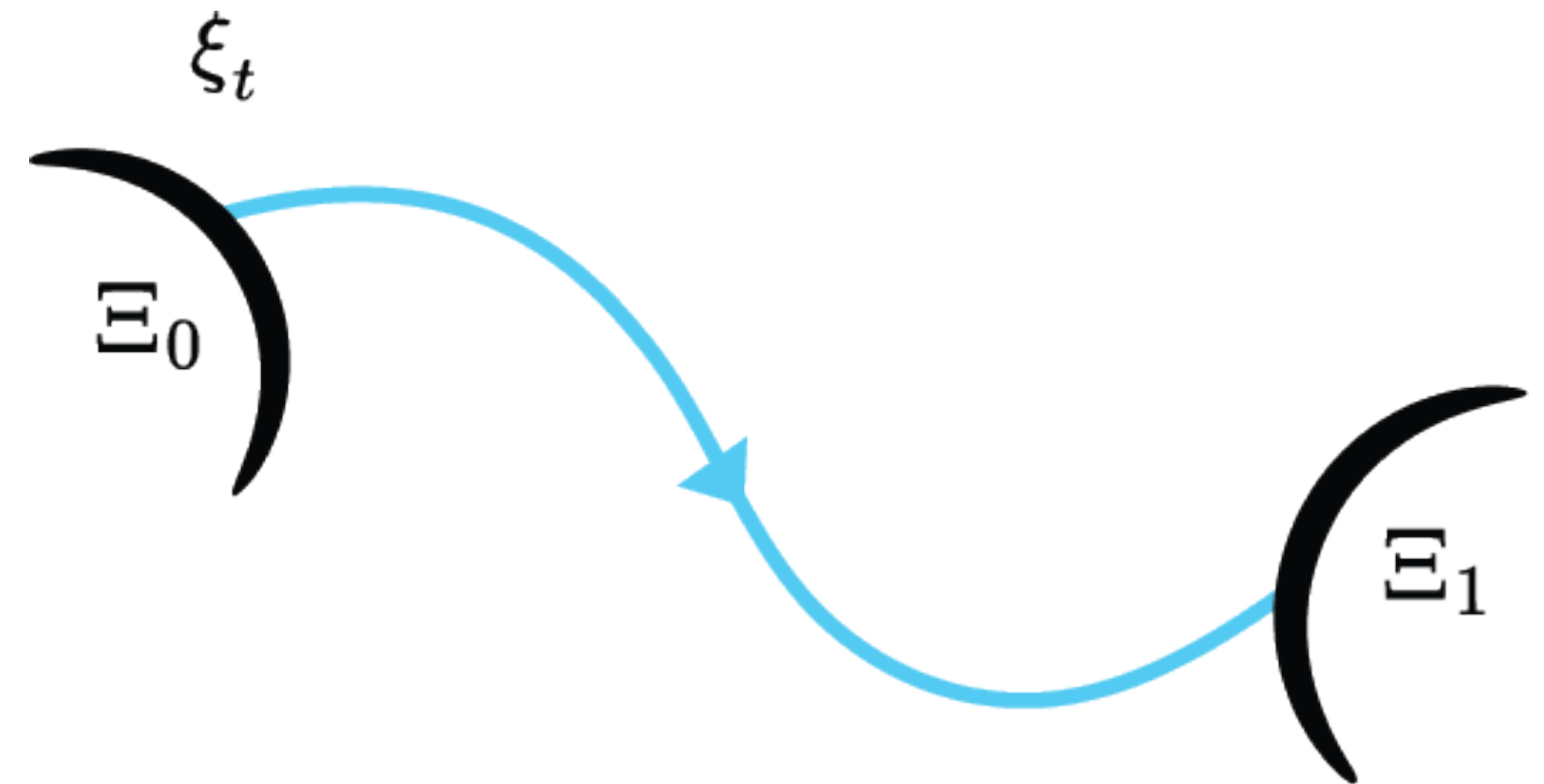
Initial and terminal boundary

$\mathcal{P} :=$

To numerically solve the OCP, we need to convert it to a finite dimensional Nonlinear Program (NLP)

Optimal Control Problem

$$\begin{aligned}
 \min_{\mathbf{u} \in \mathcal{U}} \quad & J(\mathbf{u}) := \phi(\boldsymbol{\xi}_1) + \int_0^1 \mathcal{L}(s, \boldsymbol{\xi}_s, \mathbf{u}_s) ds \\
 \text{s.t.} \quad & \boldsymbol{\xi}(t, \mathbf{u}) = \boldsymbol{\xi}_0 + \int_0^t \mathbf{f}(s, \boldsymbol{\xi}_s) ds + \int_0^t \mathbf{g}(s, \boldsymbol{\xi}_s, \mathbf{u}_s) ds \quad \forall t \in [0, 1], \\
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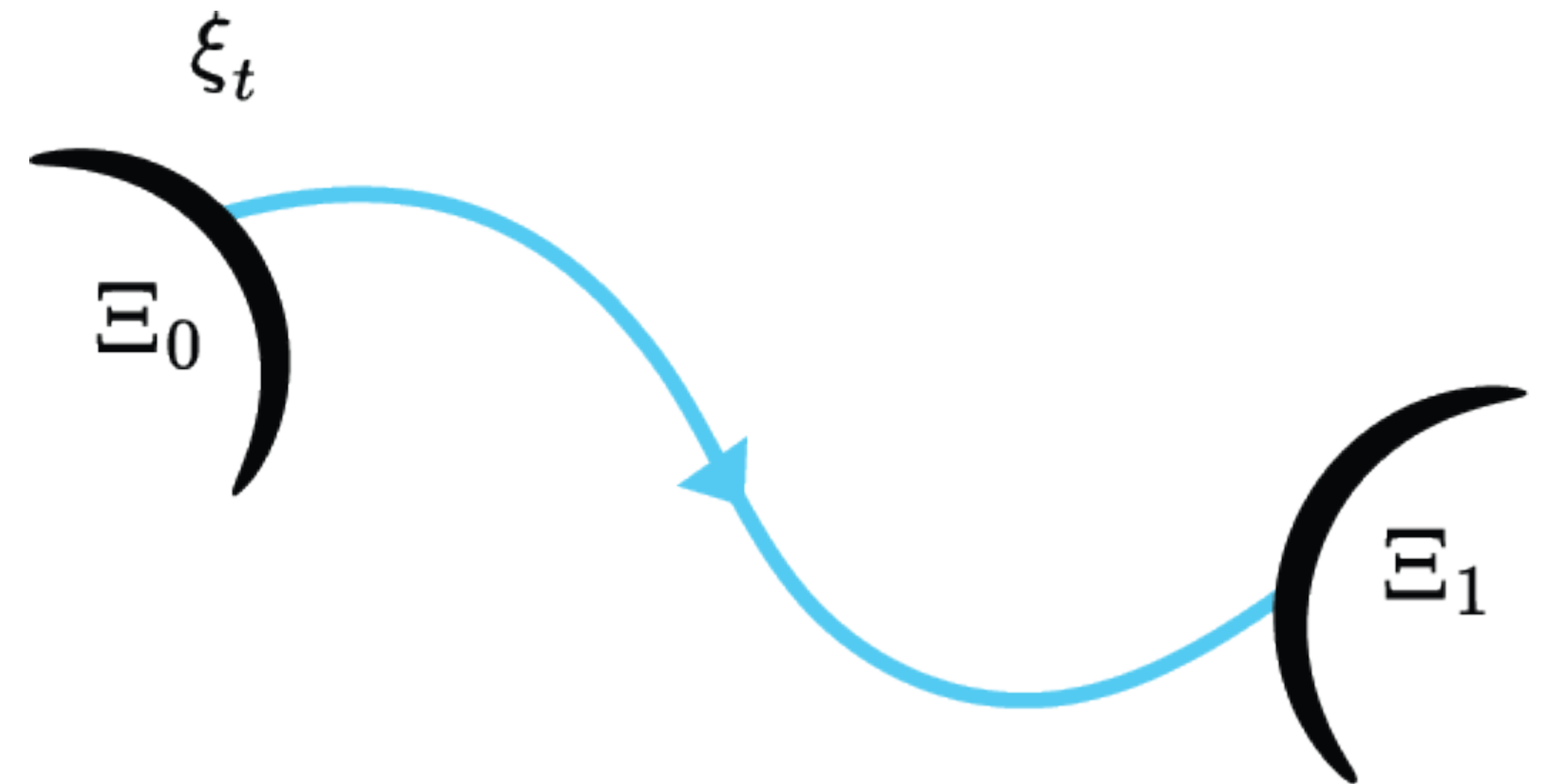
Transcription

Nonlinear Program

$$\begin{aligned} \min_{x \in \mathbb{R}^m} \quad & J(x) \\ \text{s.t.} \quad & \mathbf{c}_e(x) = \mathbf{0} \\ & \mathbf{c}_i(x) \leq \mathbf{0} \end{aligned}$$

Can be solved using standard NLP solvers e.g., SNOPT

Gill et al. (2005)



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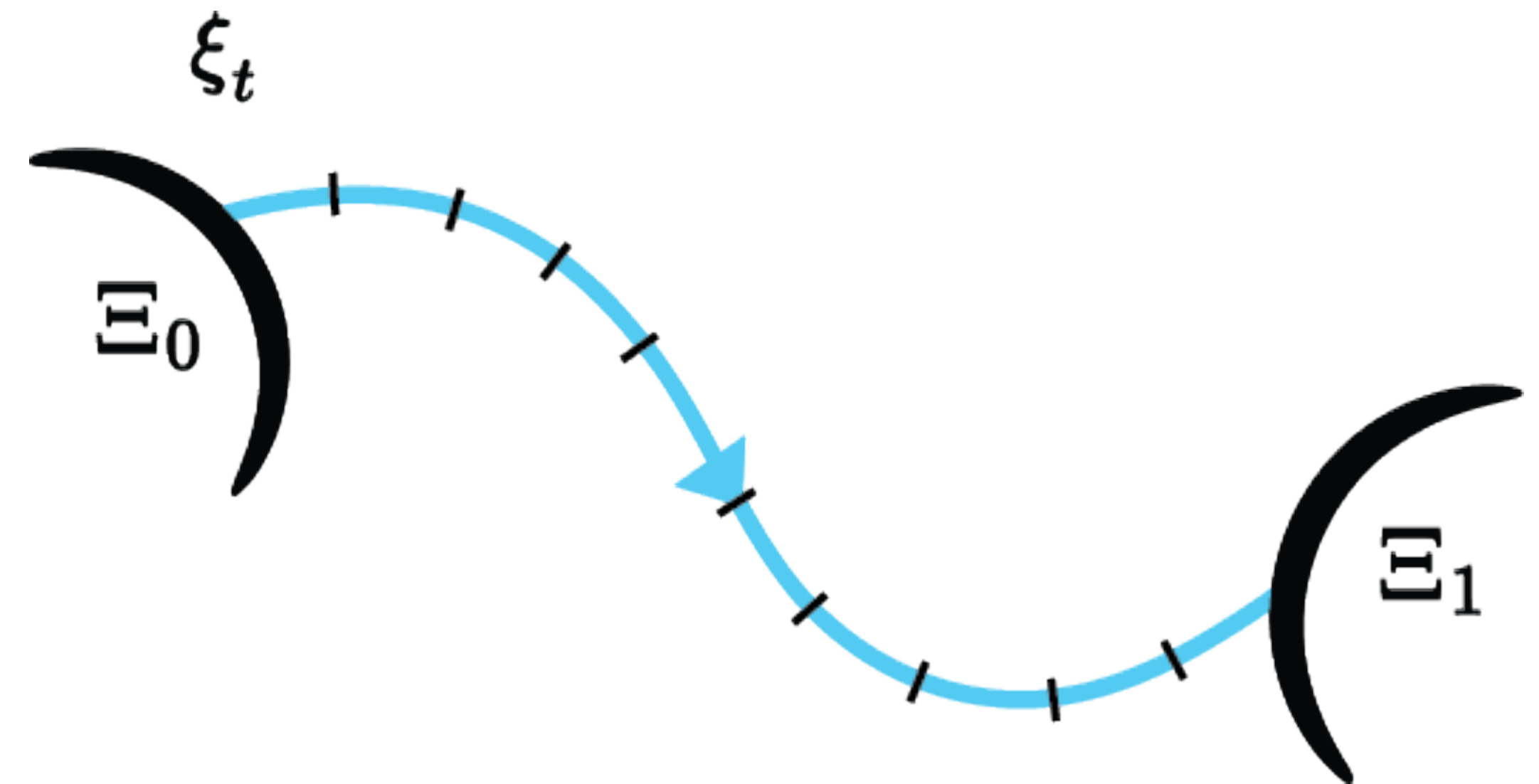
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Finite-Burn Low-Thrust Transcription



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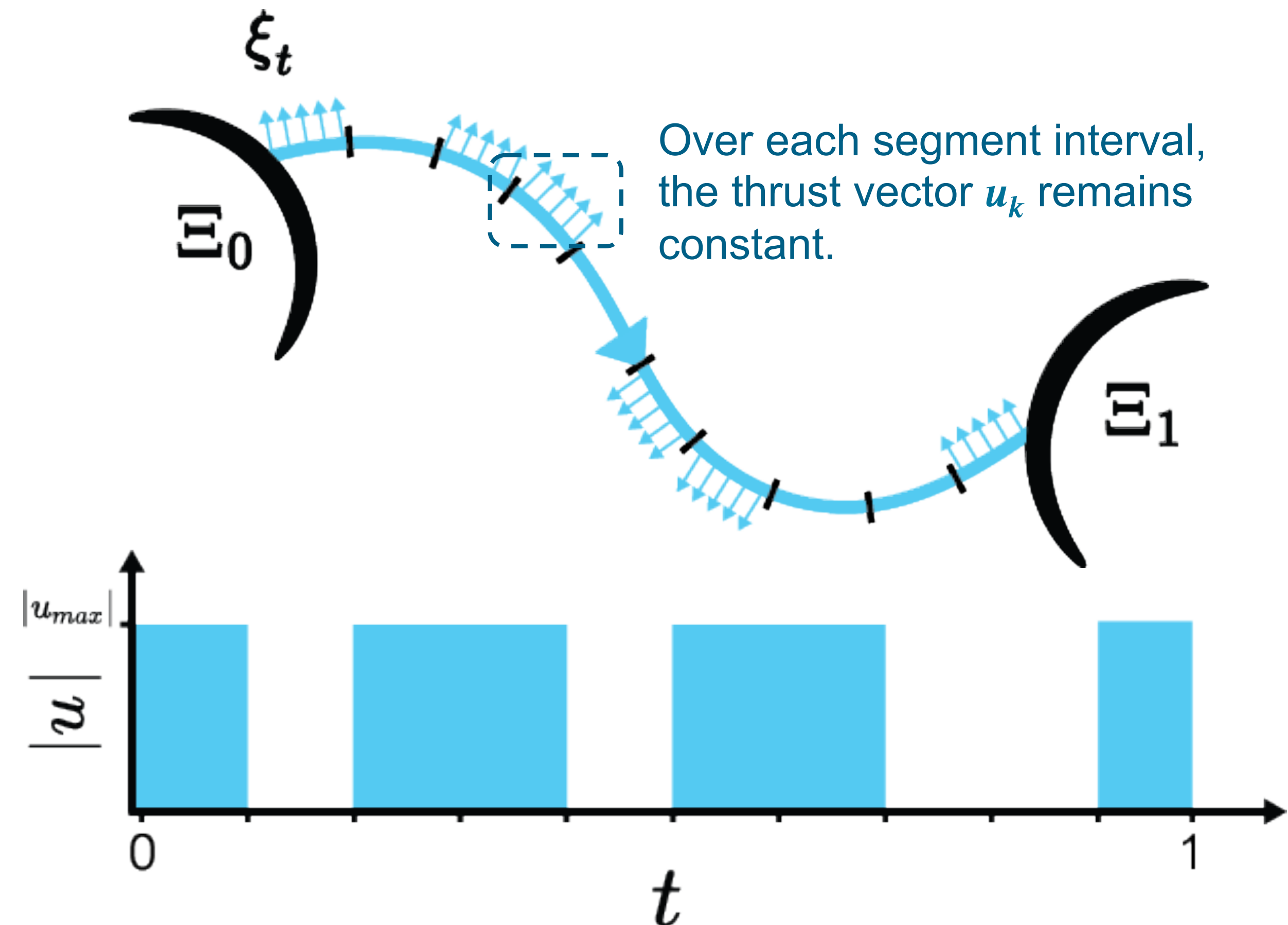
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Finite-Burn Low-Thrust Transcription



Shooting methods can be used to embed the dynamical constraints in the NLP

Recall

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$$\mathbf{x} = (\tau_i, \tau_s, \tau_f, \mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_K, m_f)$$

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Nonlinear Program

$$\begin{aligned} & \min_{x \in \mathbb{R}^m} J(x) \\ \text{s.t. } & c_e(x) = 0 \\ & c_i(x) \leq 0 \end{aligned}$$

$$x = (\tau_i, \tau_s, \tau_f, u_1, u_2, \dots, u_K, m_f)$$

Forward Shooting


 $\mathbb{I}_1$
 $\mathbb{I}_0$

Shooting methods can be used to embed the dynamical constraints in the NLP

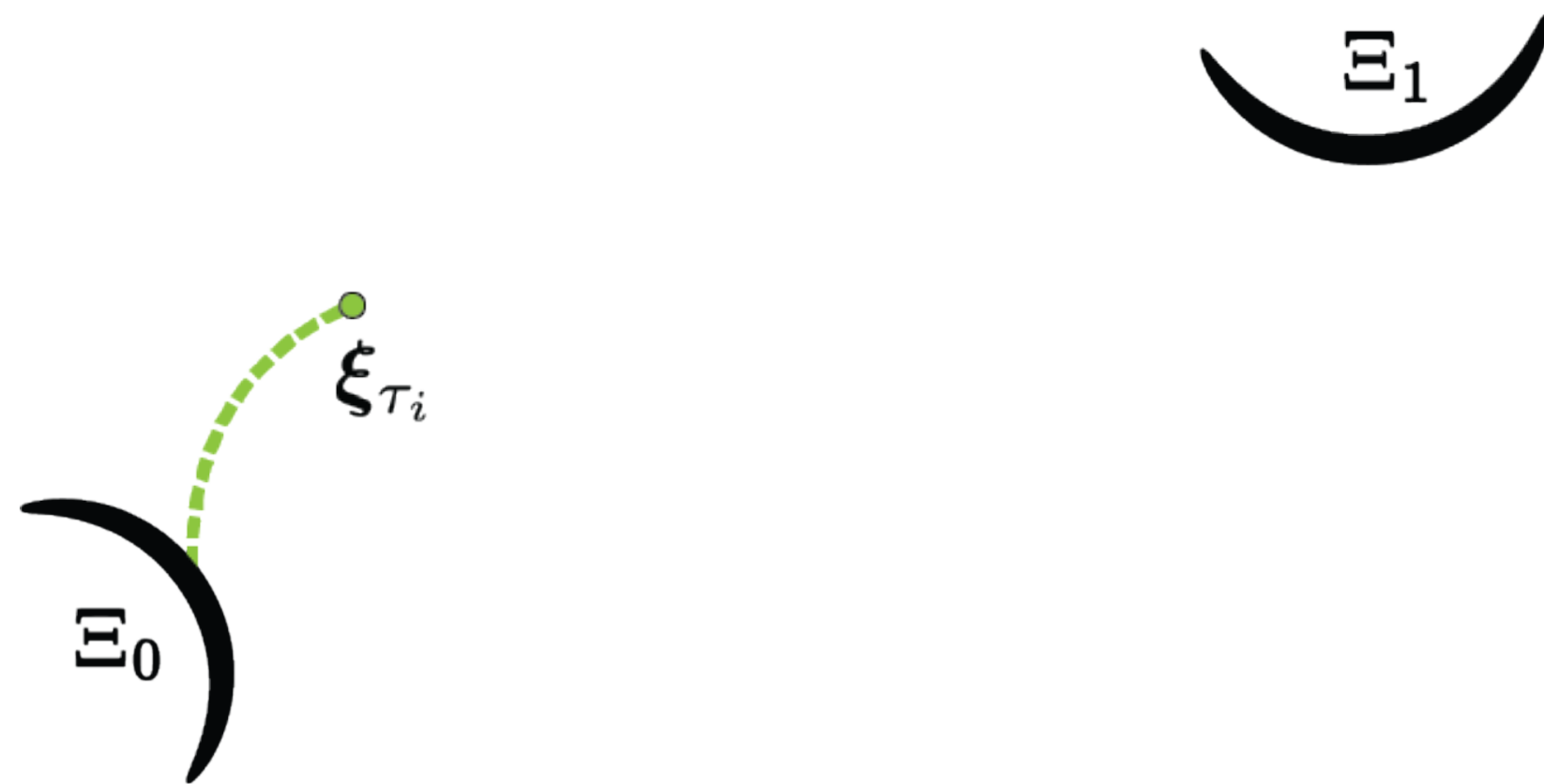
Recall

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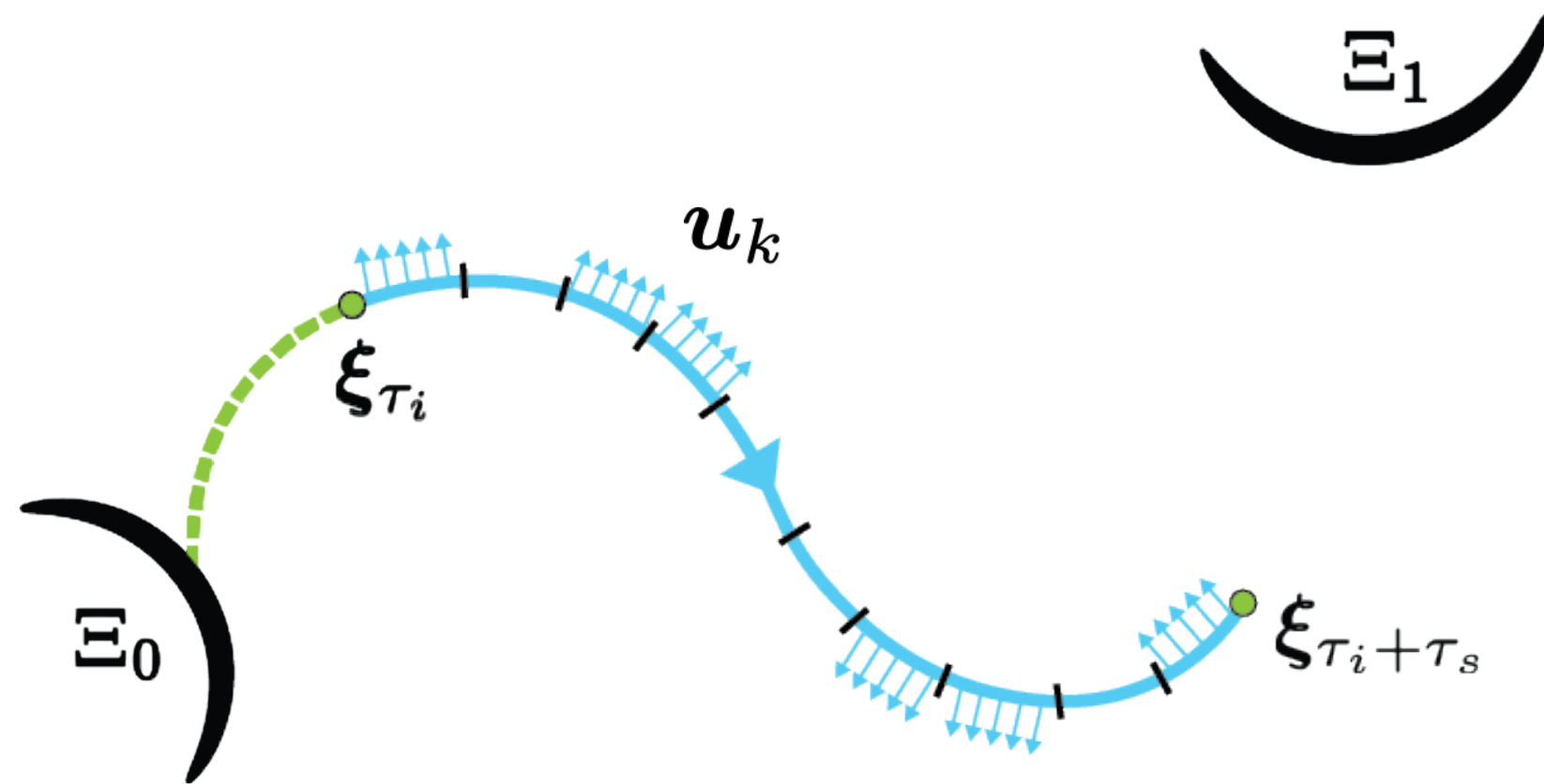
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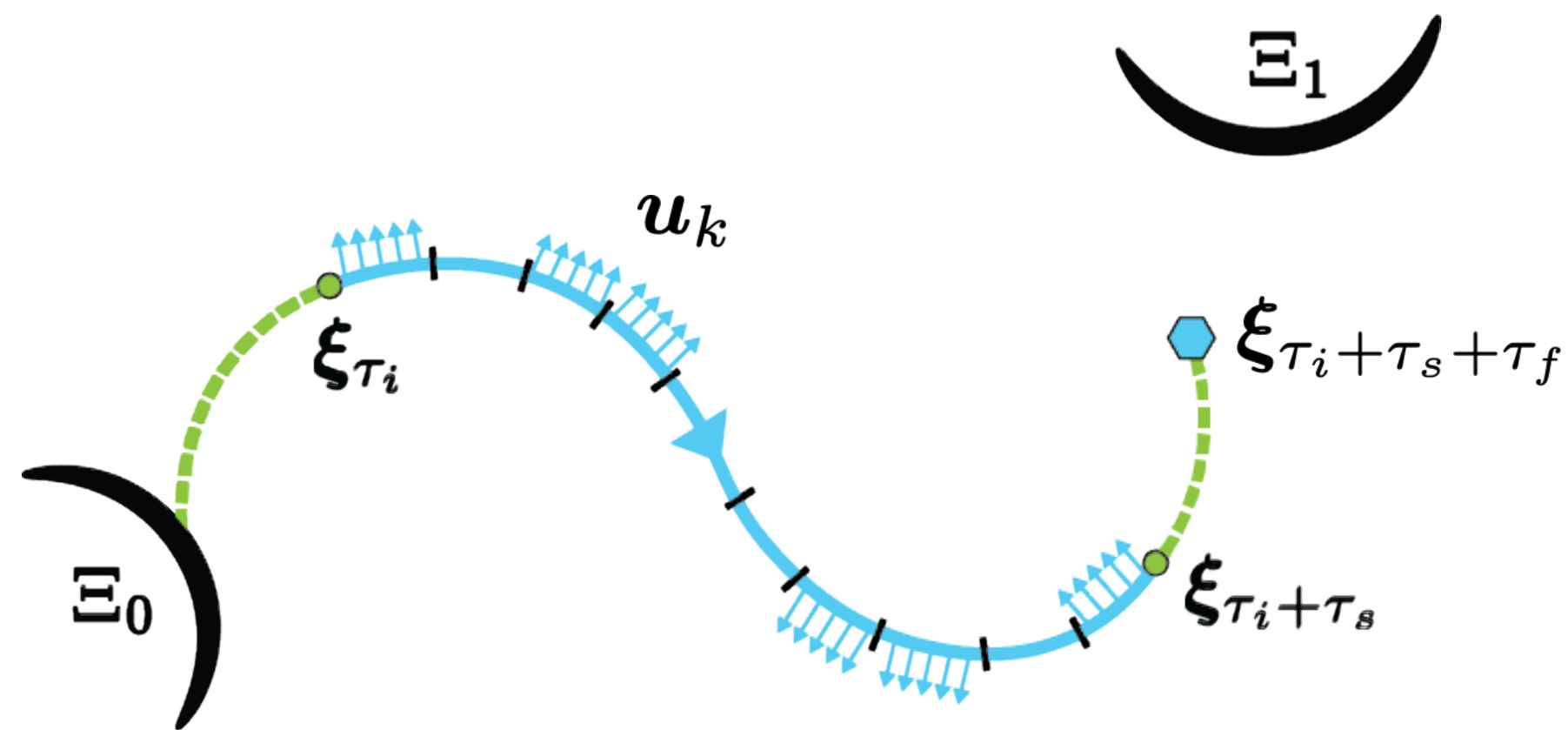
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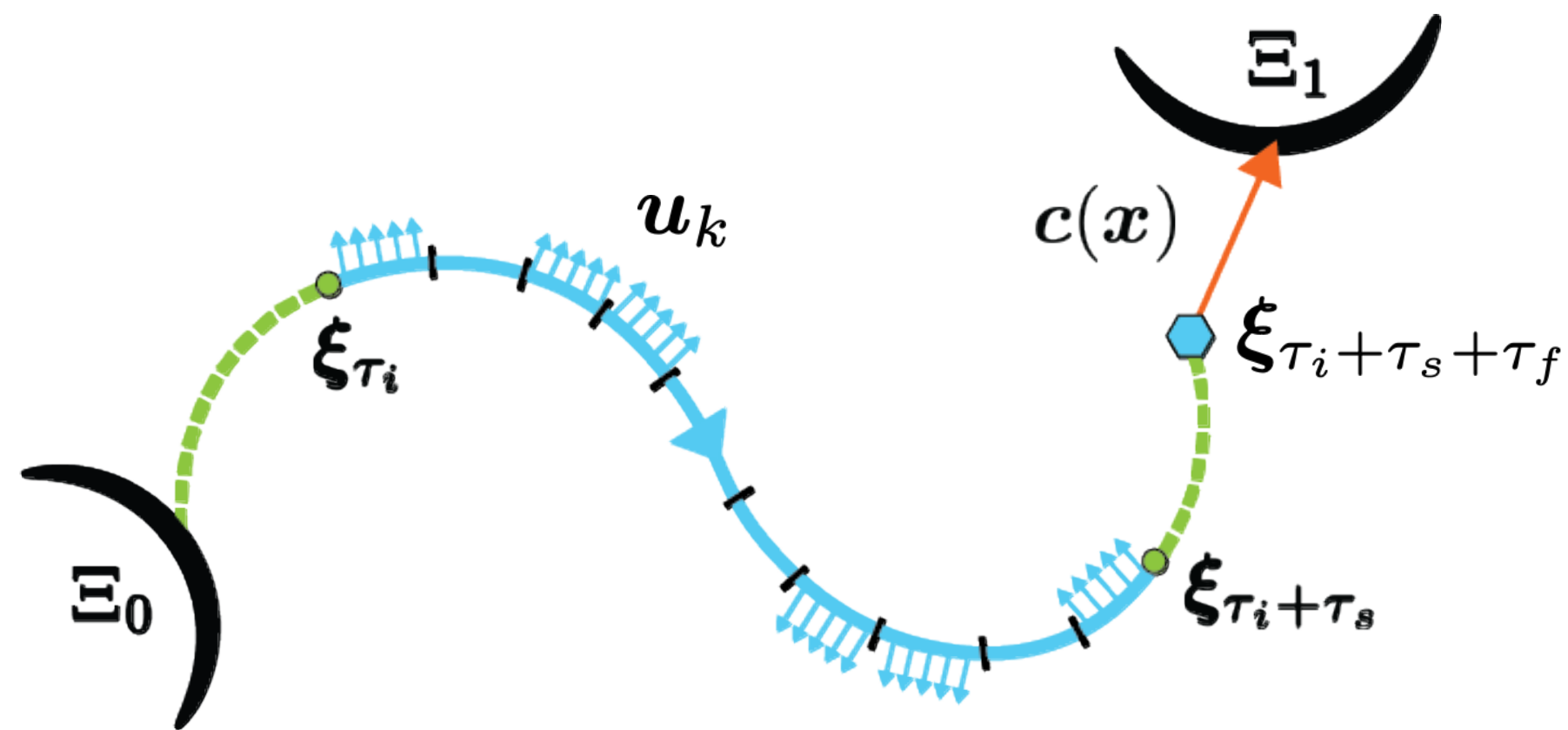
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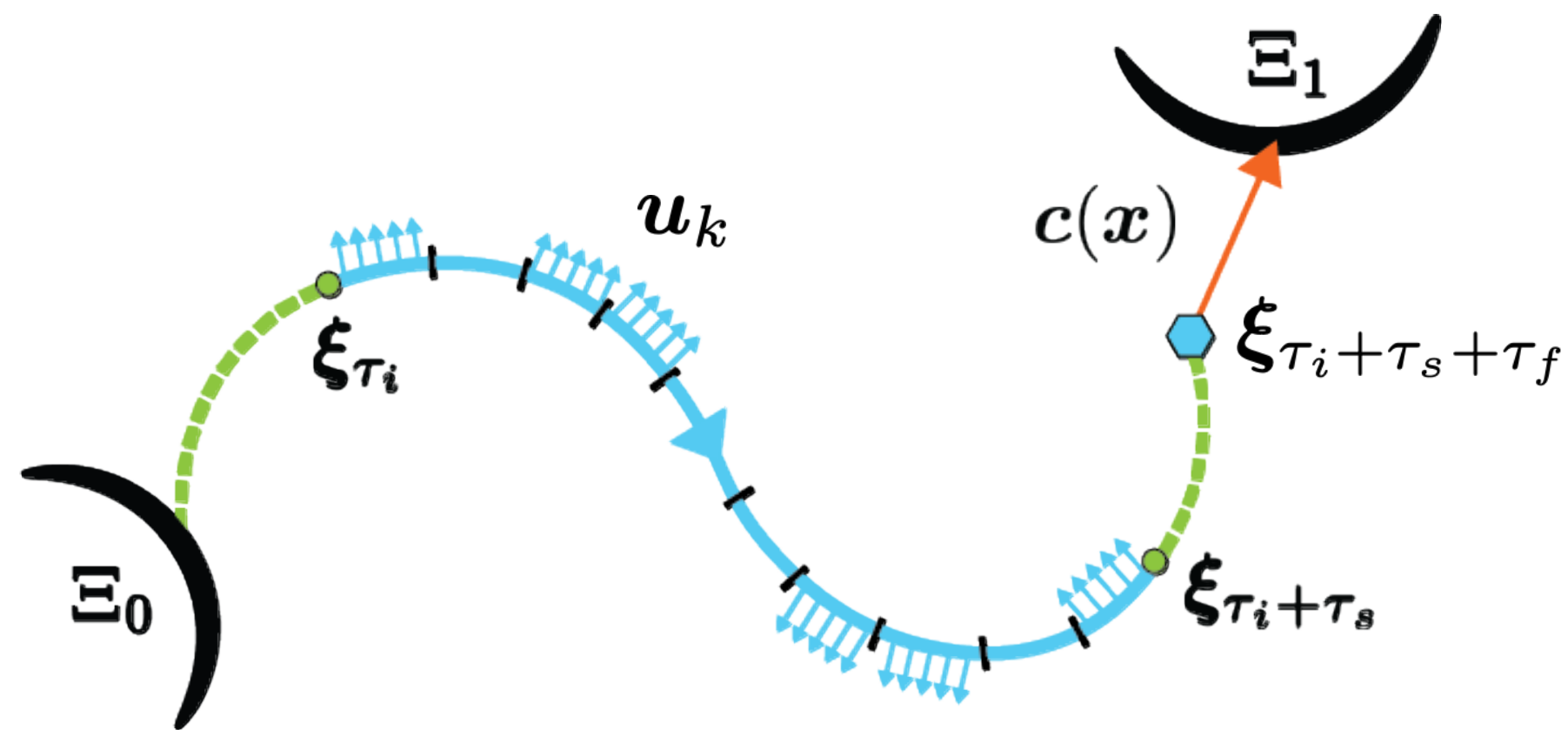
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Nonlinear Program

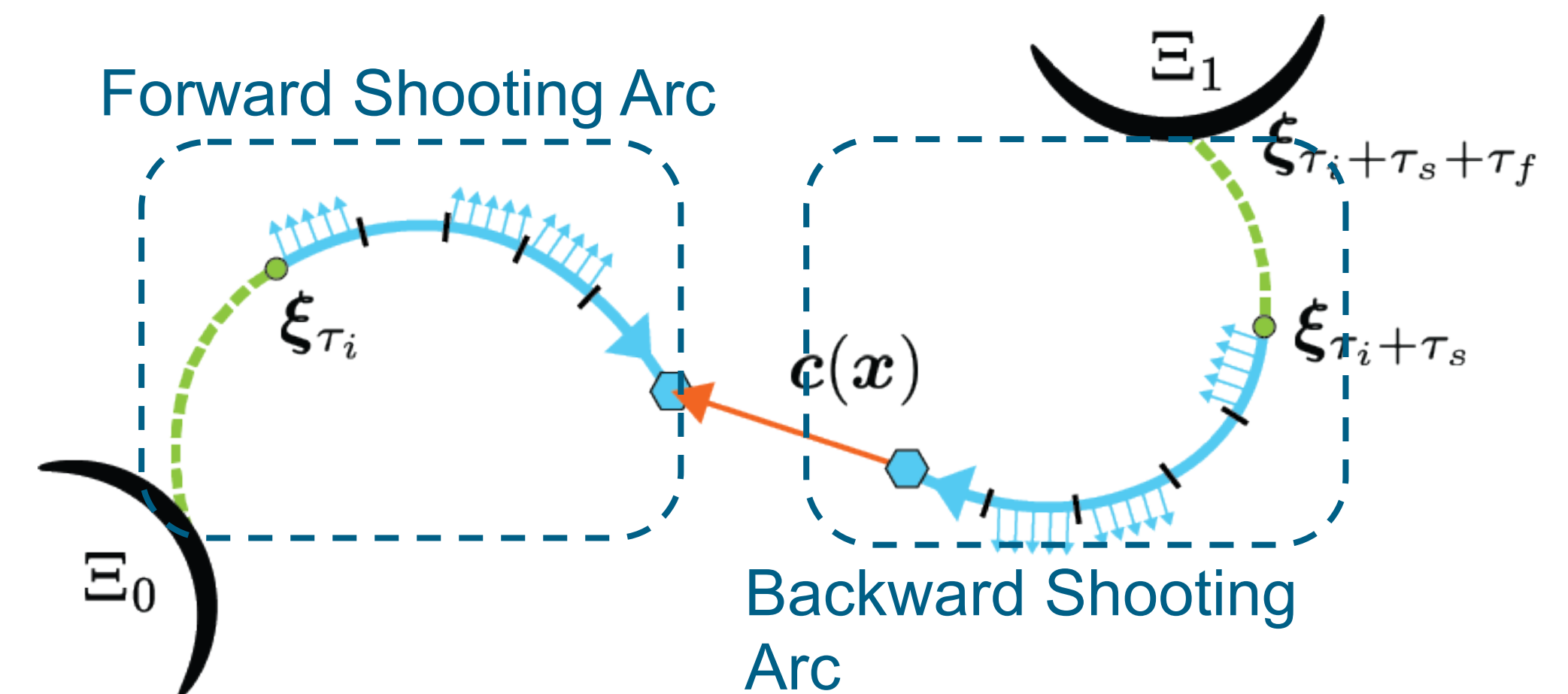
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Forward Shooting

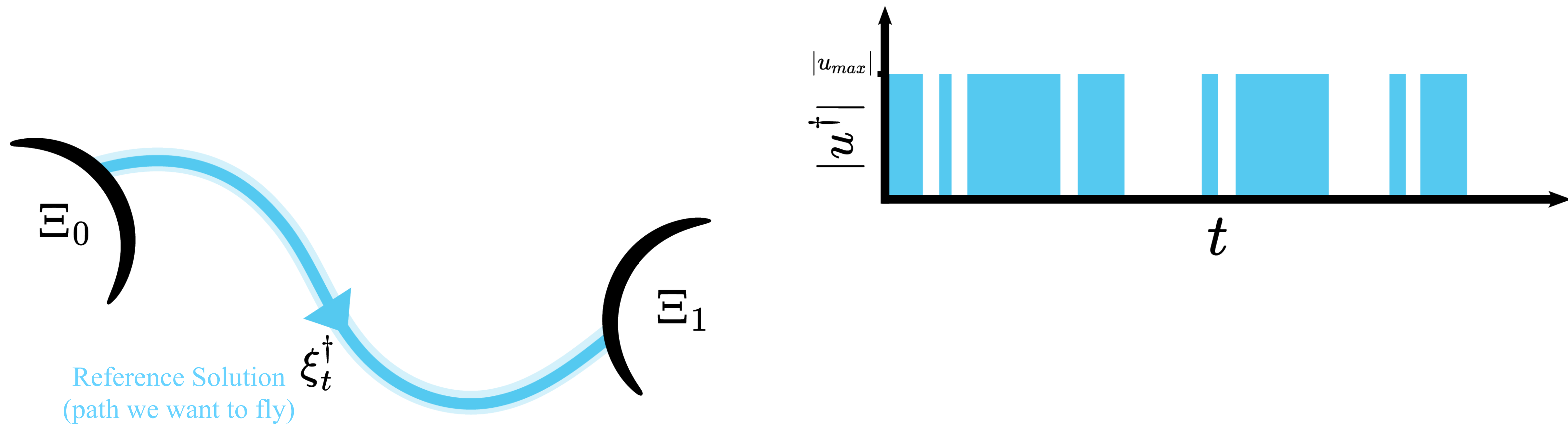


Forward-Backward Shooting

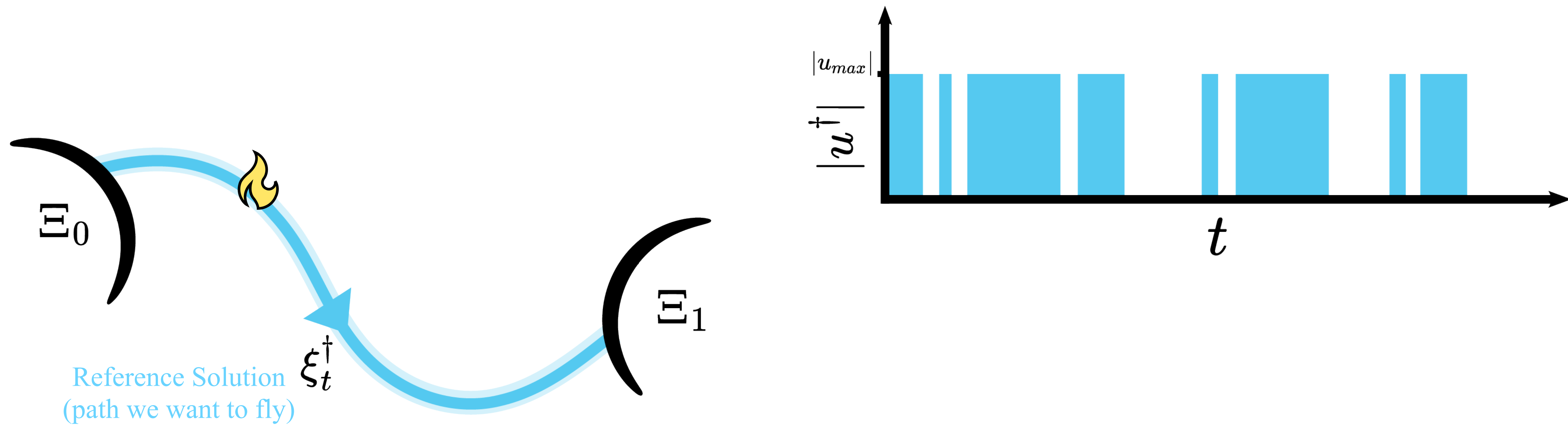


The missed thrust problem can be formulated as a coupled OCP

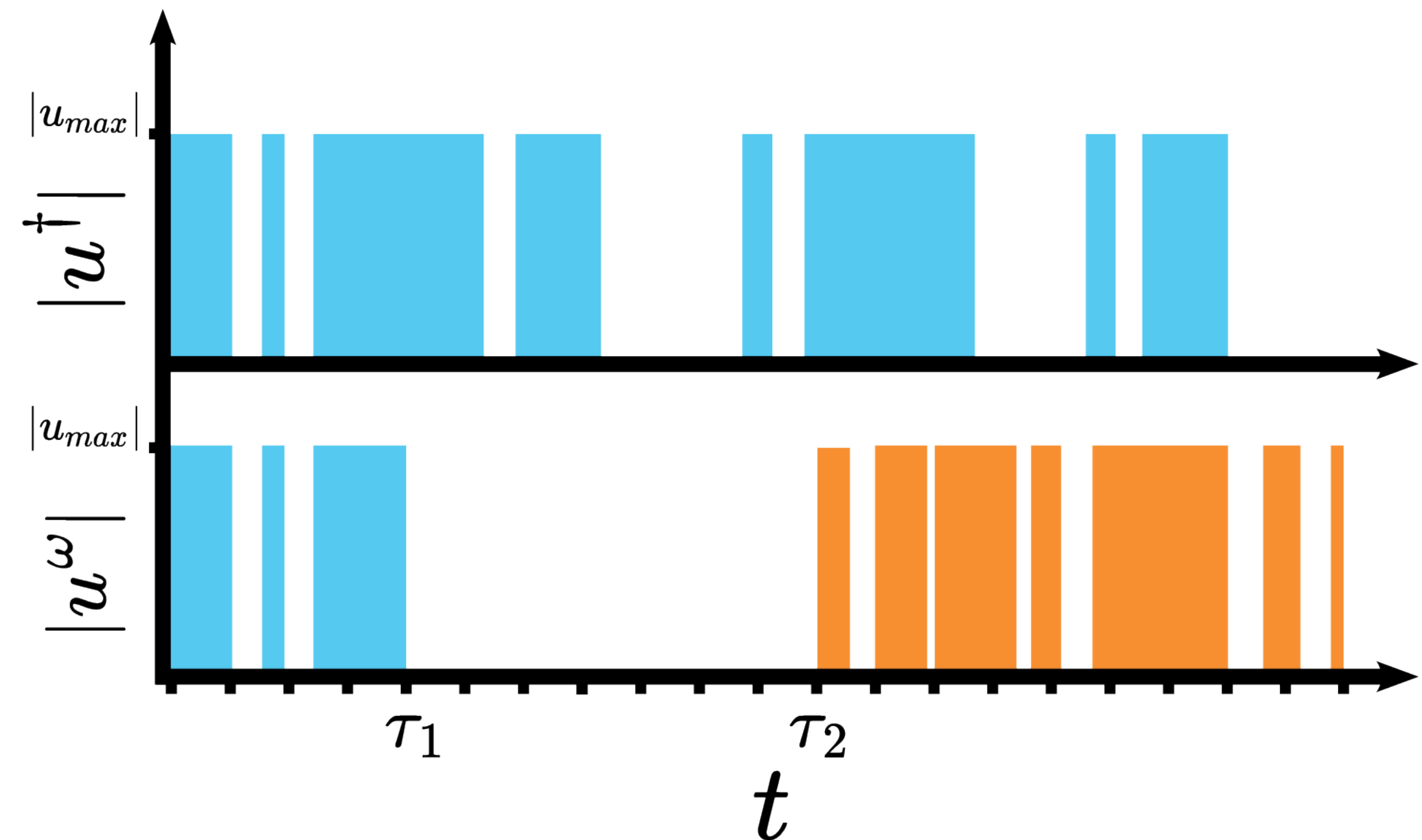
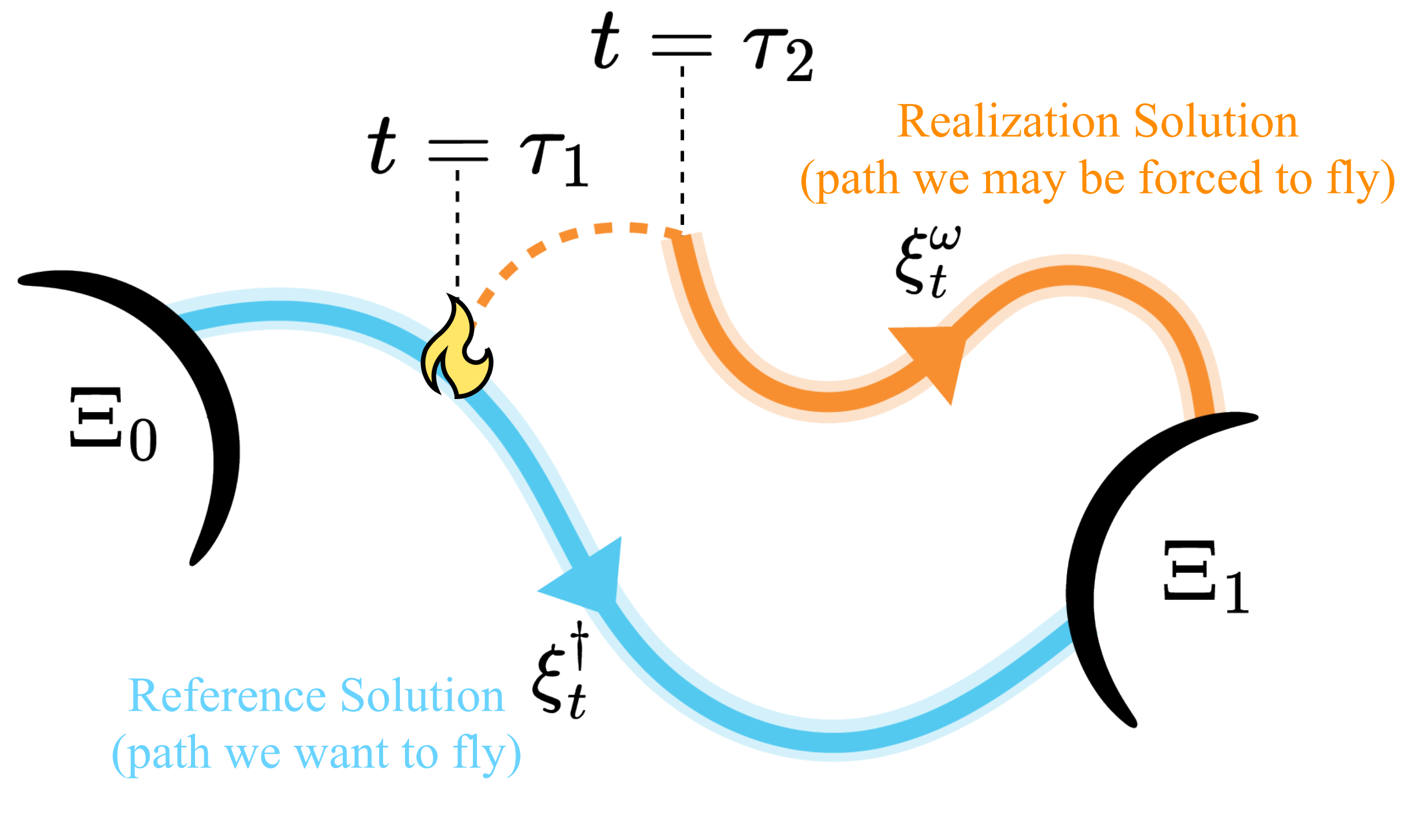
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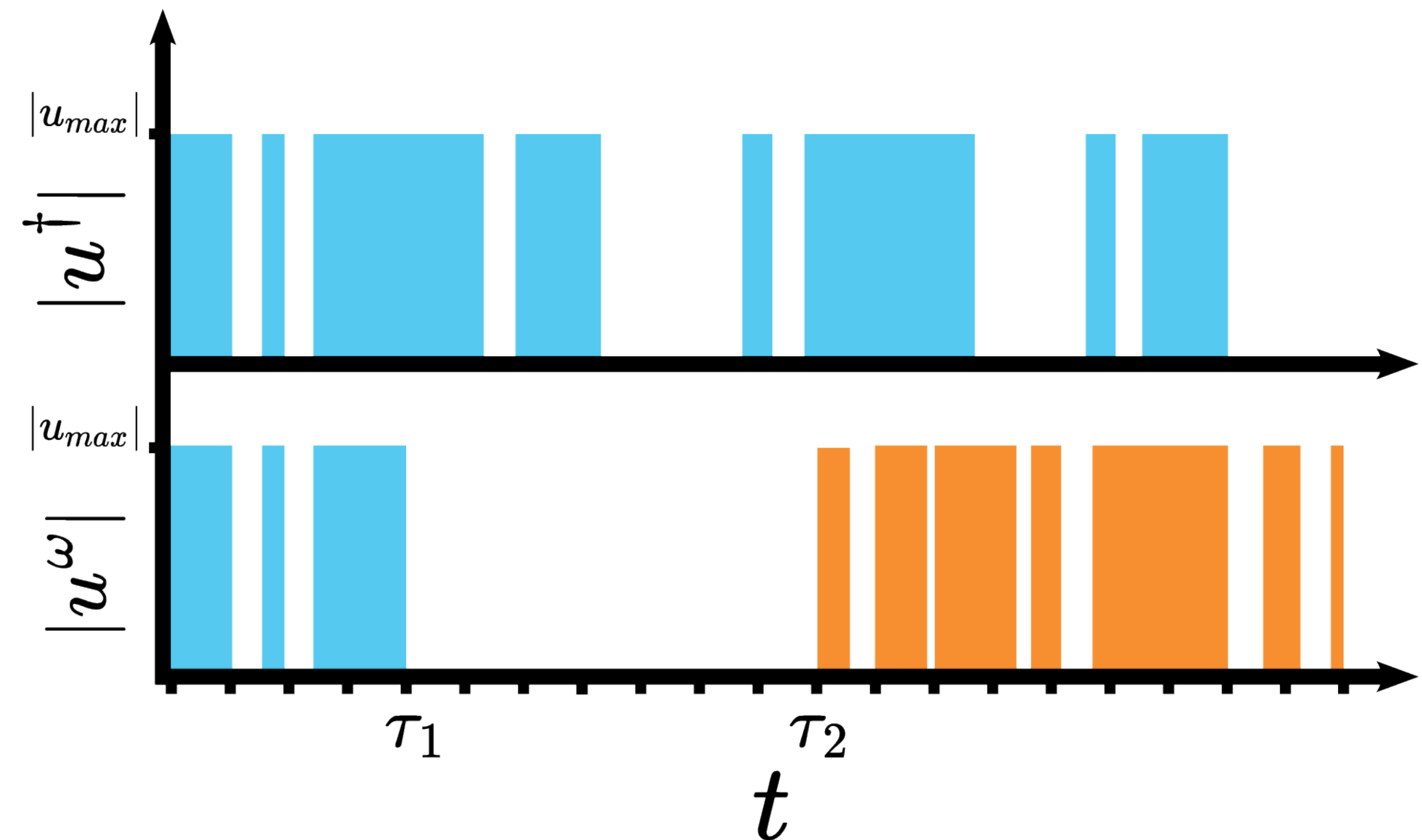
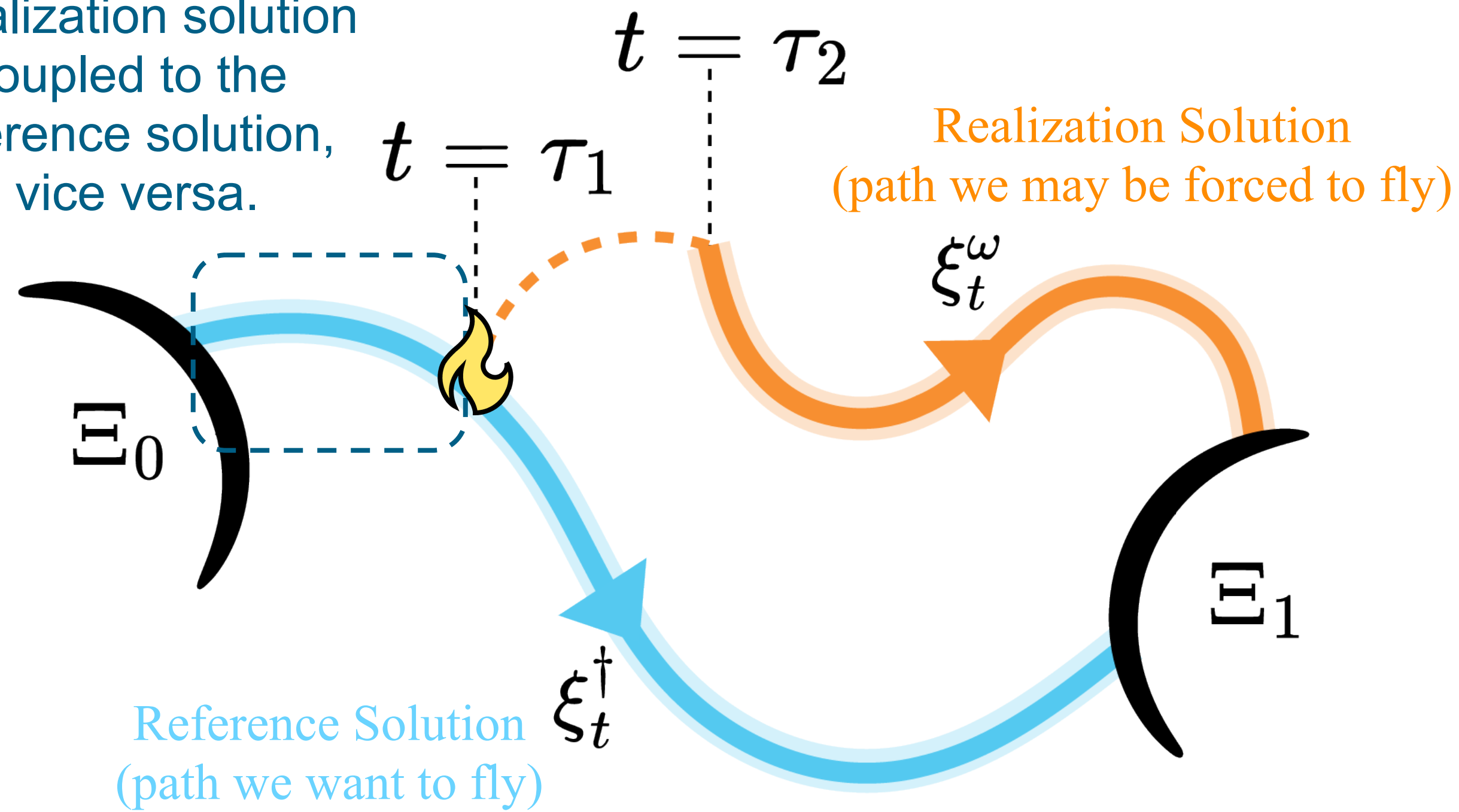


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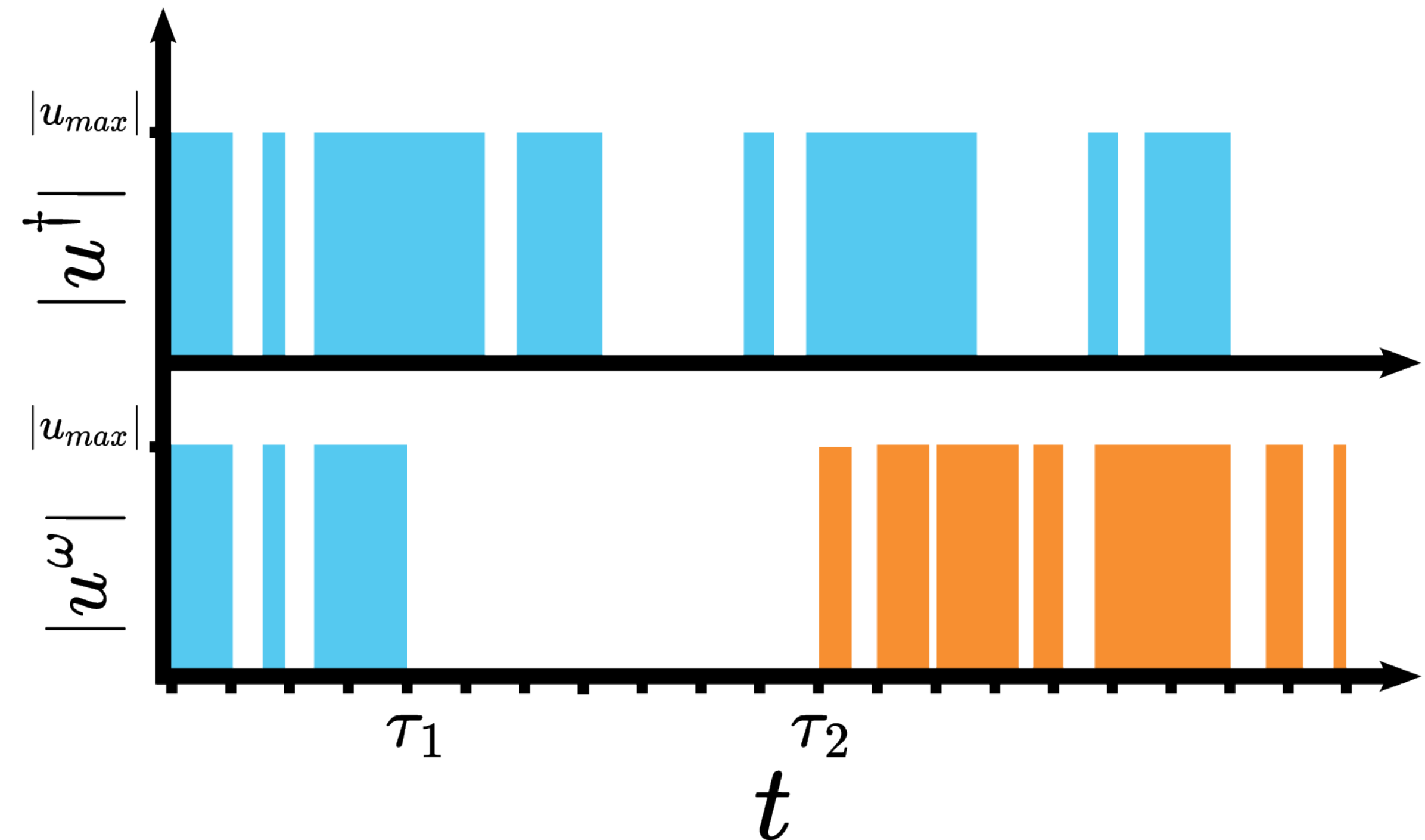
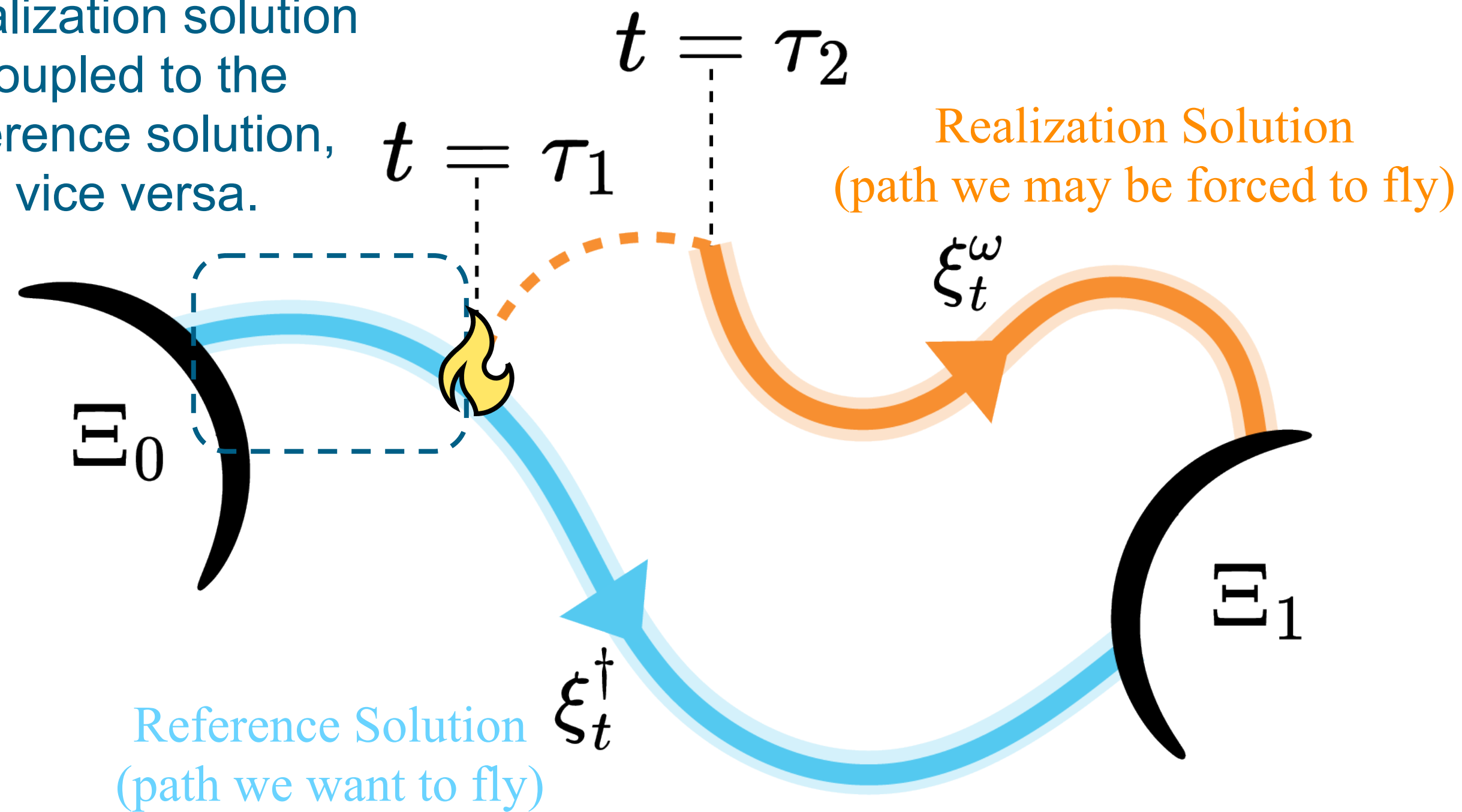
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Realization solution is coupled to the reference solution, and vice versa.



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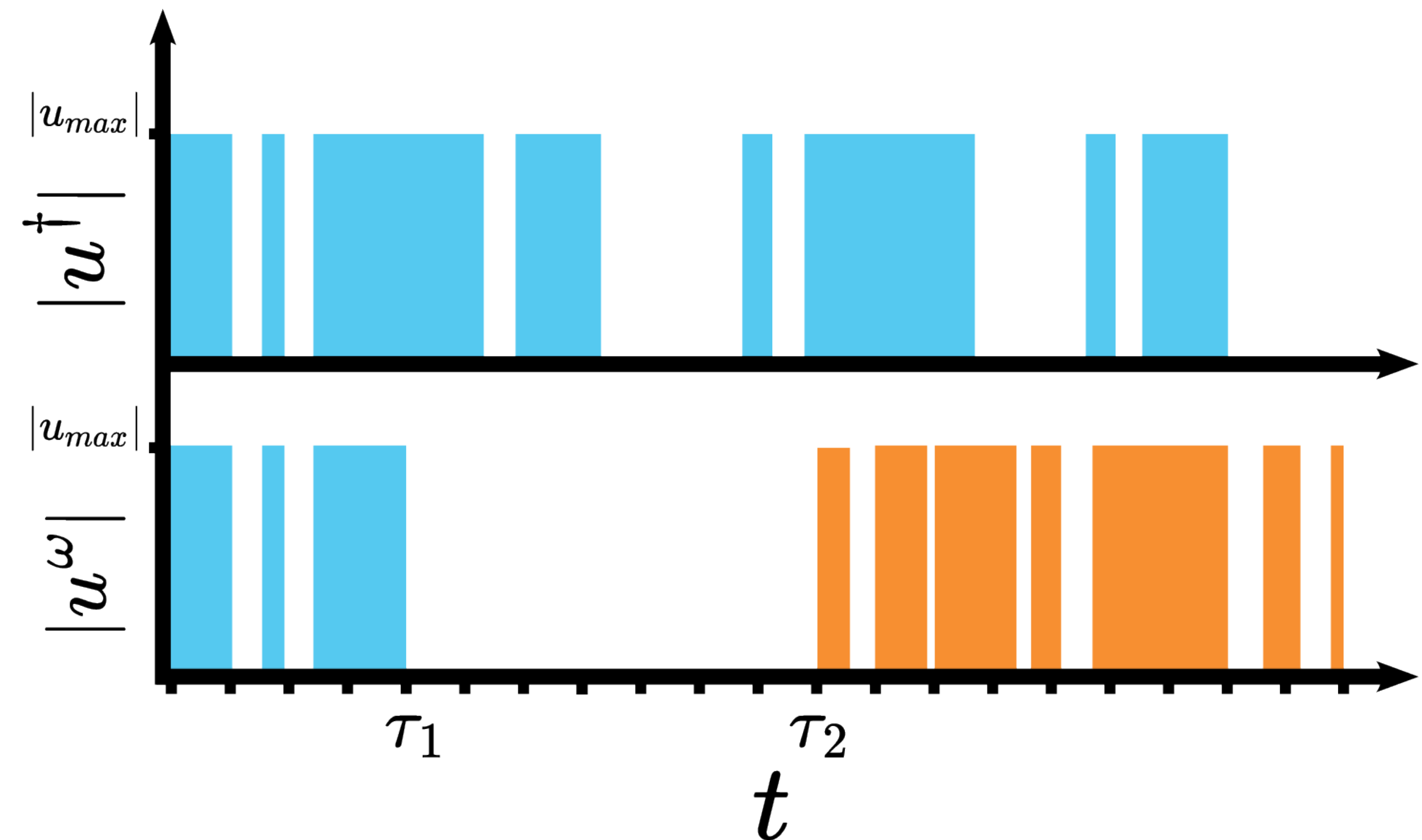
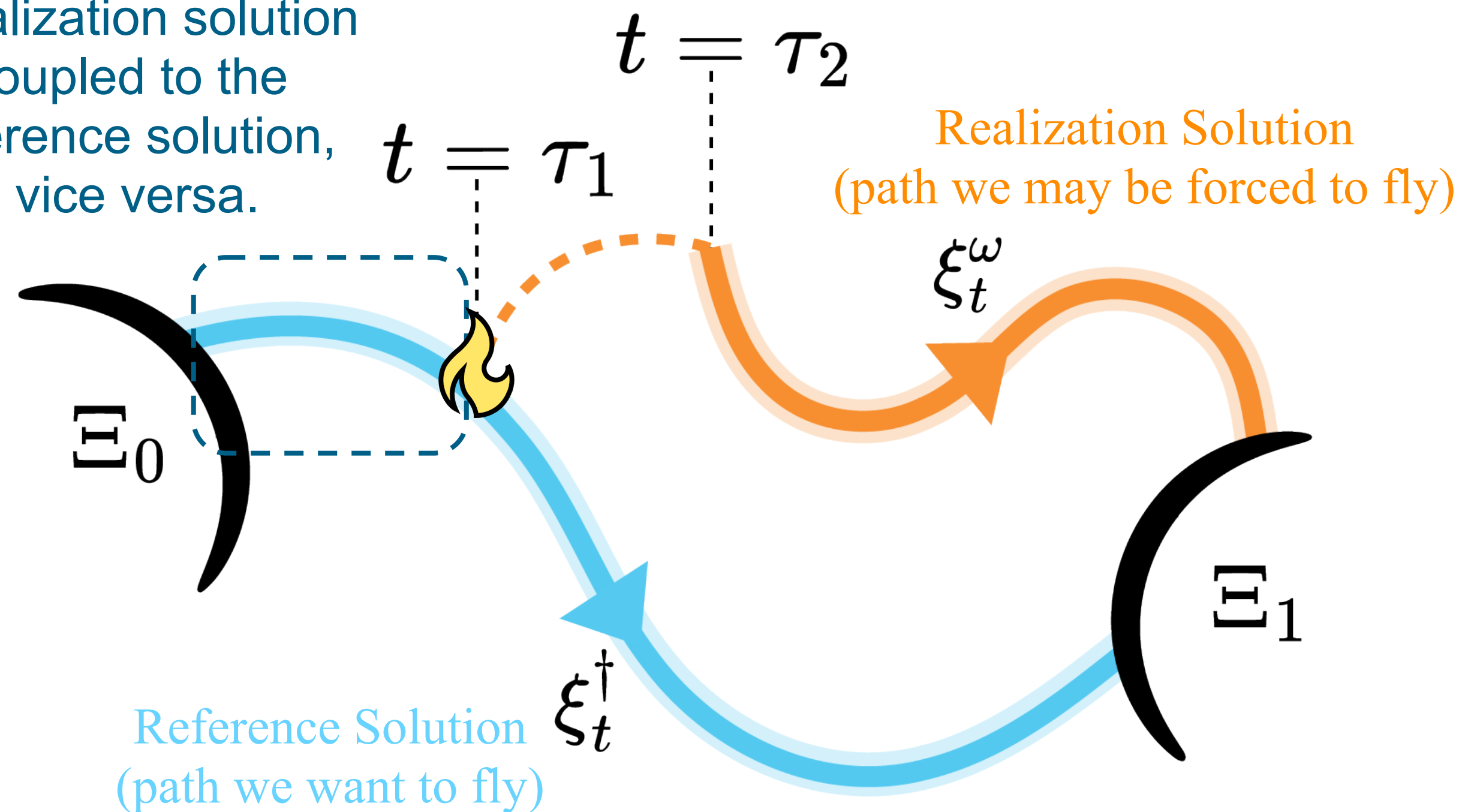


Modeling assumptions:

- 1 MTE only
- MTE can only start at the beginning of a thrust segment
- MTE can last for an arbitrary duration

The missed thrust problem can be formulated as a coupled OCP

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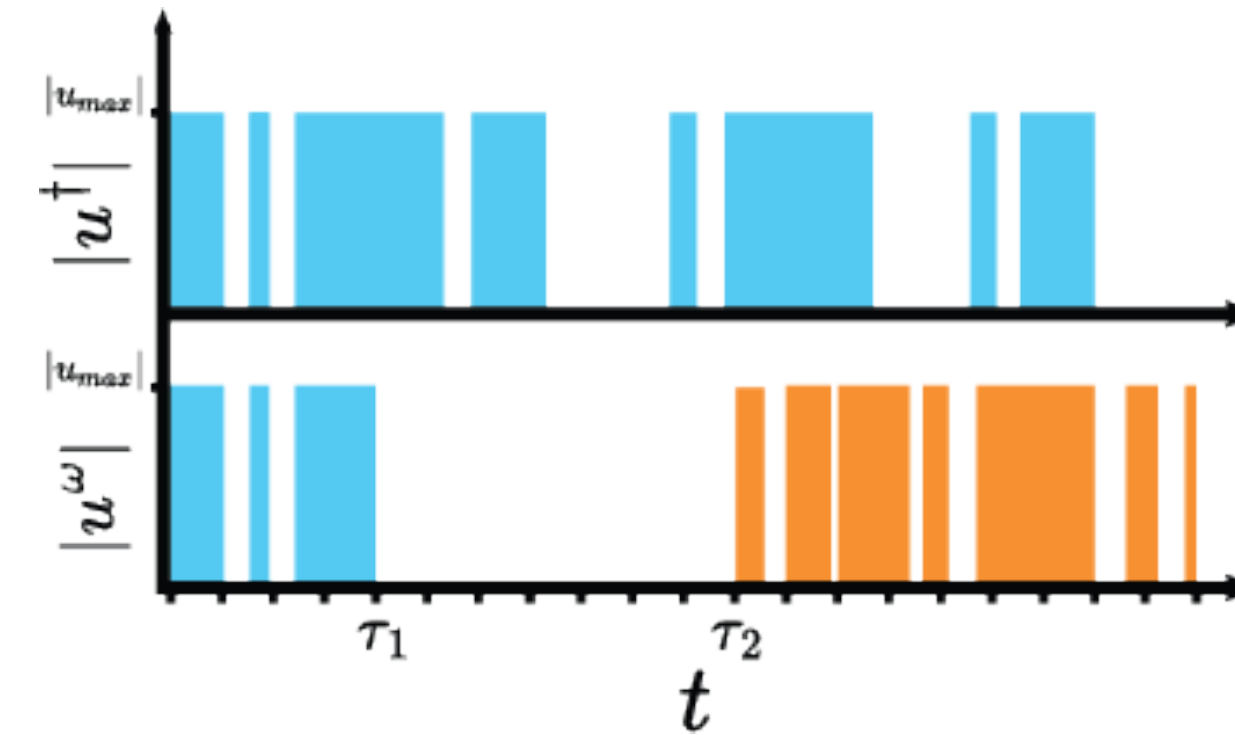
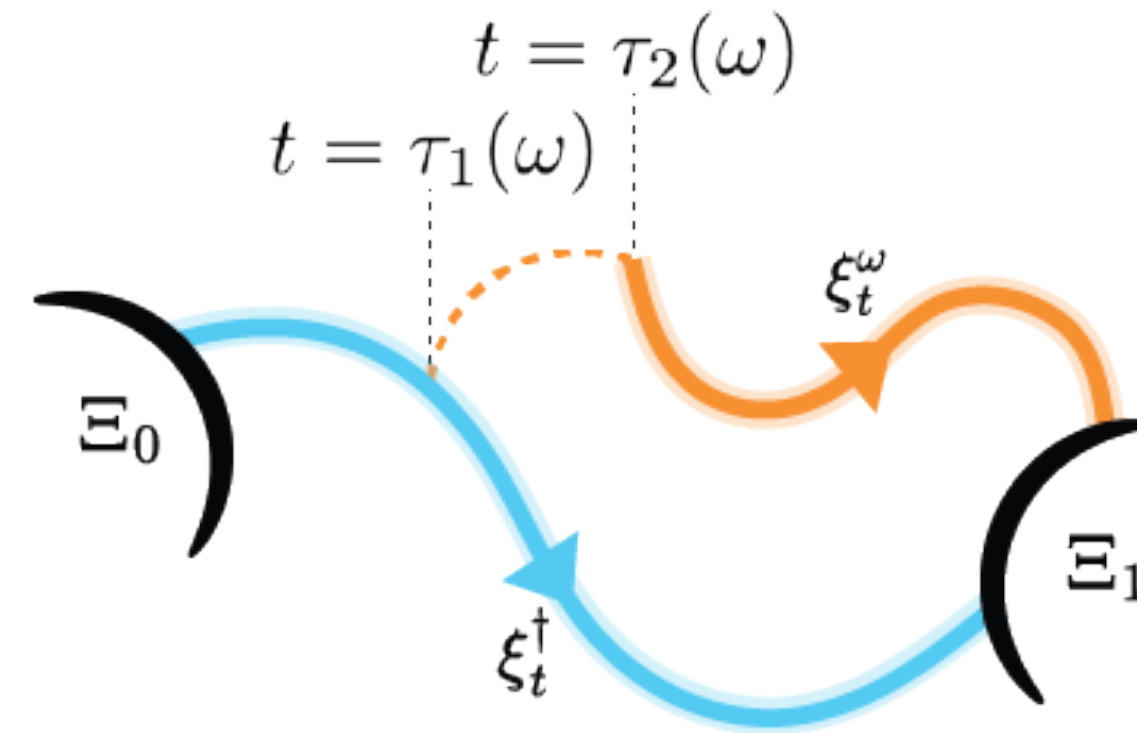


Modeling assumptions:

- 1 MTE only
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- MTE can last for an arbitrary duration

Problem dimensionality increases with number of realizations

The resulting OCP can be converted to NLP in a similar manner



Missed Thrust Design

$$\min_{\mathbf{u} \in \mathcal{U}^\Omega} \{J(\mathbf{u}^\dagger) \equiv \phi(\xi_1^\dagger) + \int_0^1 \mathcal{L}(s, \xi_s^\dagger, \mathbf{u}_s^\dagger) ds\}$$

$$\text{s.t. } \xi_t^\dagger = \xi_0^\dagger + \int_0^t \mathbf{f}(s, \xi_s^\dagger) ds + \int_0^t \mathbf{g}(s, \xi_s^\dagger, \mathbf{u}_s^\dagger) ds$$

$$\xi_t^\omega = \xi_0^\dagger + \int_0^t \mathbf{f}(s, \xi_s^\omega) ds + \int_0^{\tau_1(\omega) \wedge t} \mathbf{g}(s, \xi_s^\omega, \mathbf{u}_s^\dagger) ds + \int_{\tau_2(\omega)}^t \mathbf{g}(s, \xi_s^\omega, \mathbf{u}_s^\omega) ds$$

$$\forall t \in [0, 1], \quad \forall \omega \in \Omega$$

$$\xi_0^\dagger \in \Xi_0, \quad \xi_1^\dagger \in \Xi_1 \quad \xi_0^\omega \in \Xi_0, \quad \xi_1^\omega \in \Xi_1$$

Reference
dynamics

Realization
dynamics

$\mathcal{T}$

Nonlinear Program

$$\min_{\mathbf{x}^\dagger, \mathbf{x}^\omega} J^\dagger(\mathbf{x}^\dagger)$$

$$\text{s.t. } \mathbf{c}_e^\dagger(\mathbf{x}^\dagger) = \mathbf{0}, \quad \mathbf{c}_e^\omega(\mathbf{x}^\omega) = \mathbf{0}$$

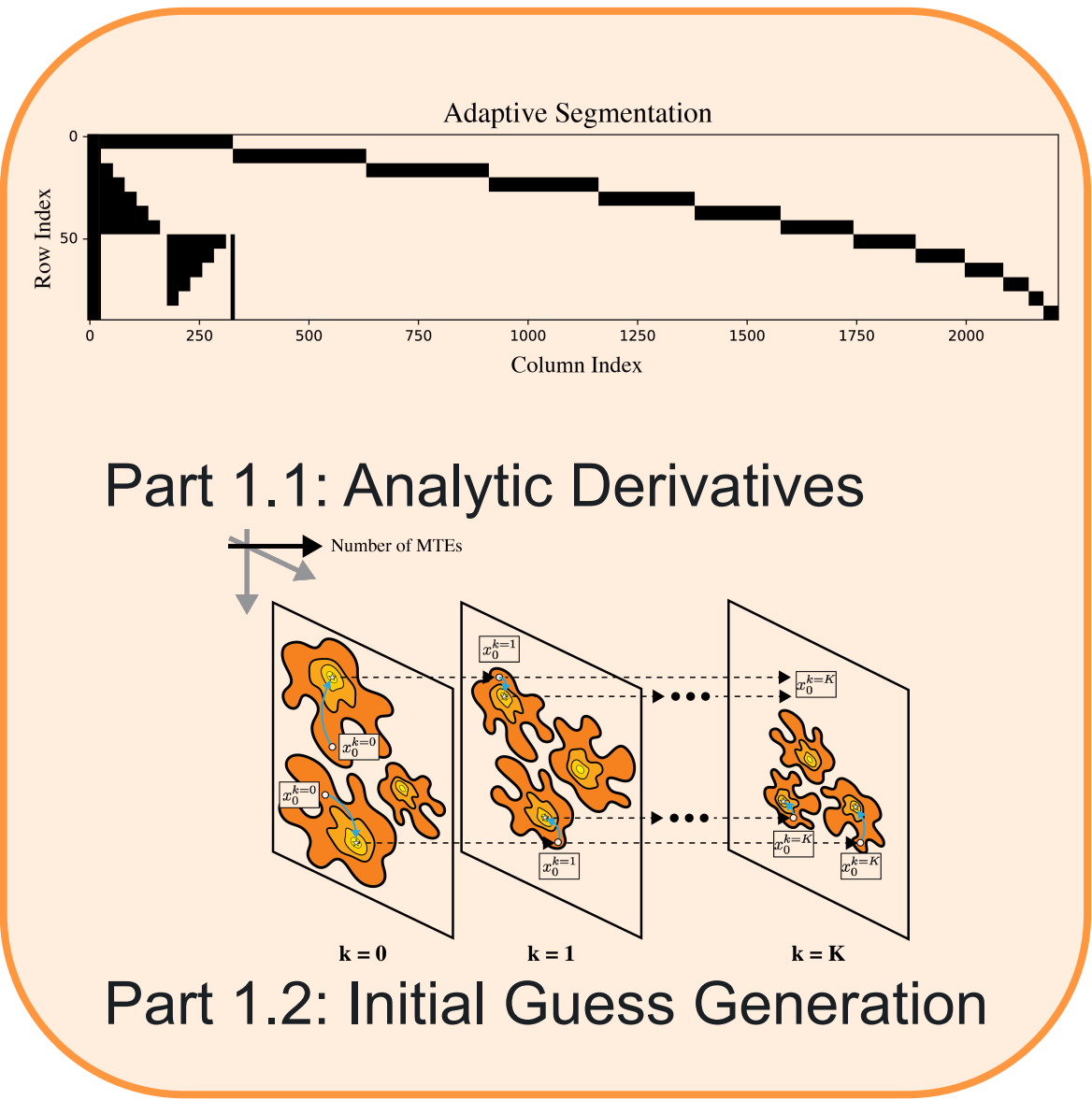
$$\mathbf{c}_i^\dagger(\mathbf{x}^\dagger) \leq \mathbf{0}, \quad \mathbf{c}_i^\omega(\mathbf{x}^\omega) \leq \mathbf{0}$$

Sinha and Beeson. JGCD. 2025
Venigalla et al. JGCD. 2020
McCarty et al. AAS. 2020

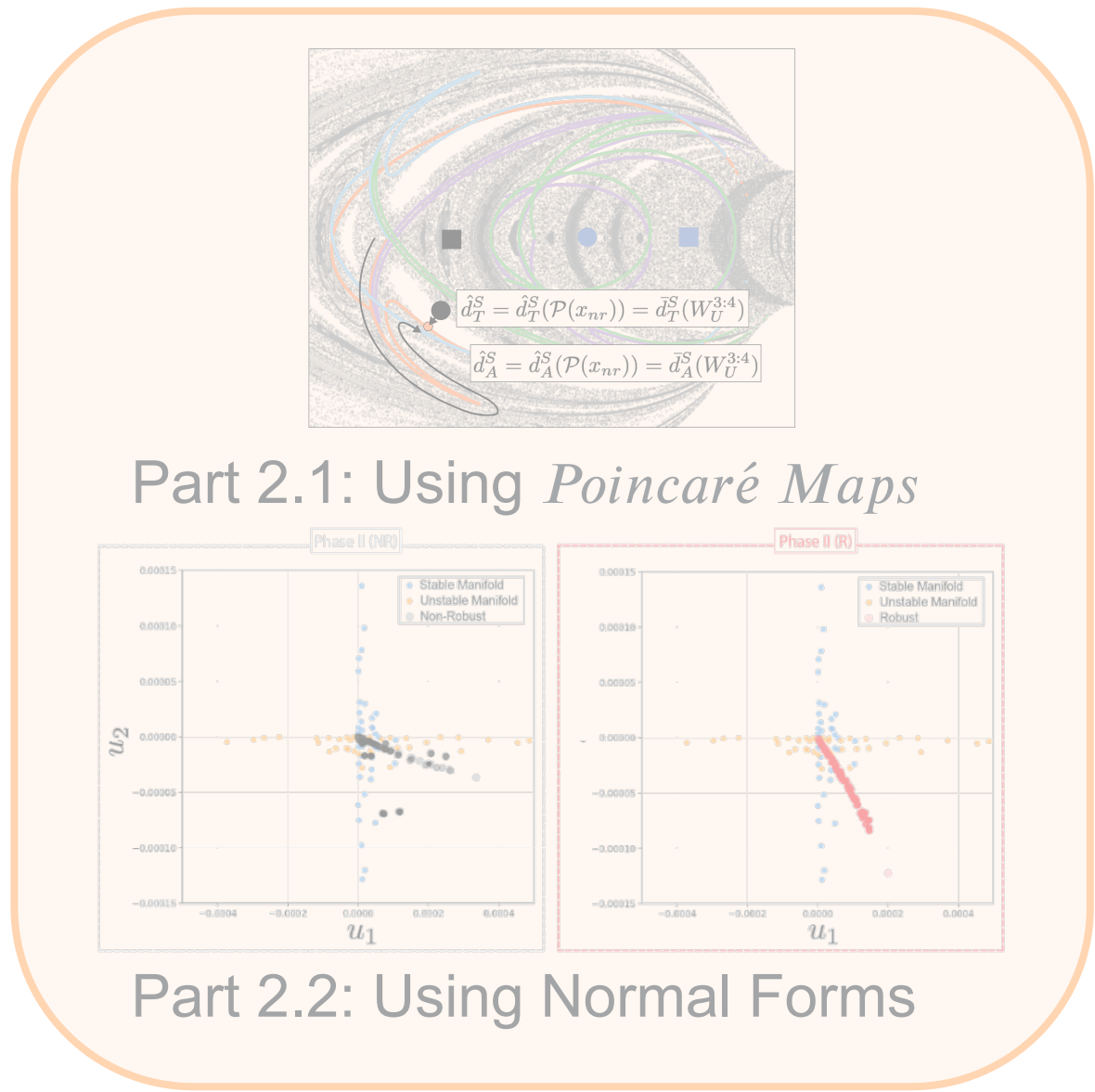


Part 1: Scalable Computational Methods

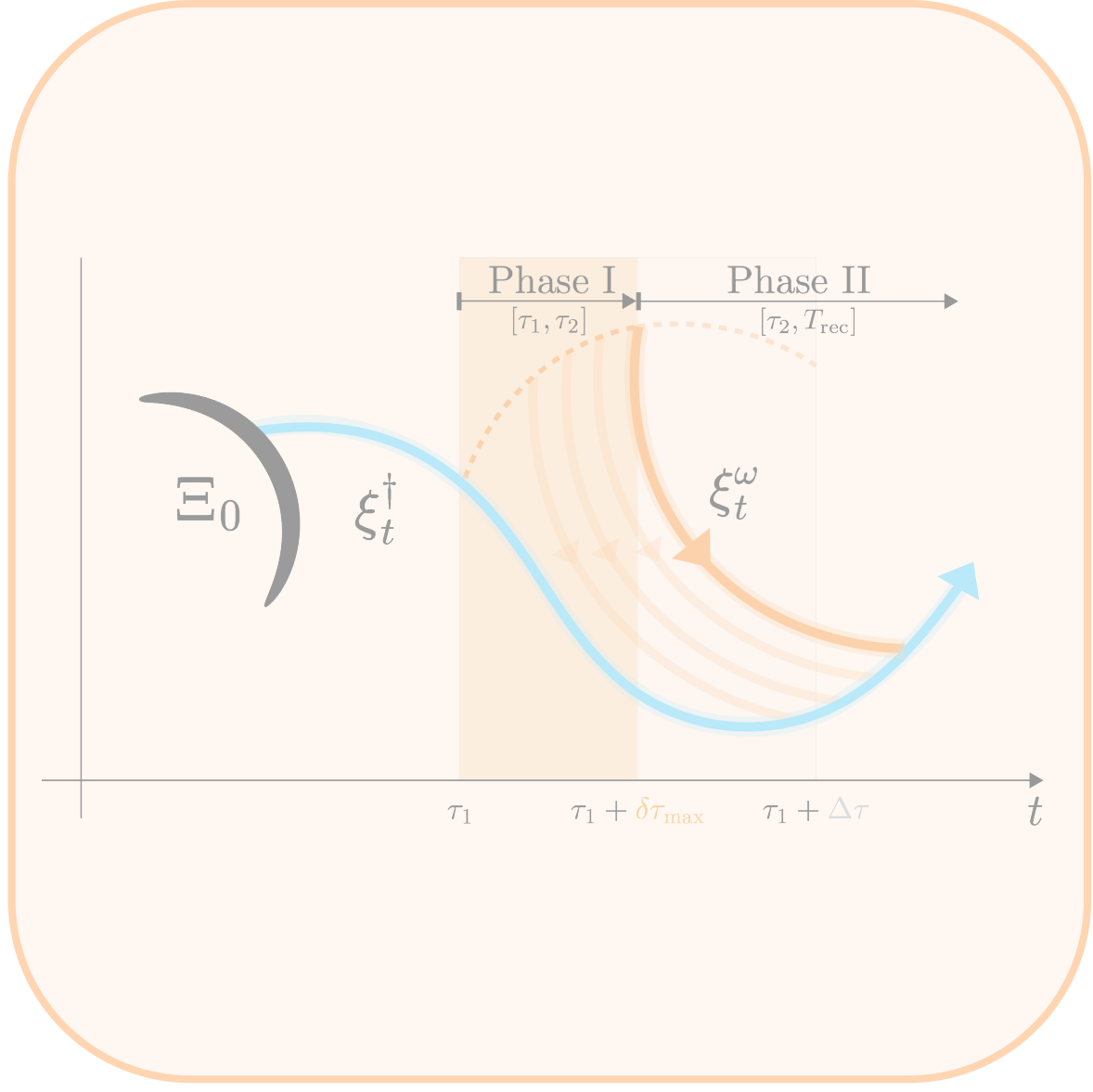
Part 1: Scalable Computational Methods



Part 2: Dynamical Systems Analysis



Part 3: Bilevel Optimal Control Framework



Part 0: Missed Thrust Design Problem Formulation

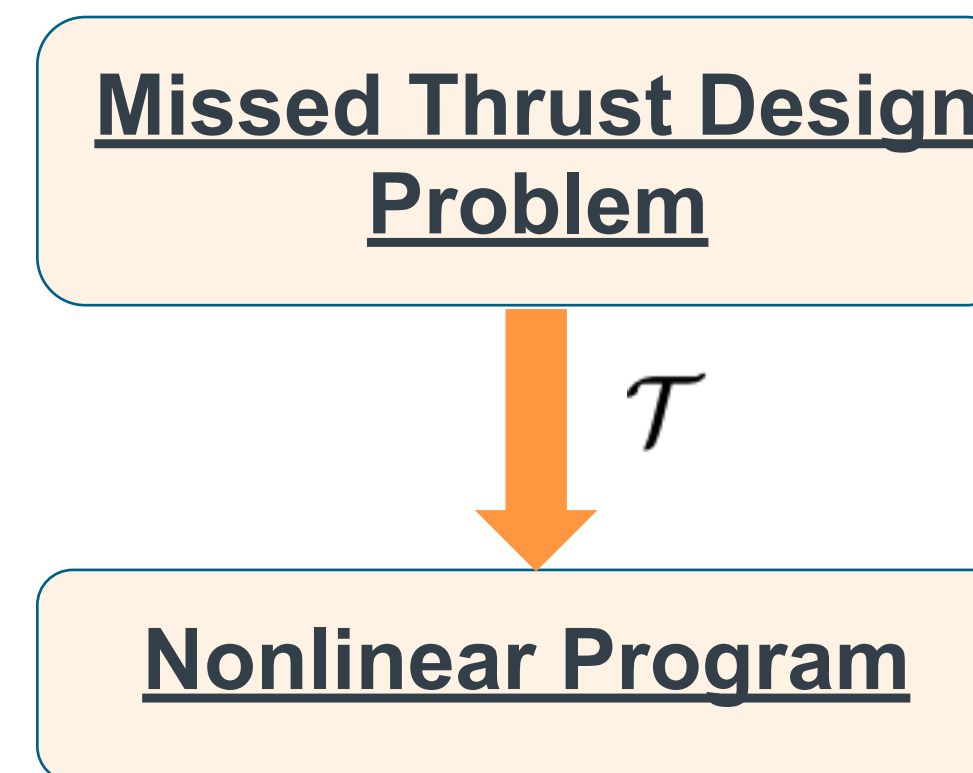


Part 1.1: Analytic Derivatives For Missed Thrust Design

Having access to analytic derivatives can significantly accelerate the solving process

Derivatives of the objective and constraints can improve performance of gradient-based numerical solvers

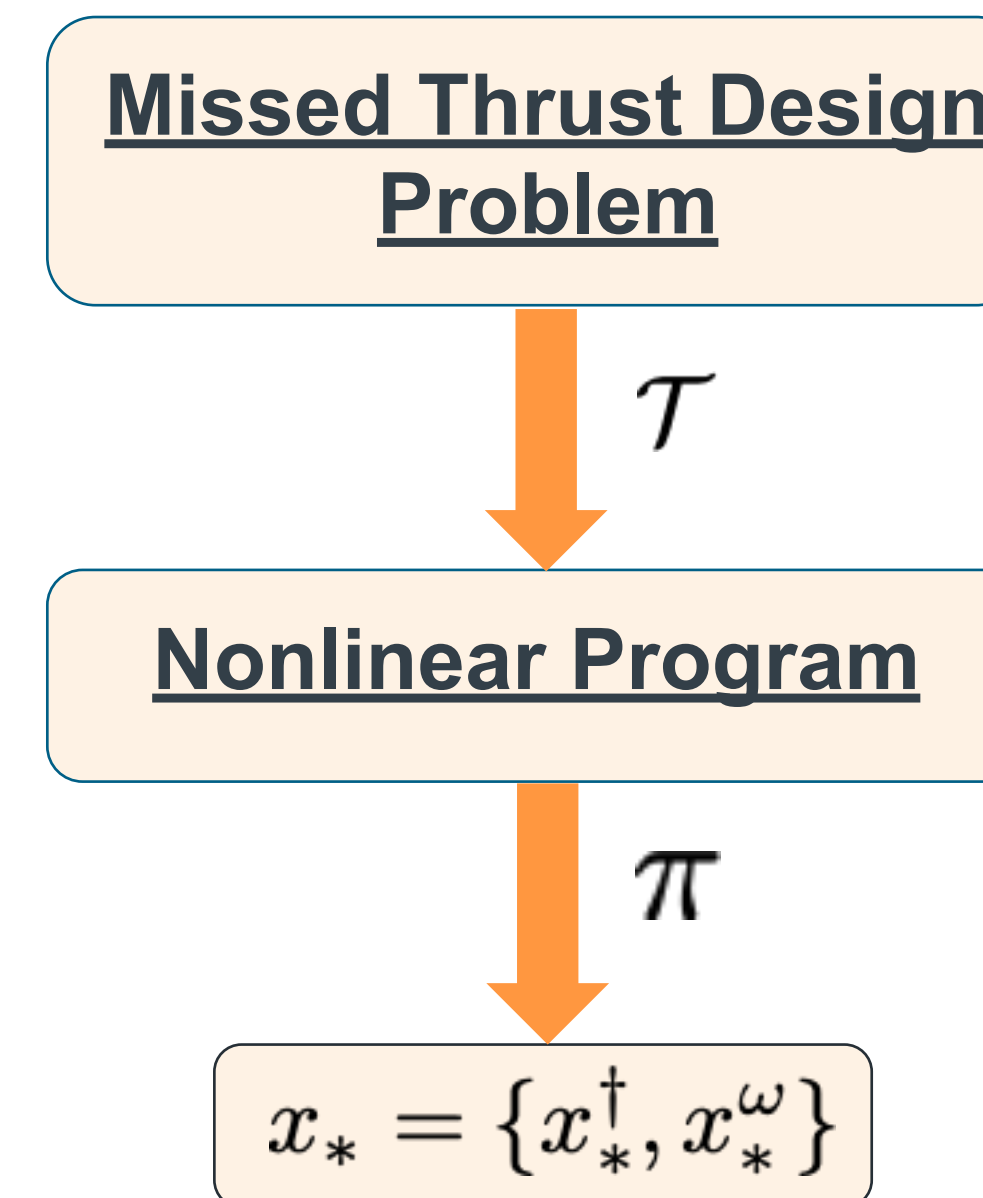
- The **Jacobian** matrix stores first-order sensitivity information
- The **non-robust** construction provides the building blocks for the **robust** case Ellison (2018)



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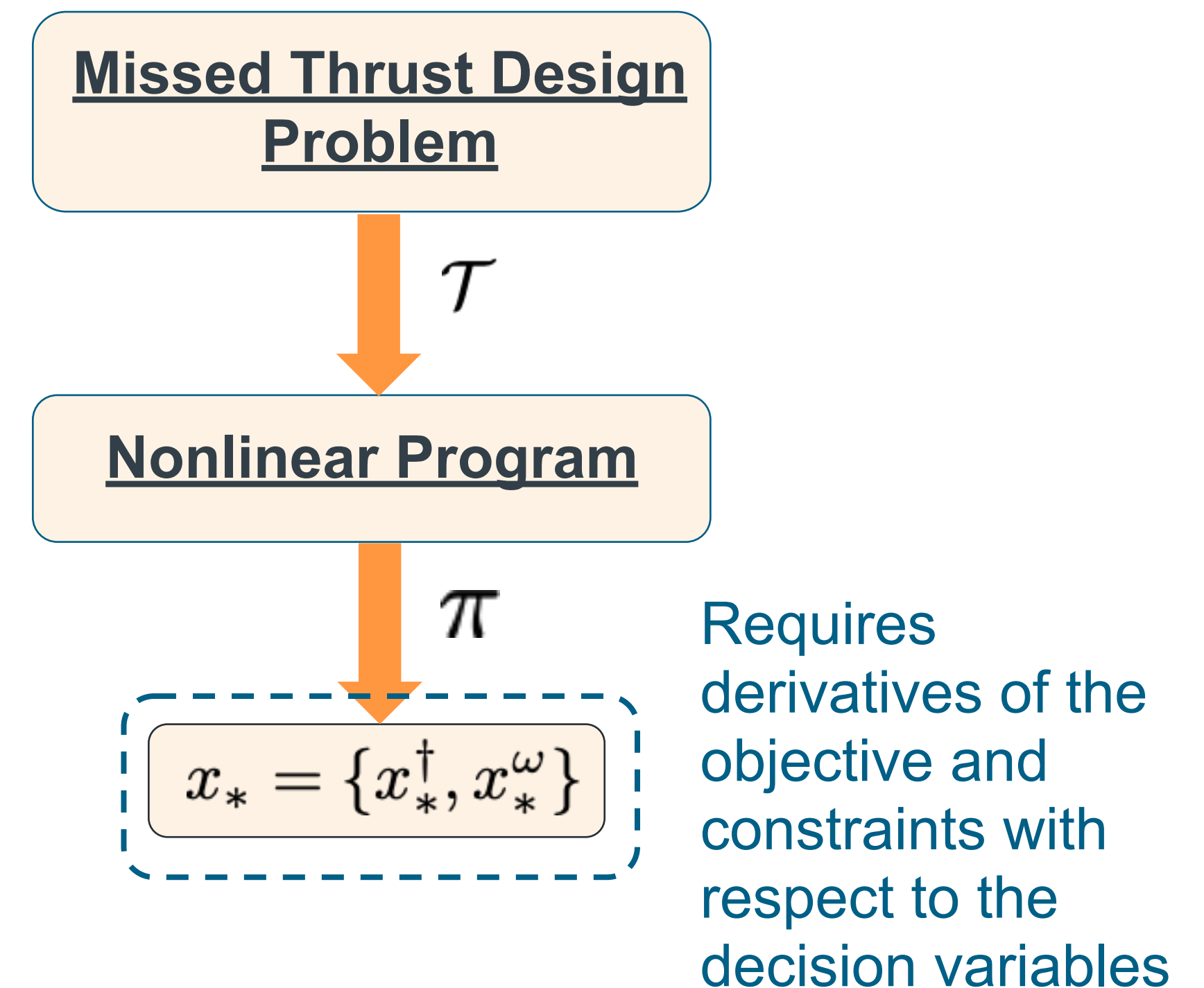
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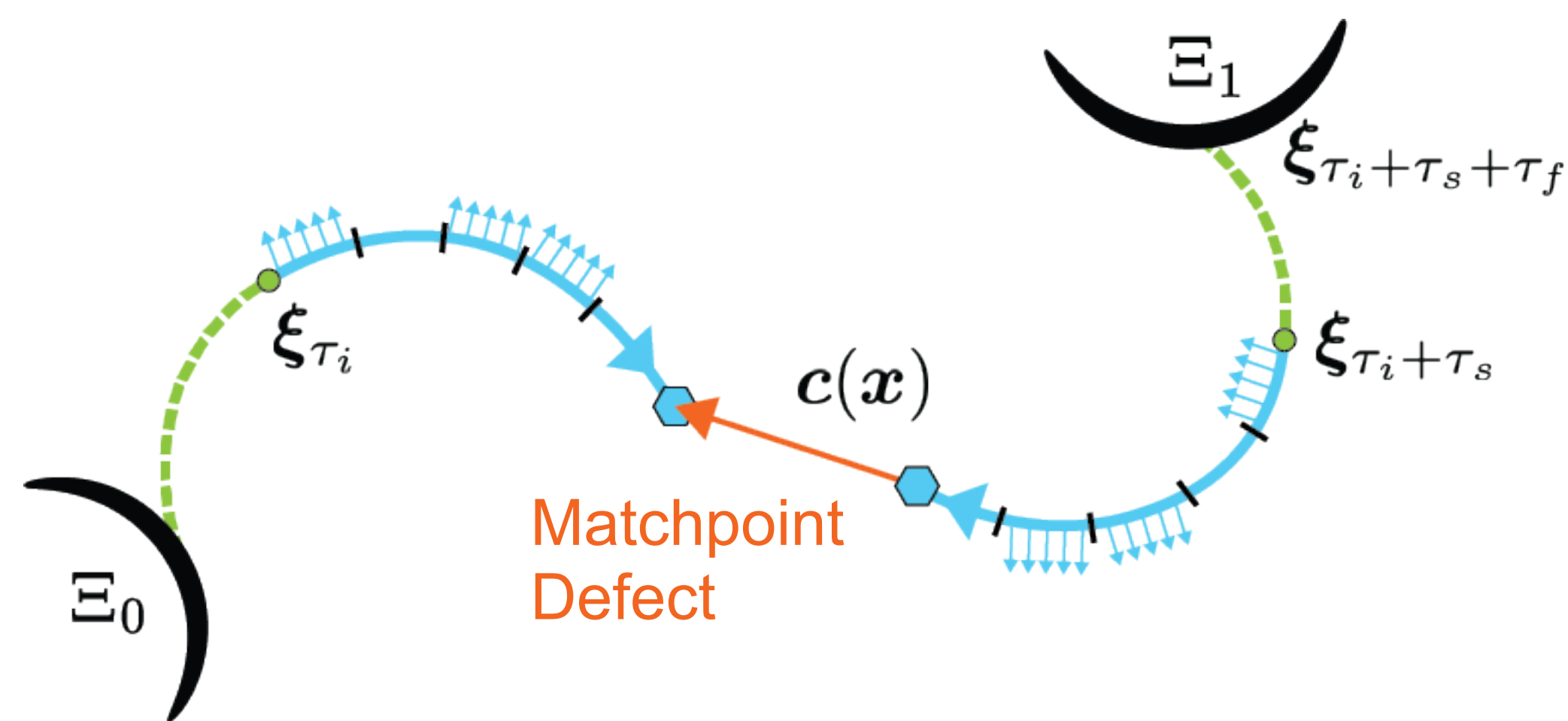
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Missed Thrust Design Problem

$\mathcal{T}$

Nonlinear Program

π

$$x_* = \{x_*^\dagger, x_*^\omega\}$$

Requires derivatives of the objective and constraints with respect to the decision variables

Analytic derivatives can be obtained by chaining together relevant state transition matrices

The augmented state η gives the local derivative objects needed for Jacobian assembly

- For a finite-burn low-thrust control segment k (with constant control u_k), define the **augmented state**

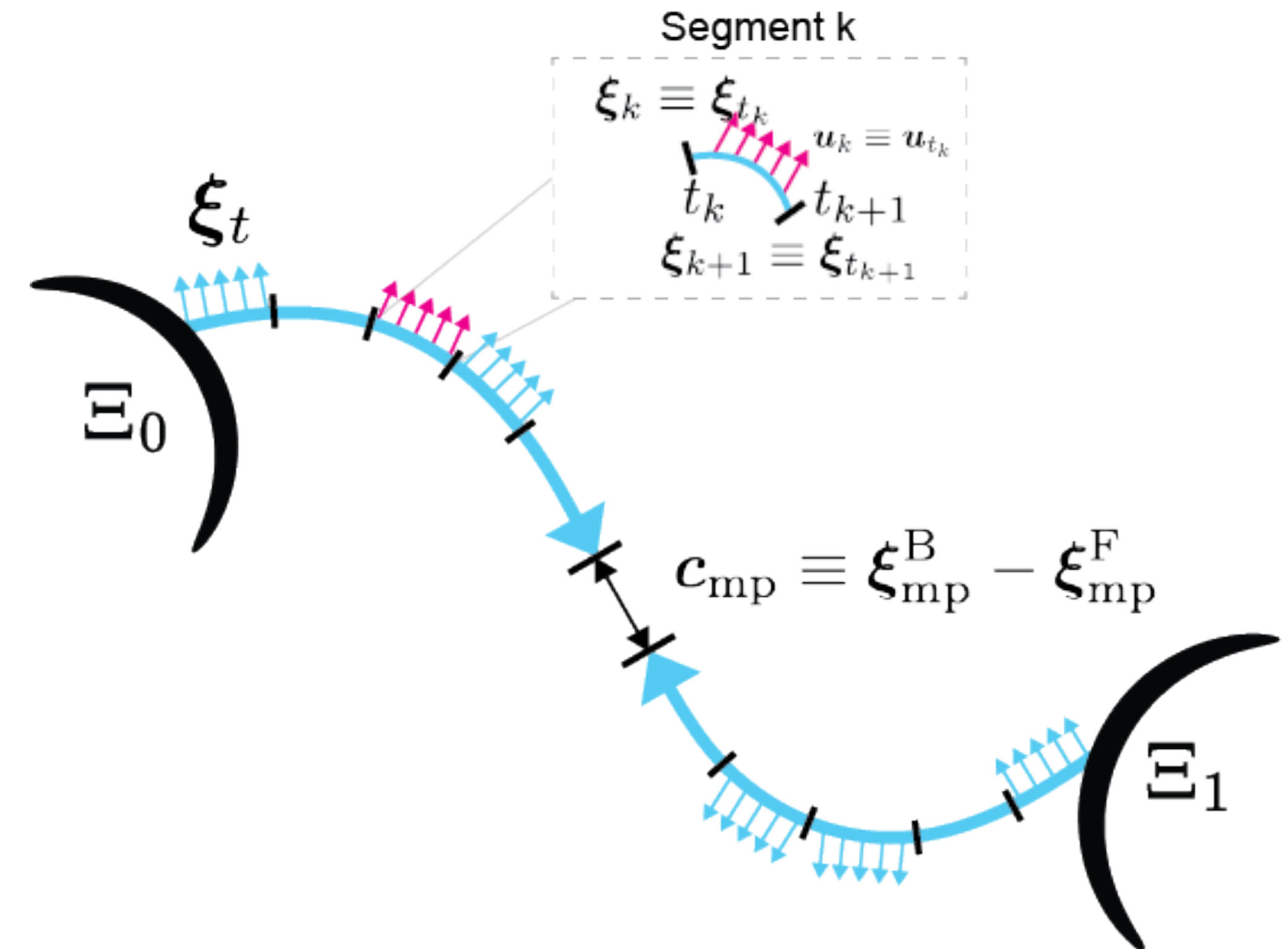
$$\eta_t \equiv [\xi_t, u_t]^\top \in \mathbb{R}^{n_\xi + n_u}.$$

$$\dot{\eta}_t = \begin{bmatrix} \tilde{f}(\xi_t, u_t) \\ 0 \end{bmatrix} = \begin{bmatrix} f(\xi_t) + g(\xi_t, u_t) \\ 0 \end{bmatrix} \quad \text{Ellison (2018)}$$

- Variational equations** for the augmented state from time t_k to time $t \in [t_k, t_{k+1}]$ is given by

$$\dot{\Phi}(t, t_k) = A(\eta_{t_k})\Phi(t, t_k), \quad A(\eta_{t_k}) \equiv \begin{bmatrix} \frac{\partial \tilde{f}}{\partial \xi} & \frac{\partial \tilde{f}}{\partial u} \\ 0 & 0 \end{bmatrix}$$

$$\Phi \equiv \begin{bmatrix} \Phi_{\xi\xi} & \Phi_{\xi u} \\ 0_{n_u \times n_\xi} & I_{n_u} \end{bmatrix}$$



For a given segment k , the perturbation δu_k induces a perturbation in the state at the end of the segment

$$\delta \xi_{k+1} = \Phi_{\xi u}(\xi_k, u_k) \delta u_k$$

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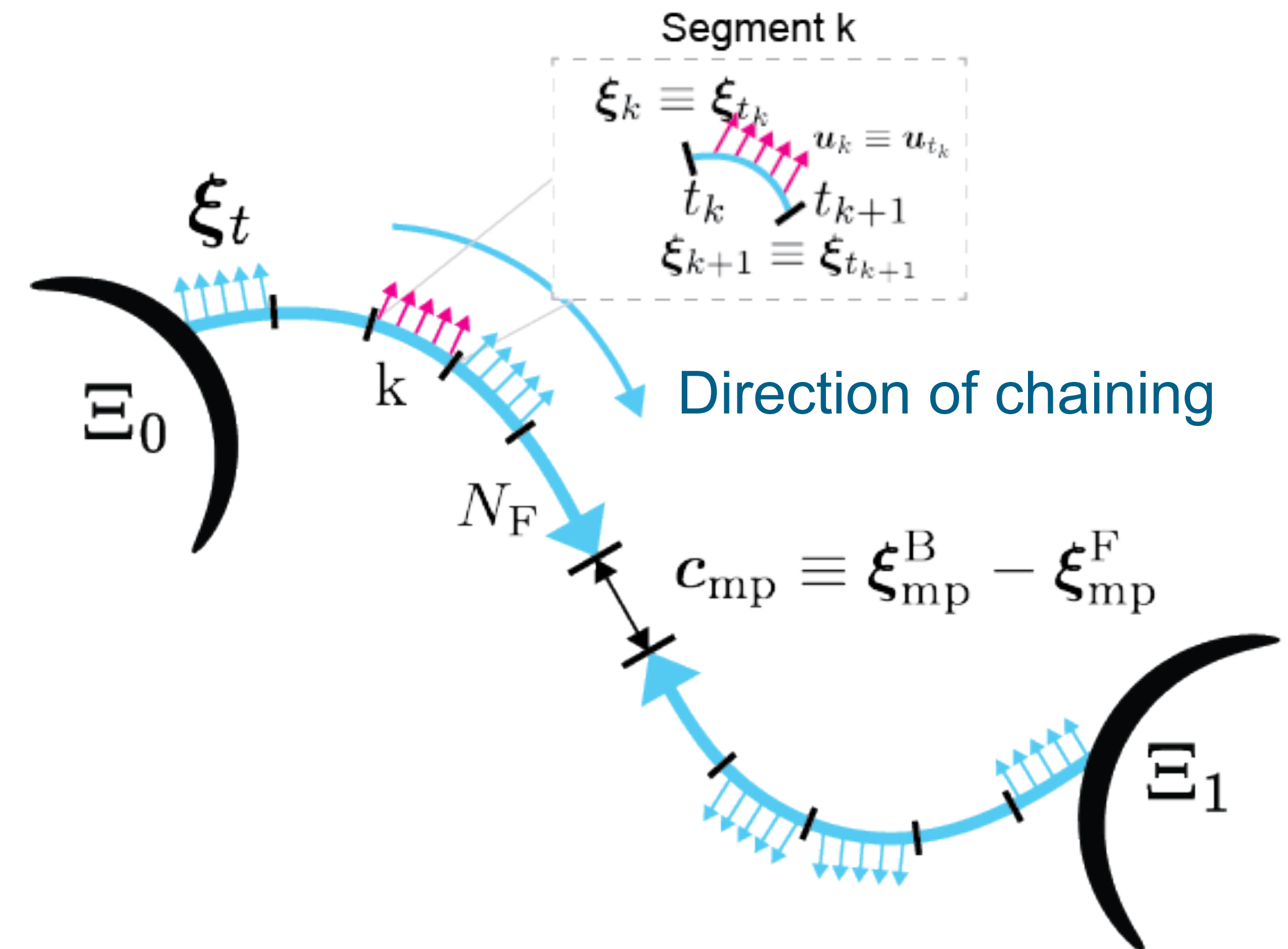
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The robust extension uses the same local machinery but adds reference-to-realization couplings

The perturbation $\delta u_k^\dagger$ induces a perturbation in the match point state for both the reference and the realization solutions

- Reference-to-reference sensitivity

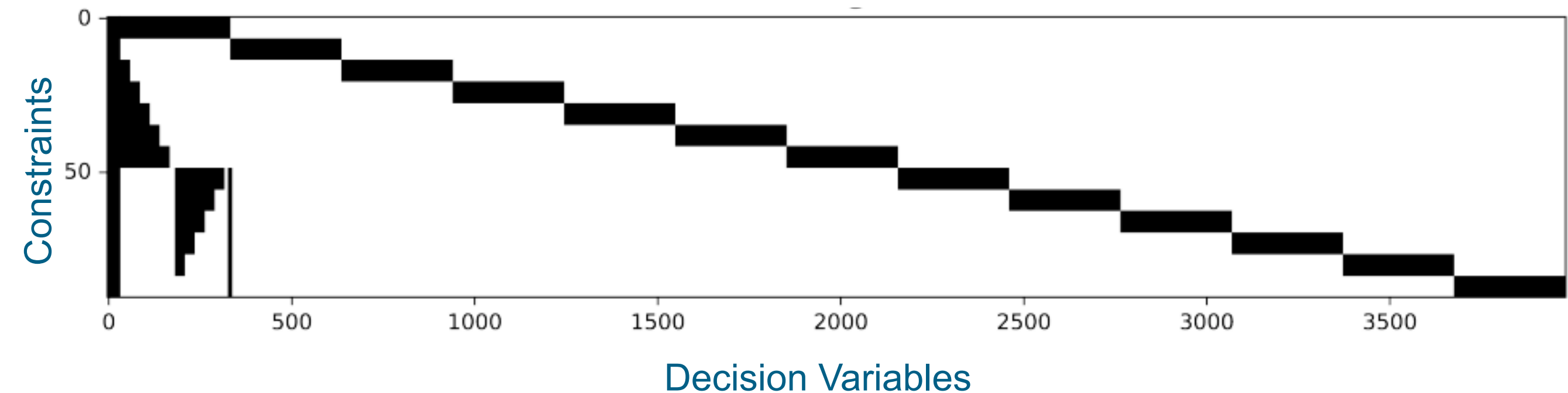
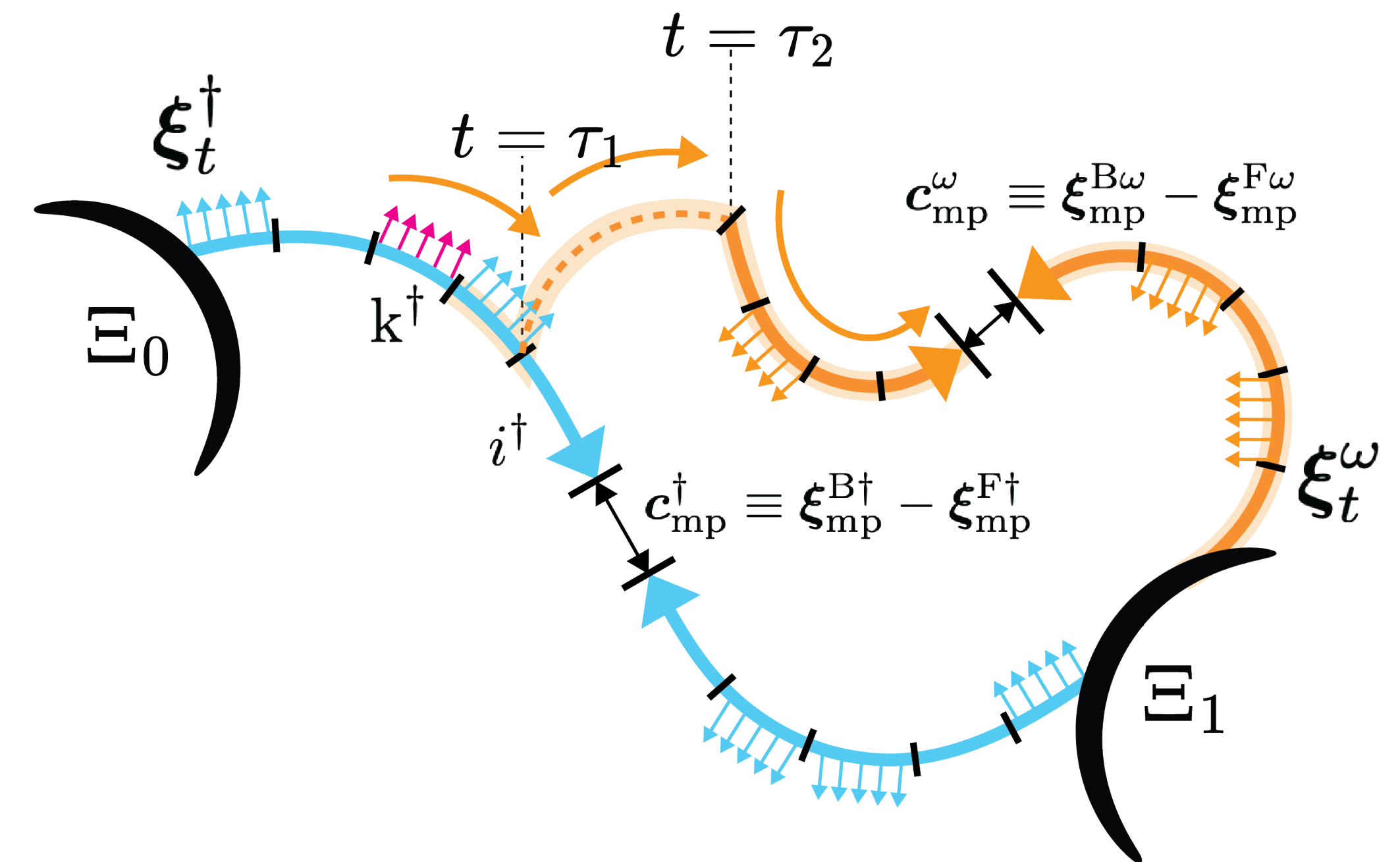
$$\frac{\partial \eta_{\text{mp}}^{\dagger, \text{F}}}{\partial u_k^{\dagger}} = \tilde{\Phi}_{N_{\text{F}}^{\dagger}}^{\dagger} \tilde{\Phi}_{N_{\text{F}}^{\dagger}-1}^{\dagger} \cdots \tilde{\Phi}_{k+1}^{\dagger} \Phi_k^{\dagger}$$

$$\frac{\partial \eta_{\text{mp}}^{\omega, \text{B}}}{\partial u_k^\dagger} = \tilde{\Phi}_{N_{\text{B}}^\dagger}^\dagger \tilde{\Phi}_{N_{\text{B}}^\dagger - 1}^\dagger \cdots \tilde{\Phi}_{k+1}^\dagger \Phi_k^\dagger$$

- Reference-to-realization sensitivity

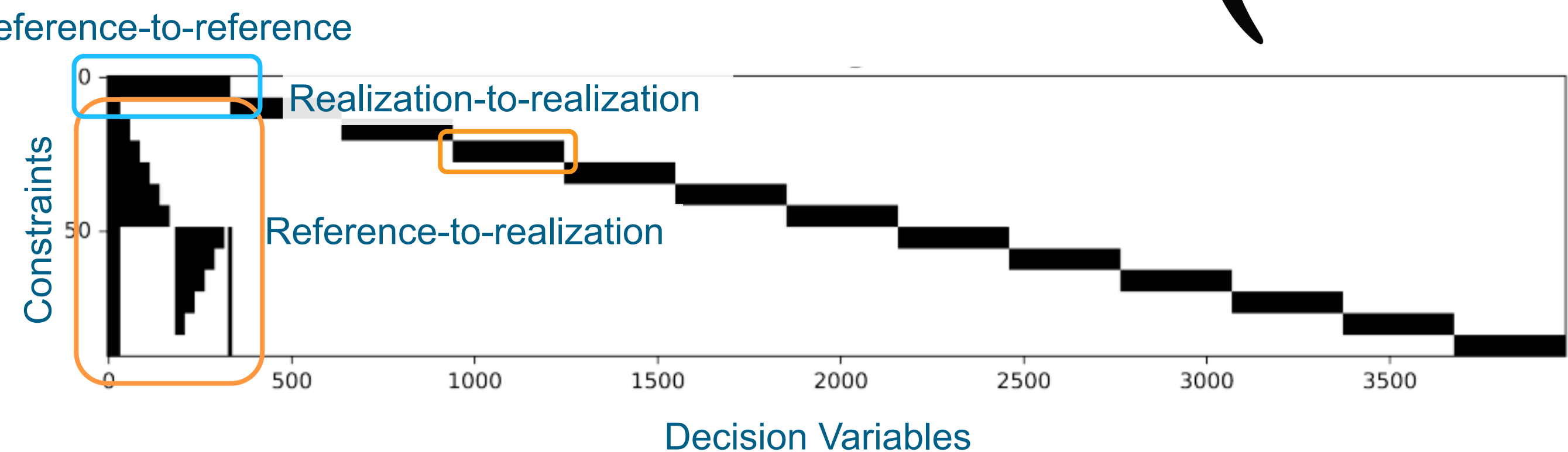
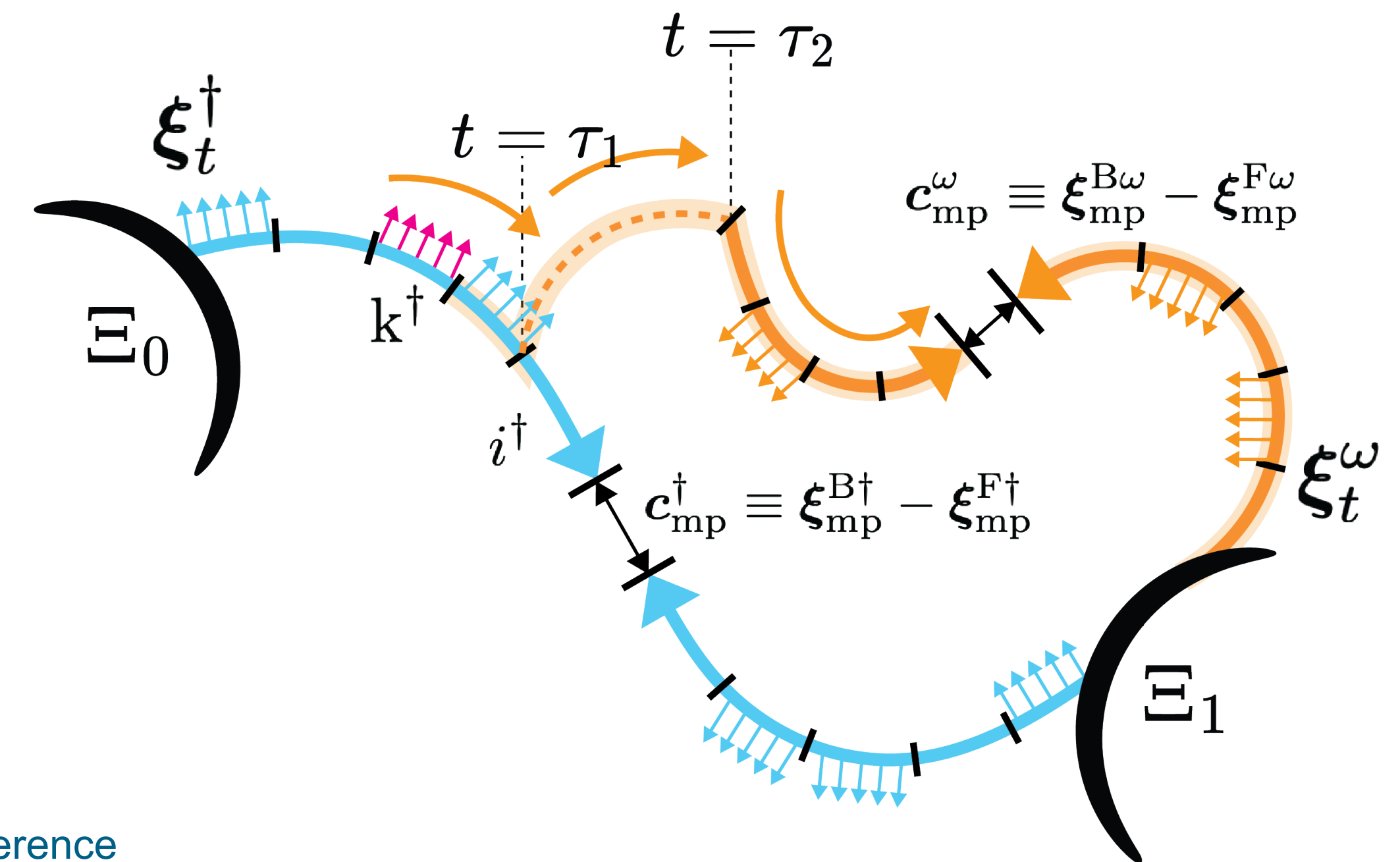
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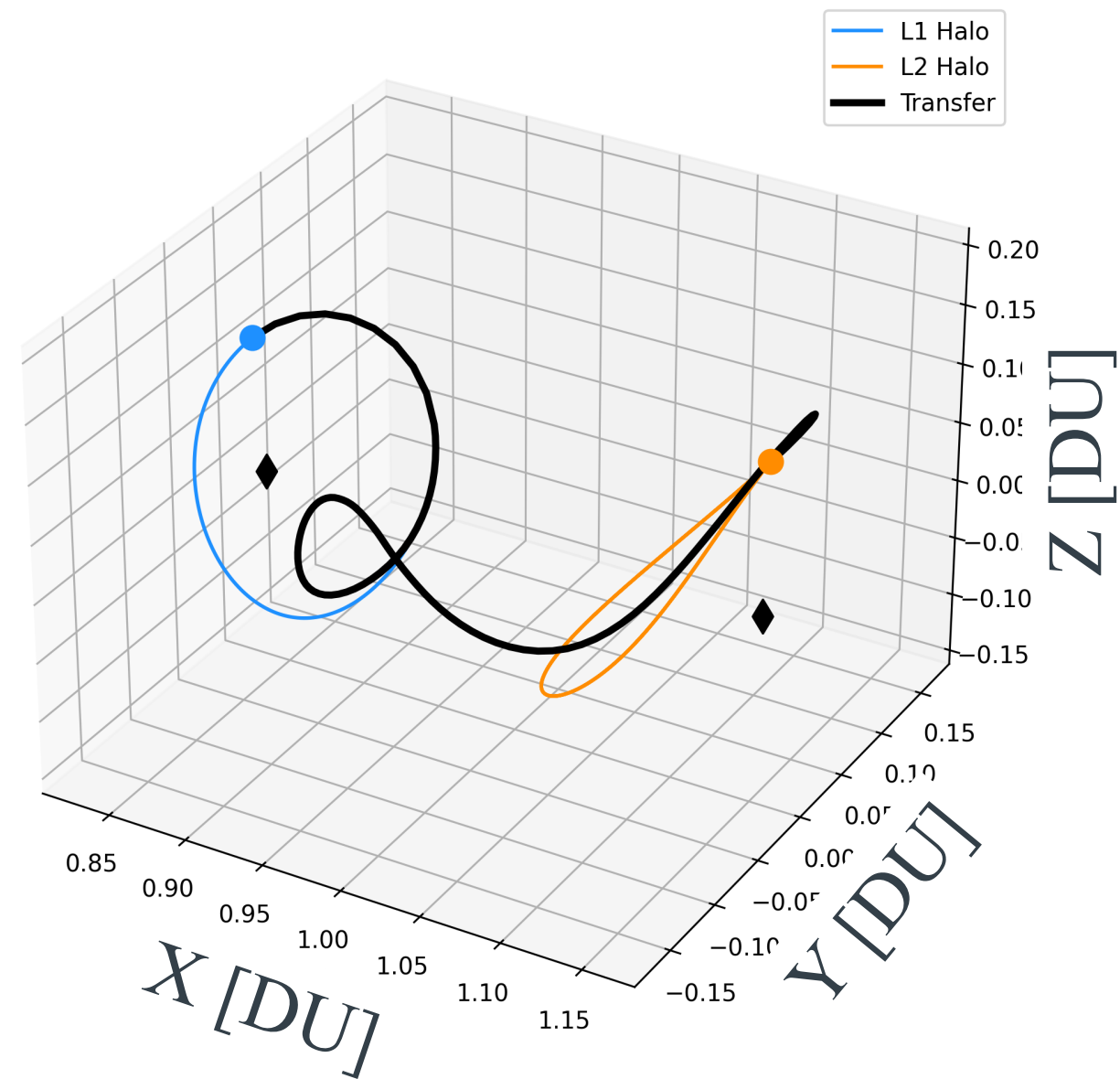


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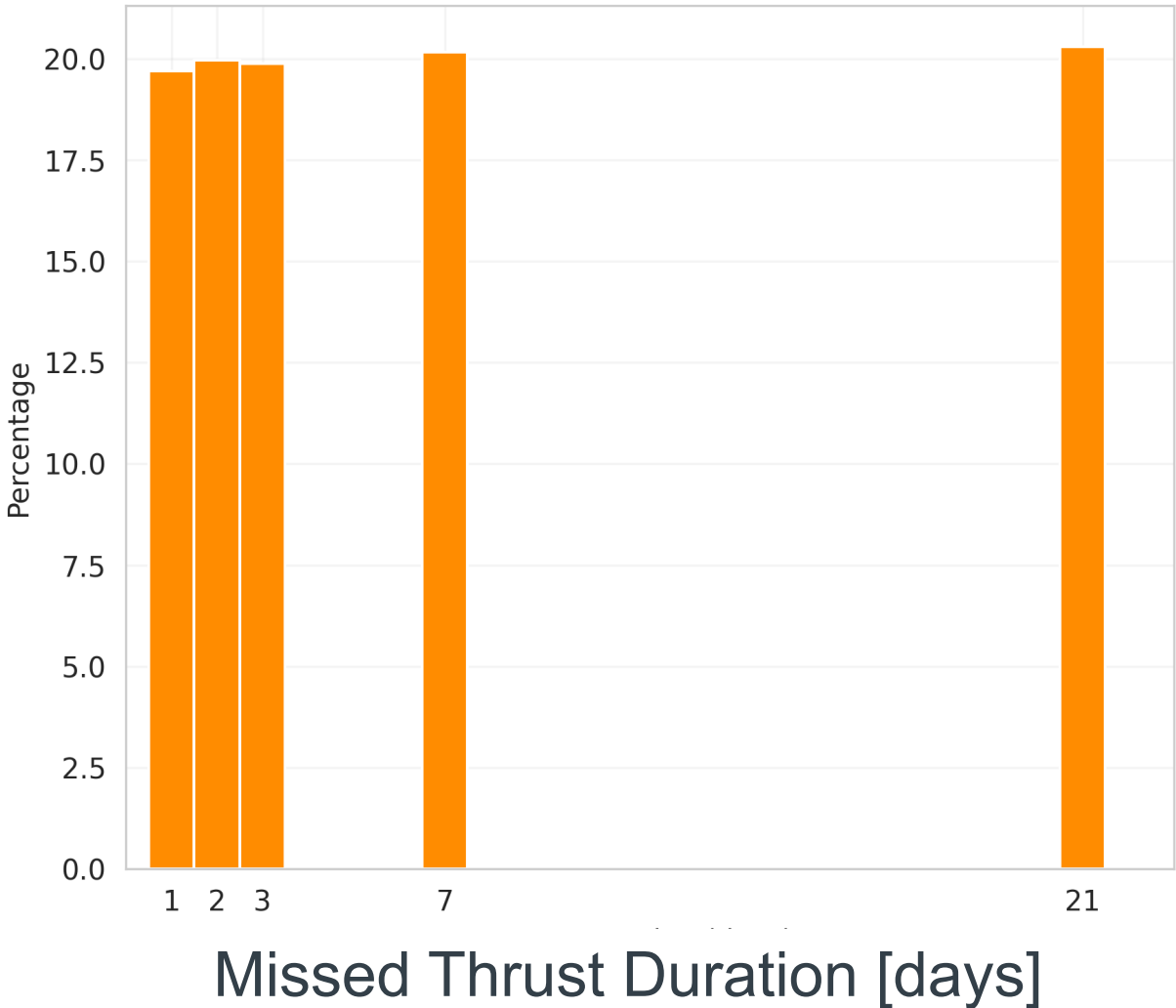
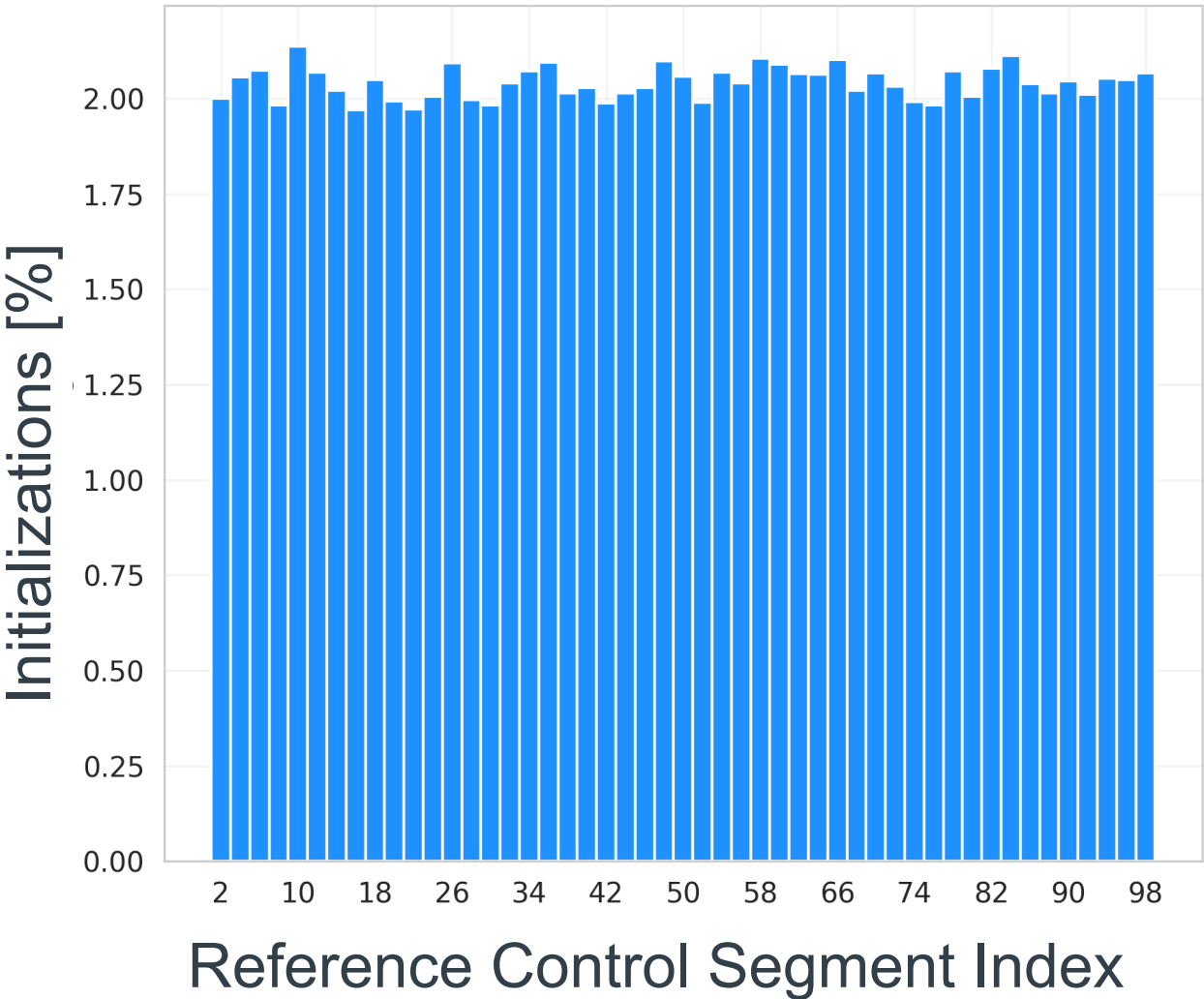
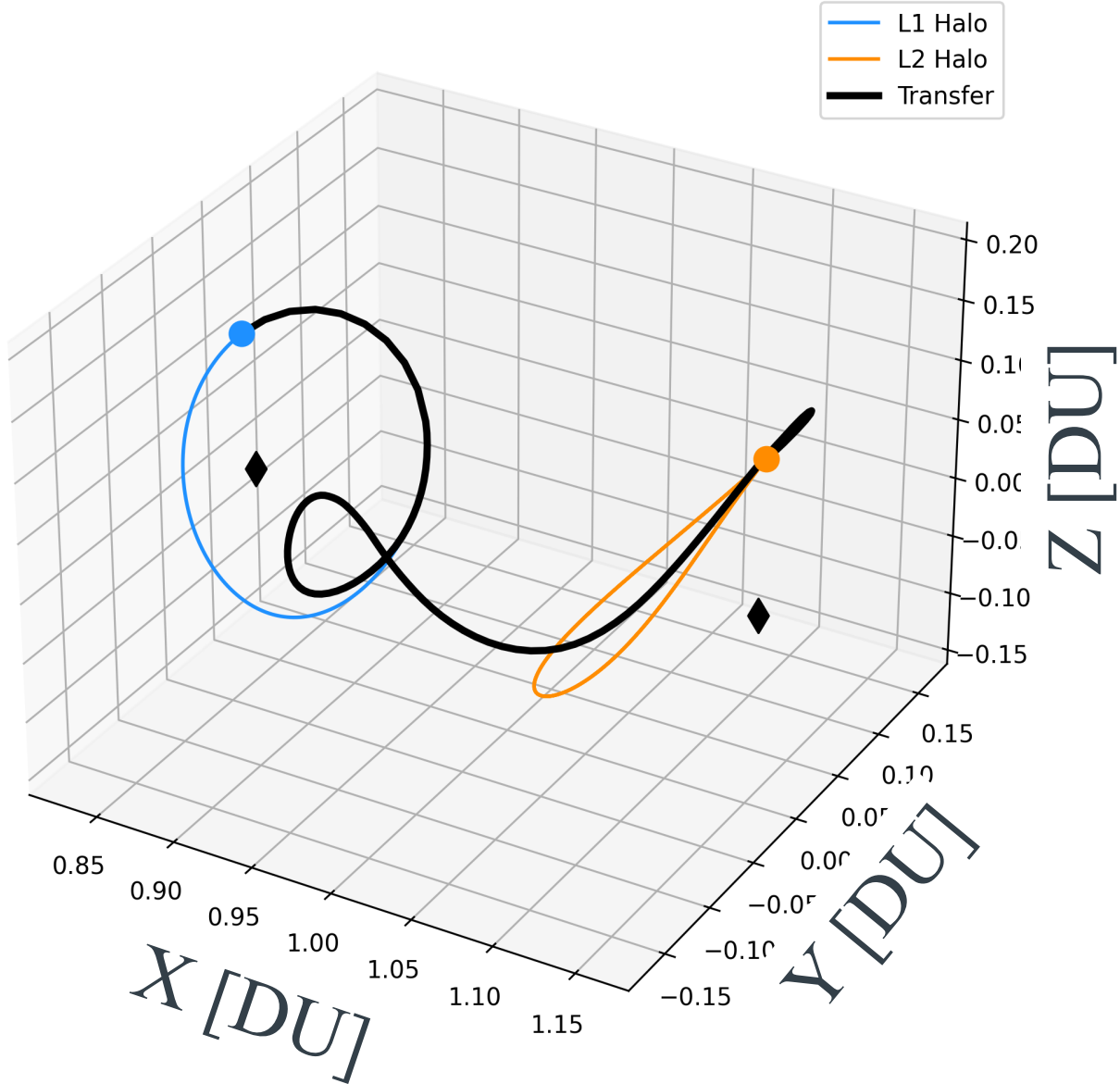


Using analytic derivatives not only helps us uncover more solutions, it helps us do so quickly.



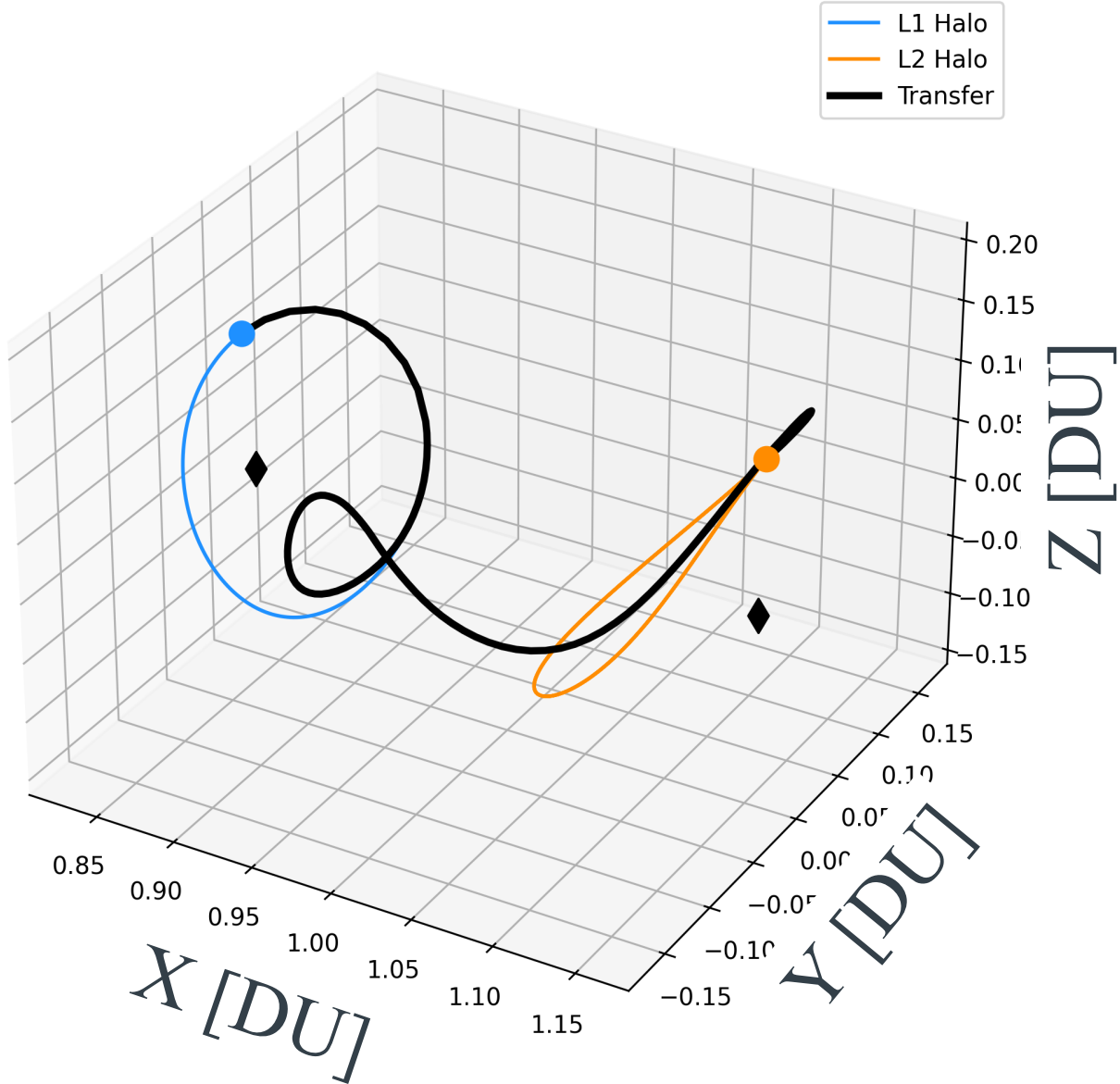
Earth-Moon L1 Halo to L2 Halo Low-Thrust Transfer

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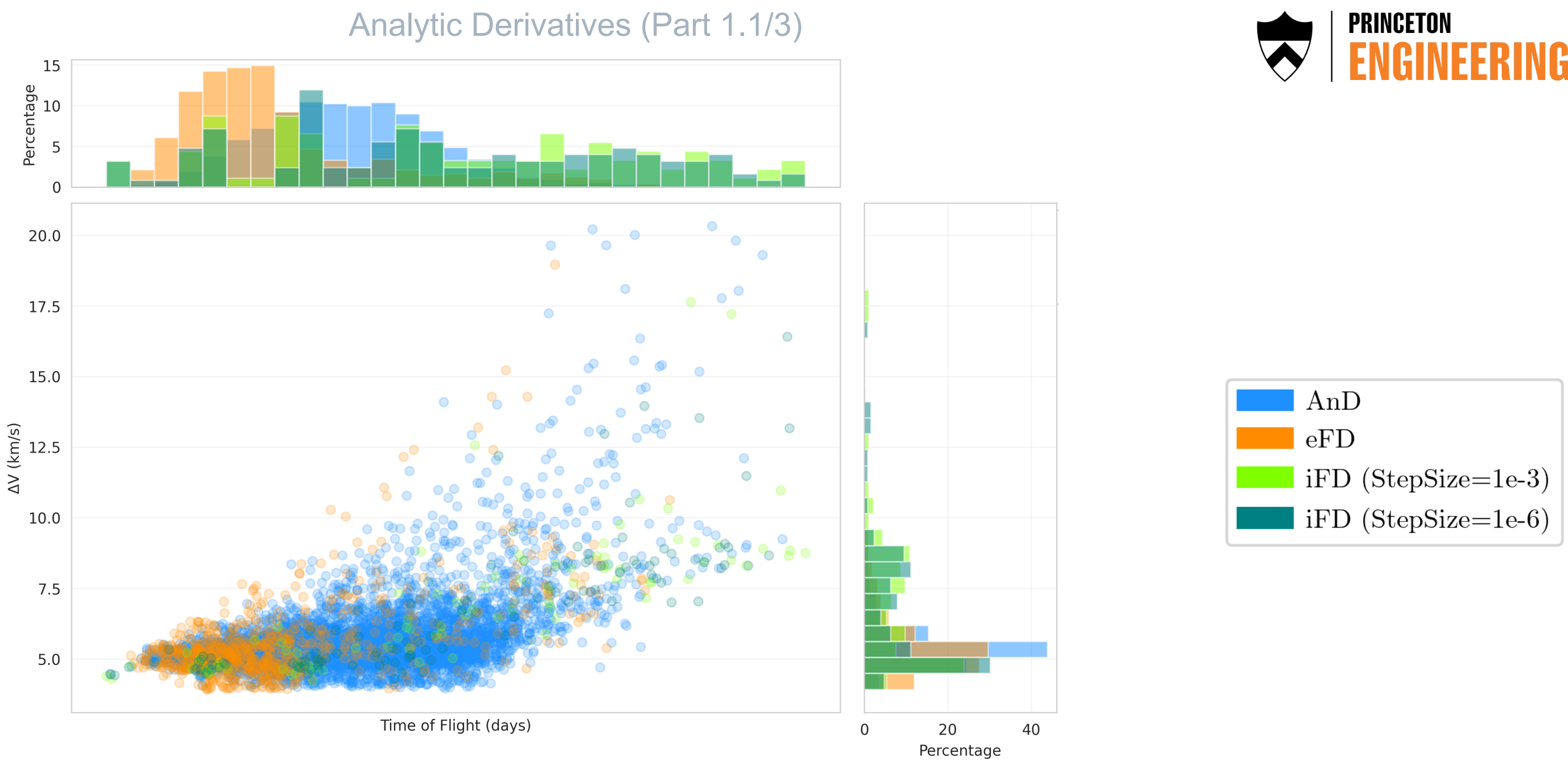
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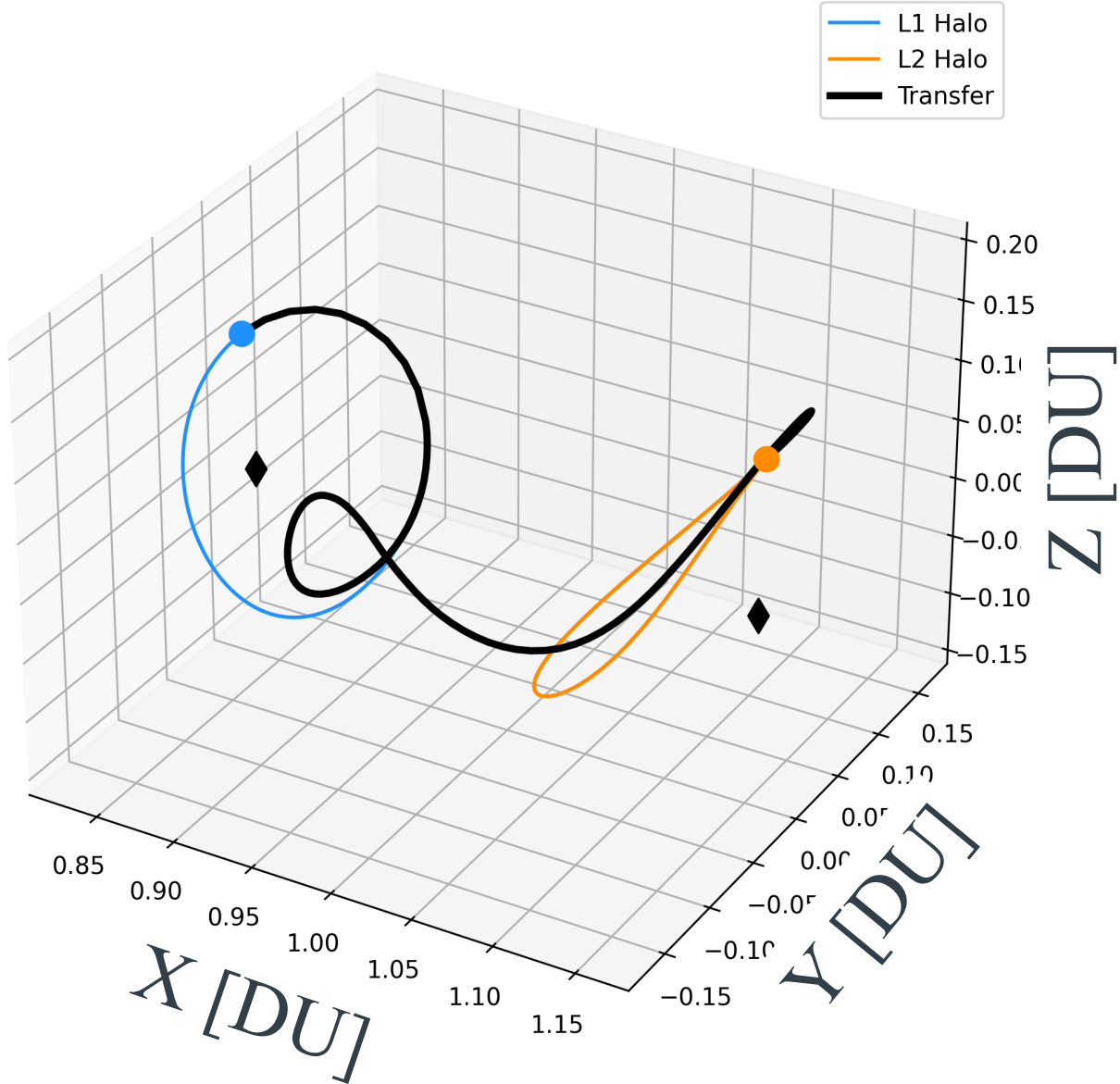


Earth-Moon L1 Halo to L2 Halo Low-Thrust Transfer

Number of Realizations = 1



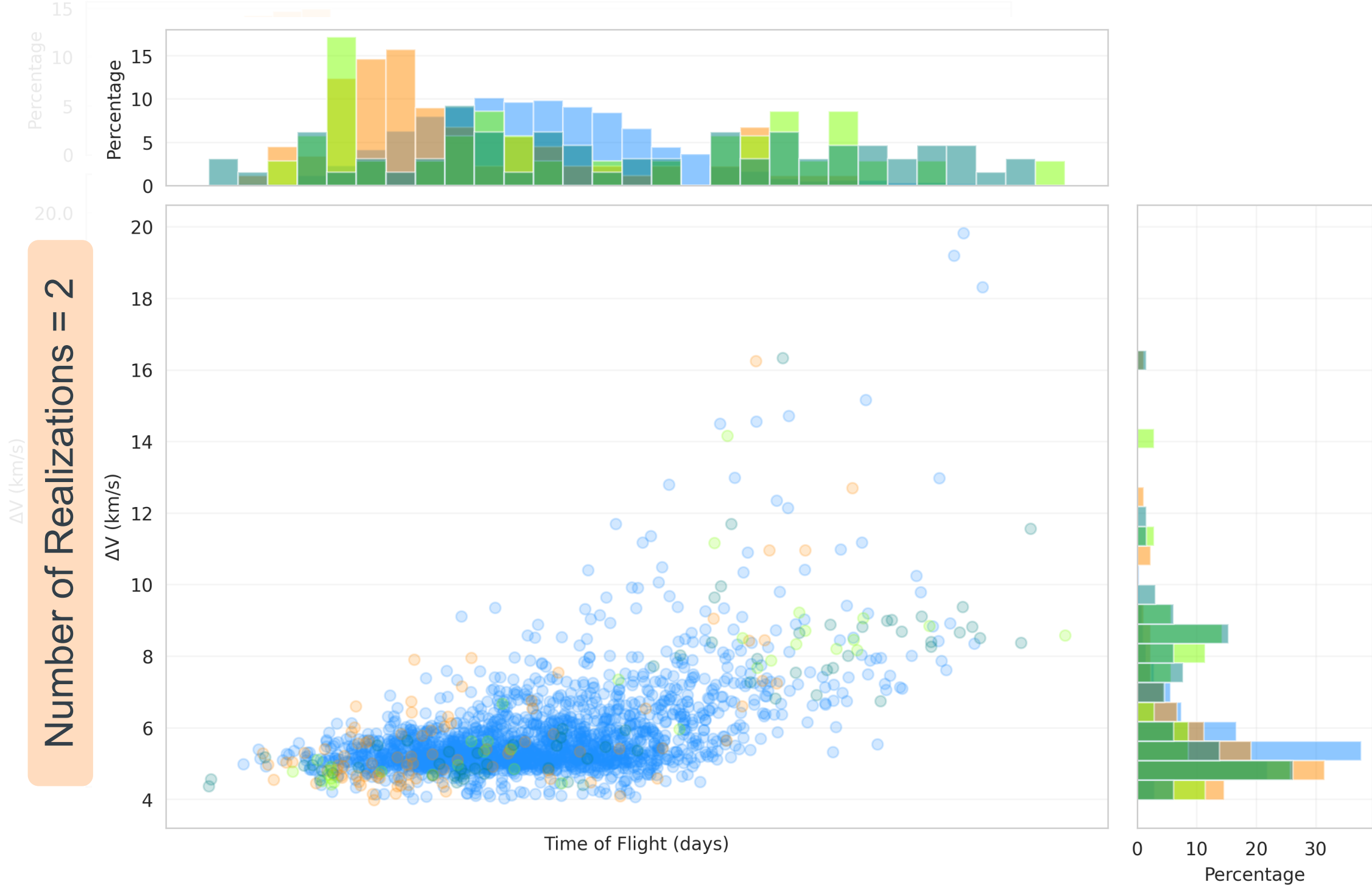
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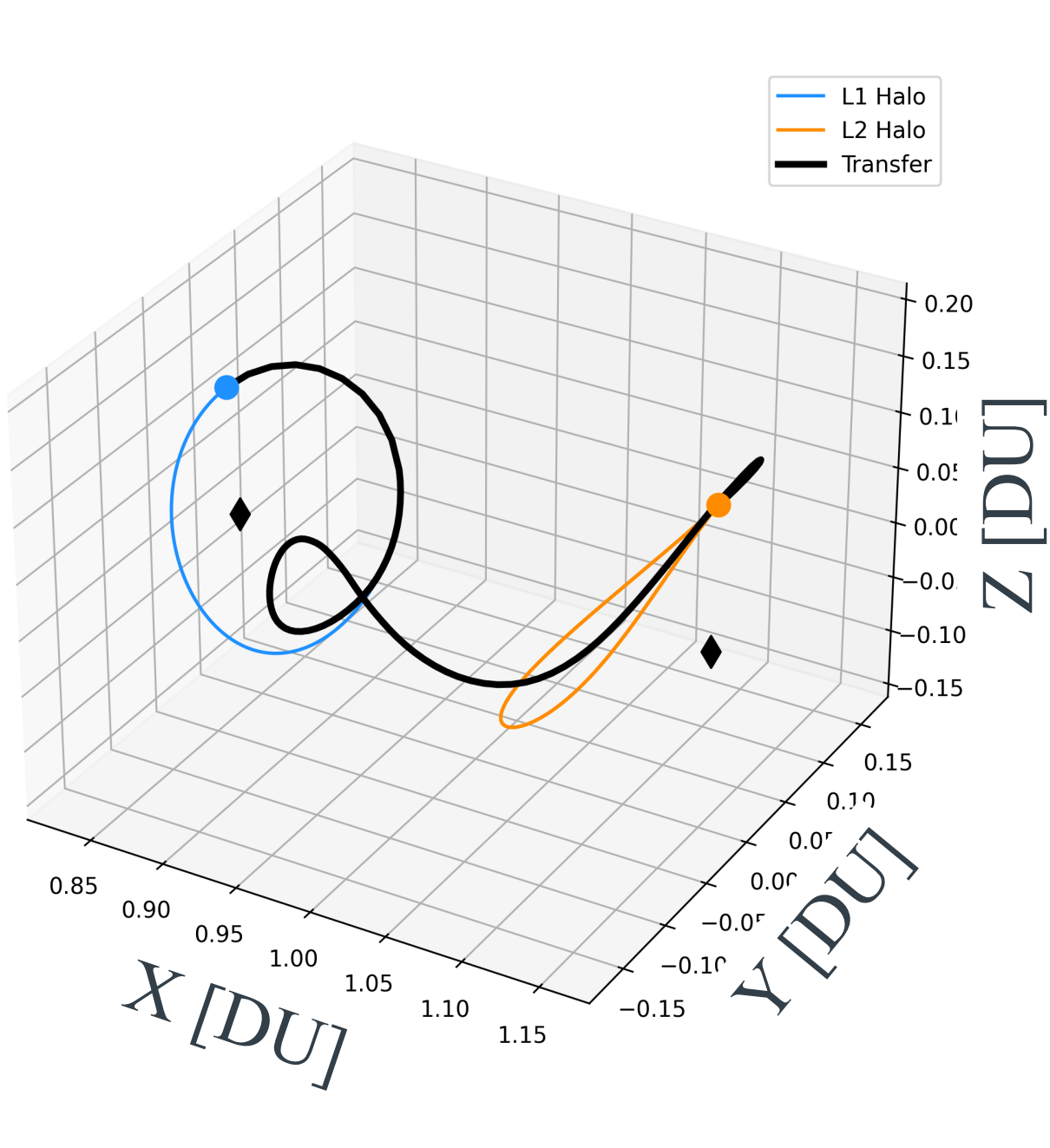
Earth-Moon L1 Halo to L2 Halo Low-Thrust Transfer

Number of Realizations = 1

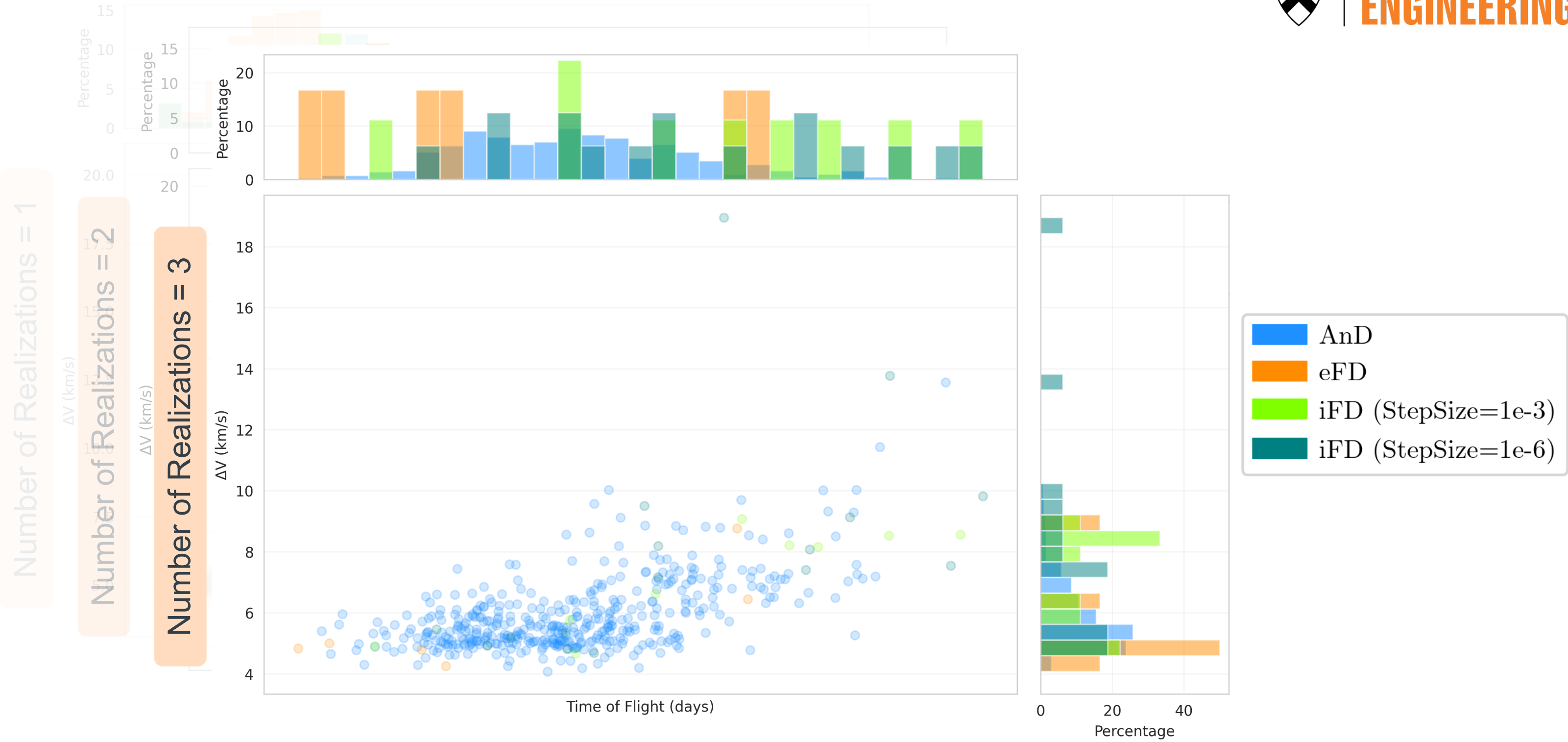
Number of Realizations = 2



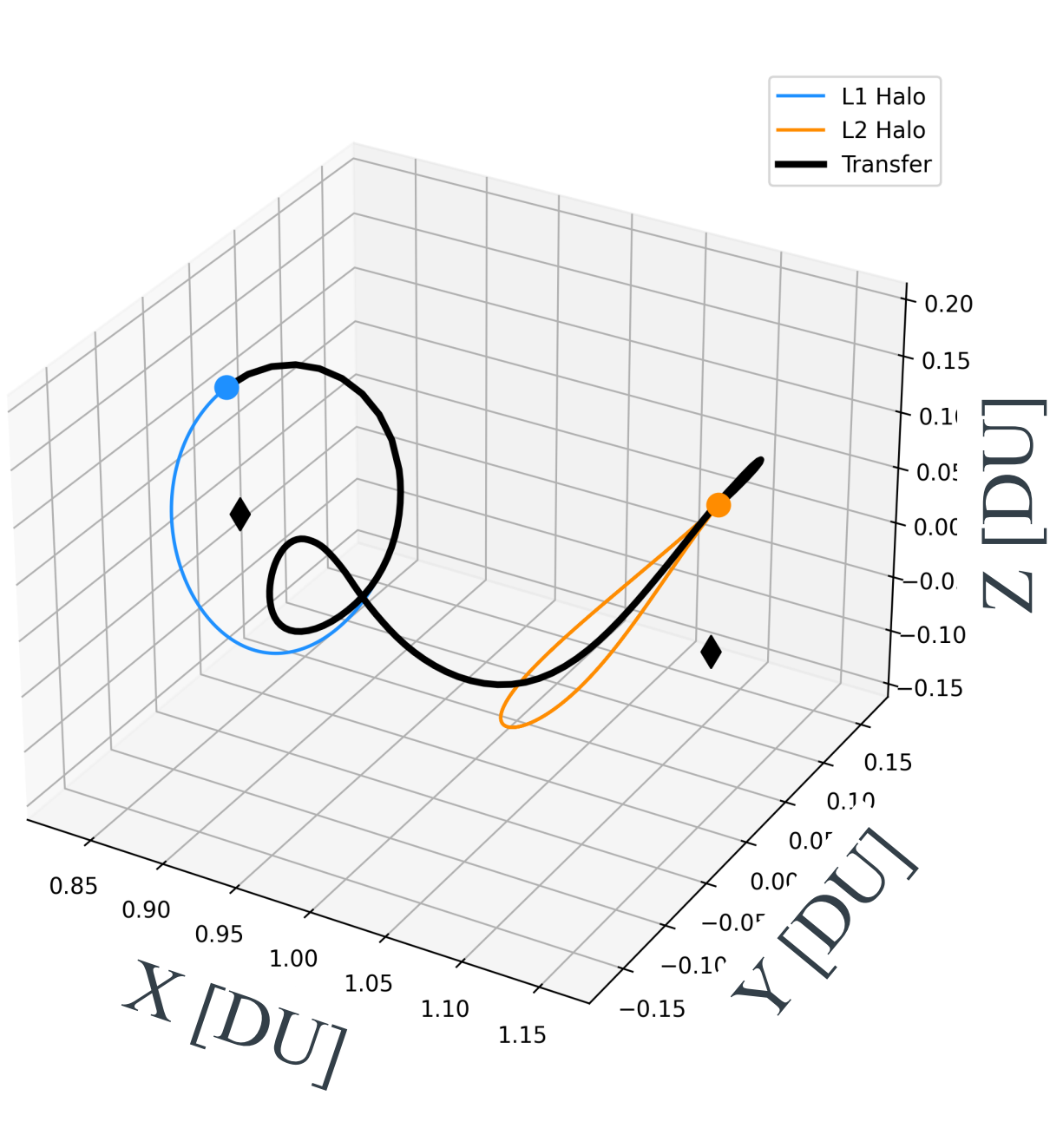
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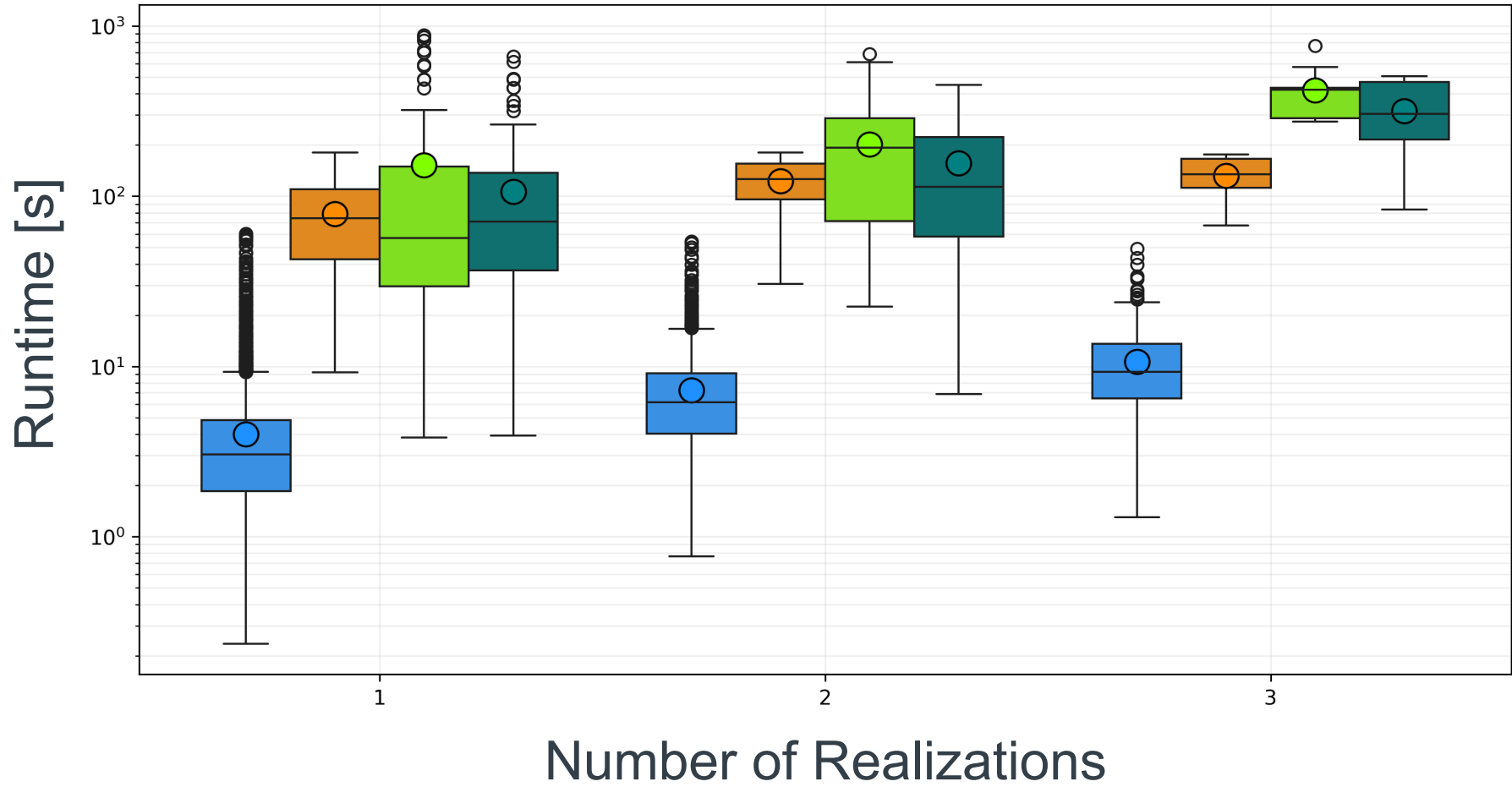
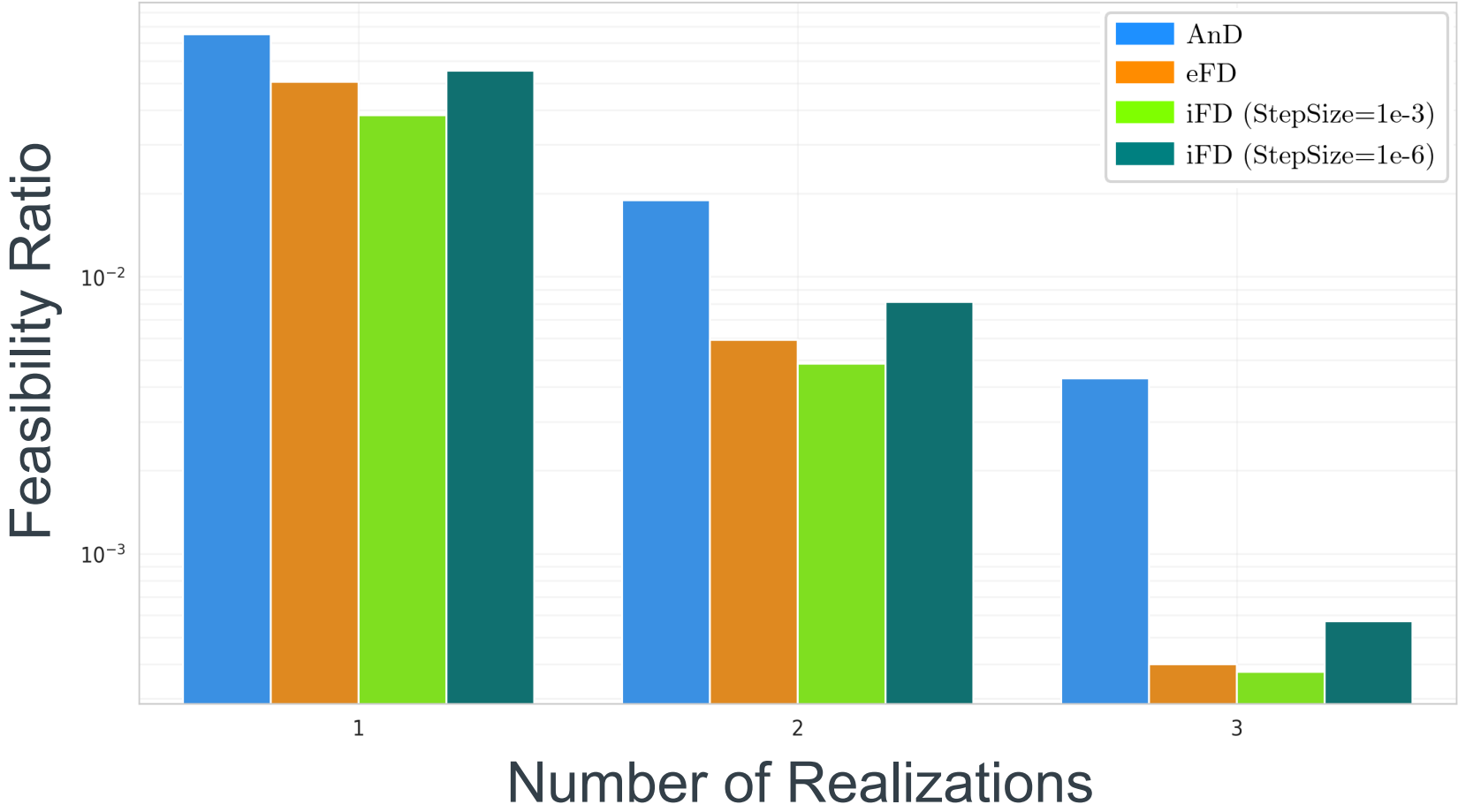
Earth-Moon L1 Halo to L2 Halo Low-Thrust Transfer



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Earth-Moon L1 Halo to L2 Halo Low-Thrust Transfer





Part 1.2: Initial Guess Generation Strategies For Missed Thrust Design

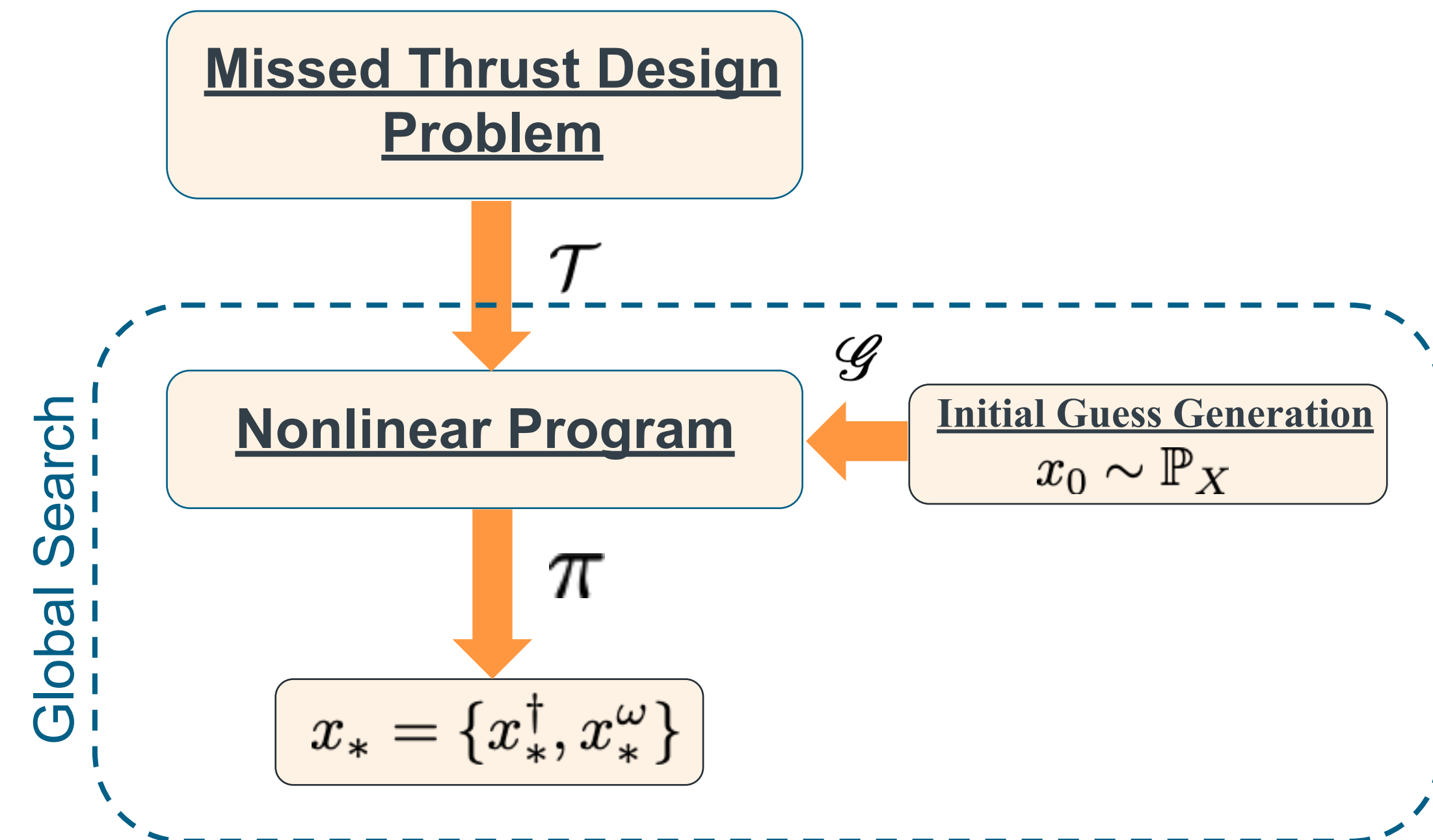
Amlan Sinha and Ryne Beeson, "Initial Guess Generation for Low-Thrust Trajectory Design with Robustness to Missed-Thrust-Events", Journal of Guidance, Control, and Dynamics, Vol. 48, No. 12 (2025), pp. 2831-2848, doi: doi/abs/10.2514/1.G009050.



The global search problem becomes more difficult when we incorporate robustness against missed thrust events

Preliminary mission design requires global exploration of the solution space

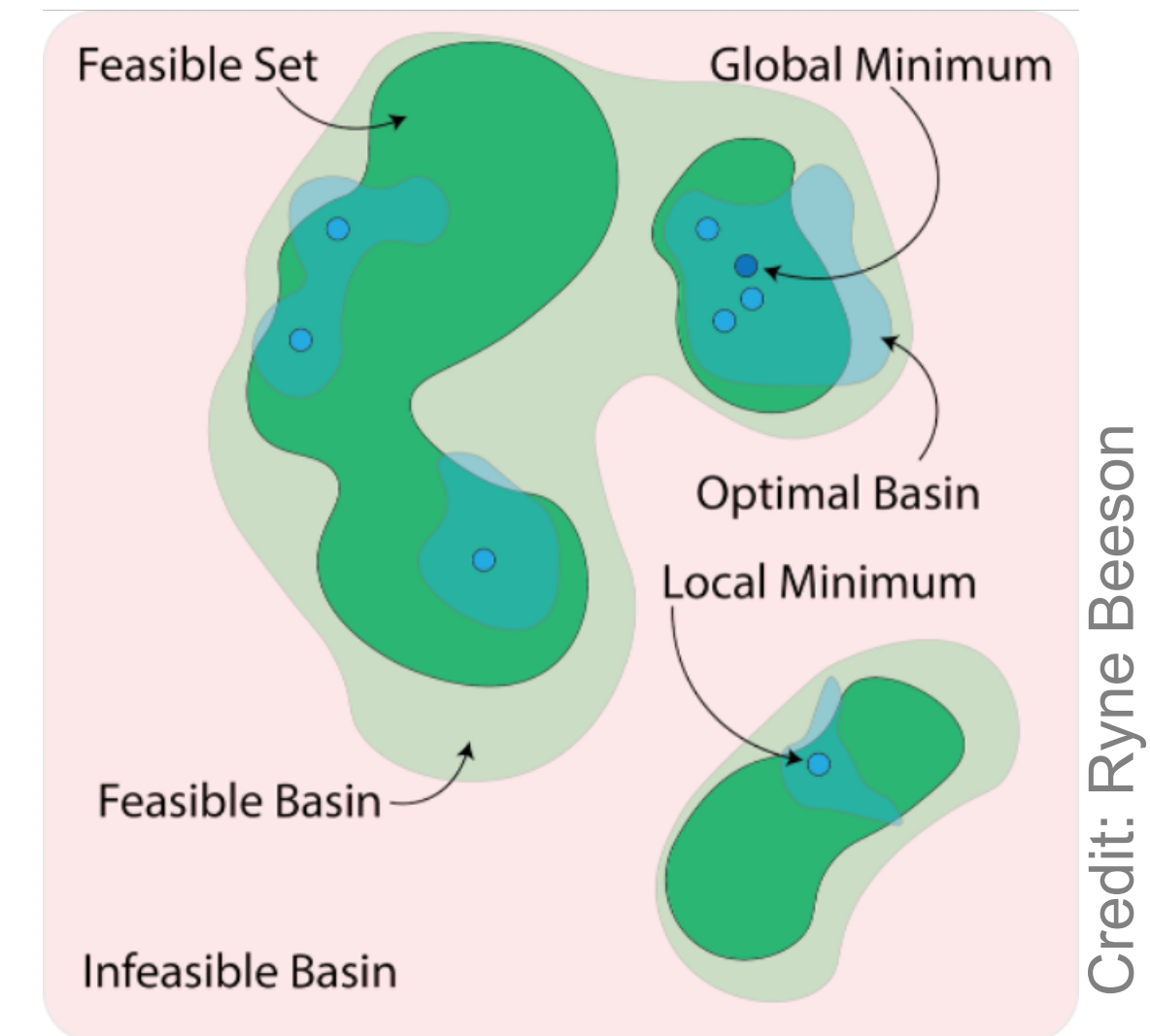
- The solution landscape is **highly structured**
- Incorporating robustness **increases**
 - Problem dimensionality
 - Topological complexity
- As **number of realizations** increases,
 - Problem complexity increases,
 - Nominal basins can deform,
 - Previous local minima may no longer be feasible



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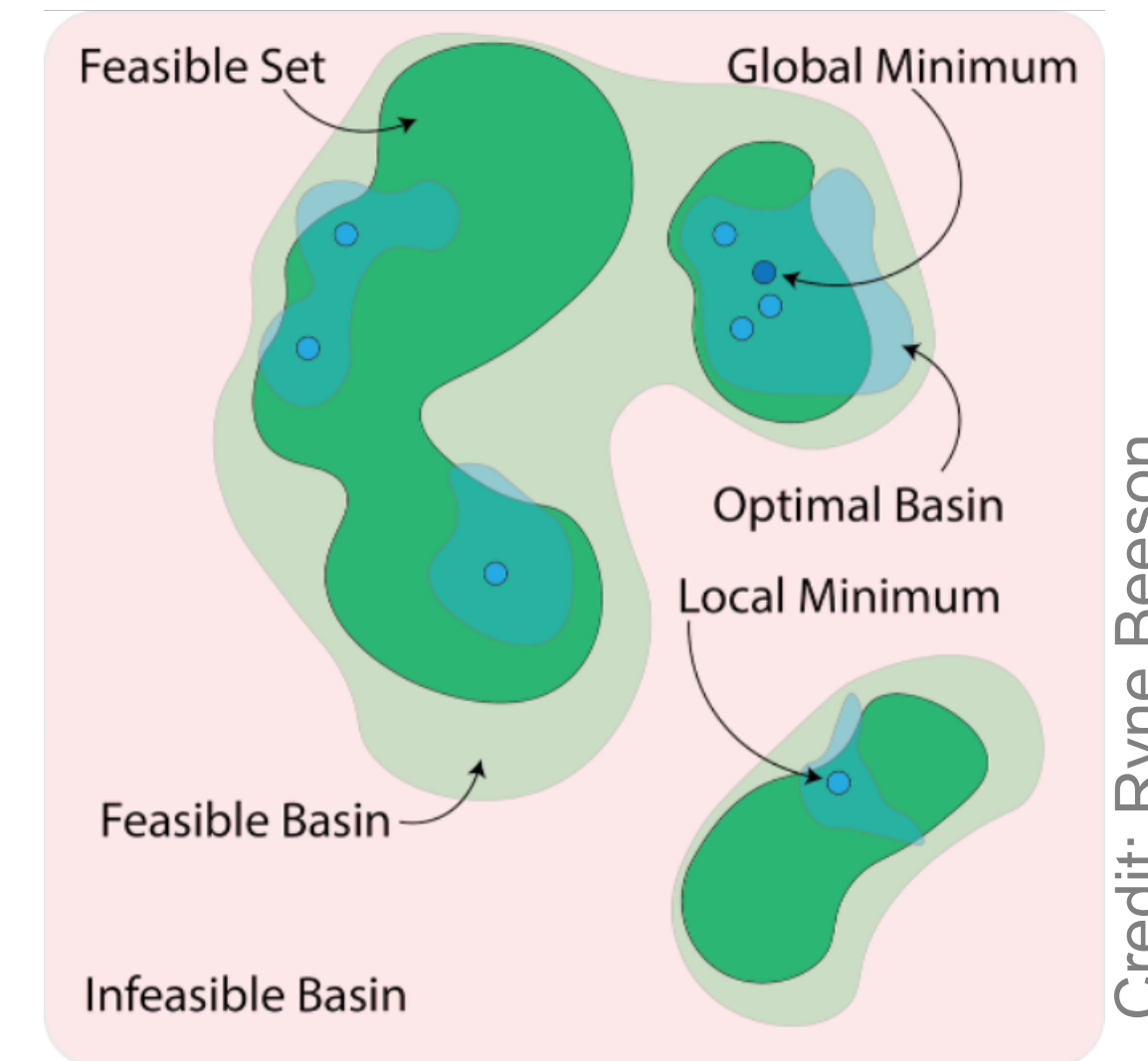
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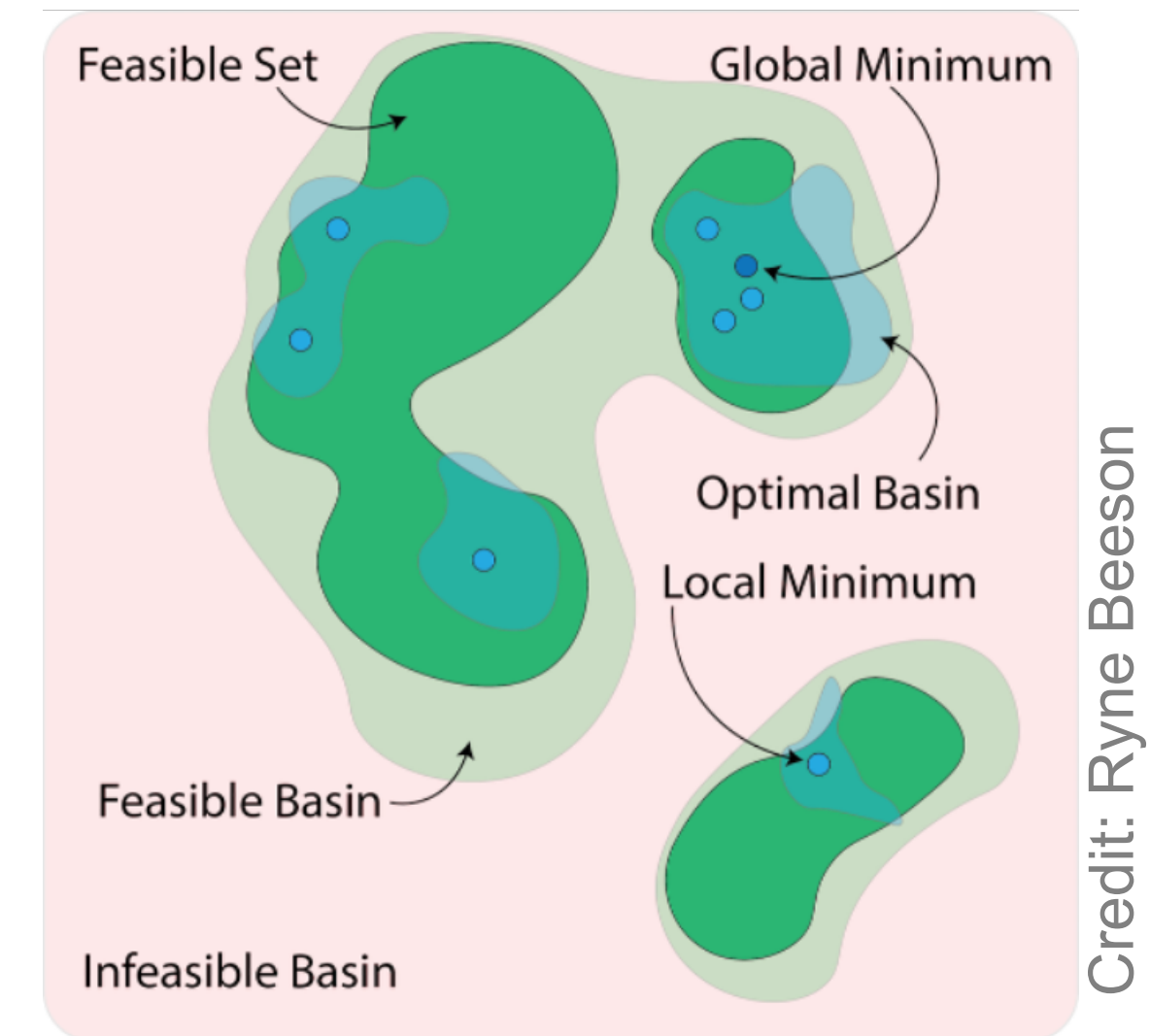


Non-Robust

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Non-Robust

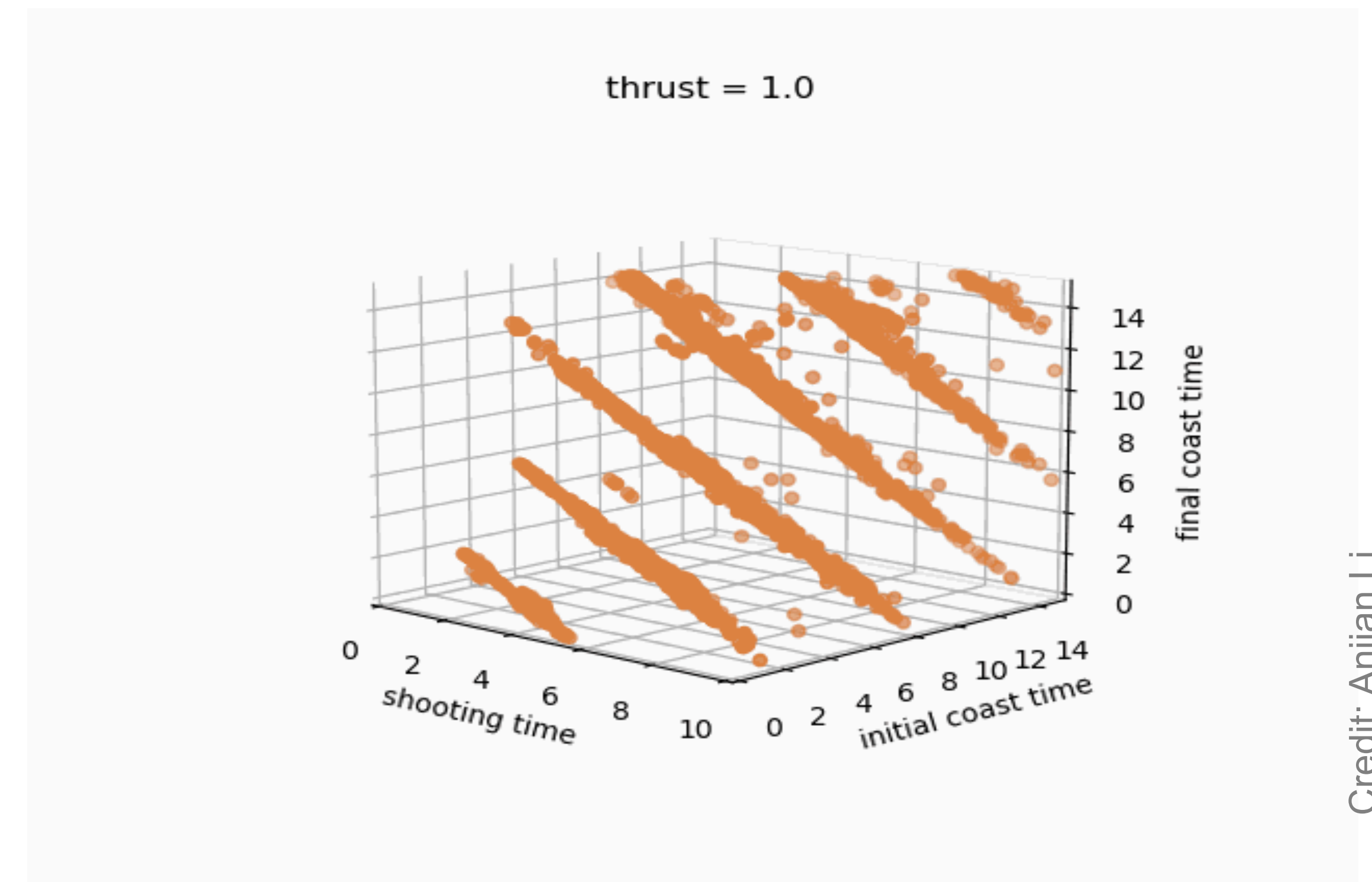
Robust

The non-robust low-thrust solution space exhibits hyperplane structure

Recall: GEO to Earth-Moon L1 Halo Transfer

Recall

$$\mathbf{x} = (\tau_i, \tau_s, \tau_f, \mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_K, m_f)$$



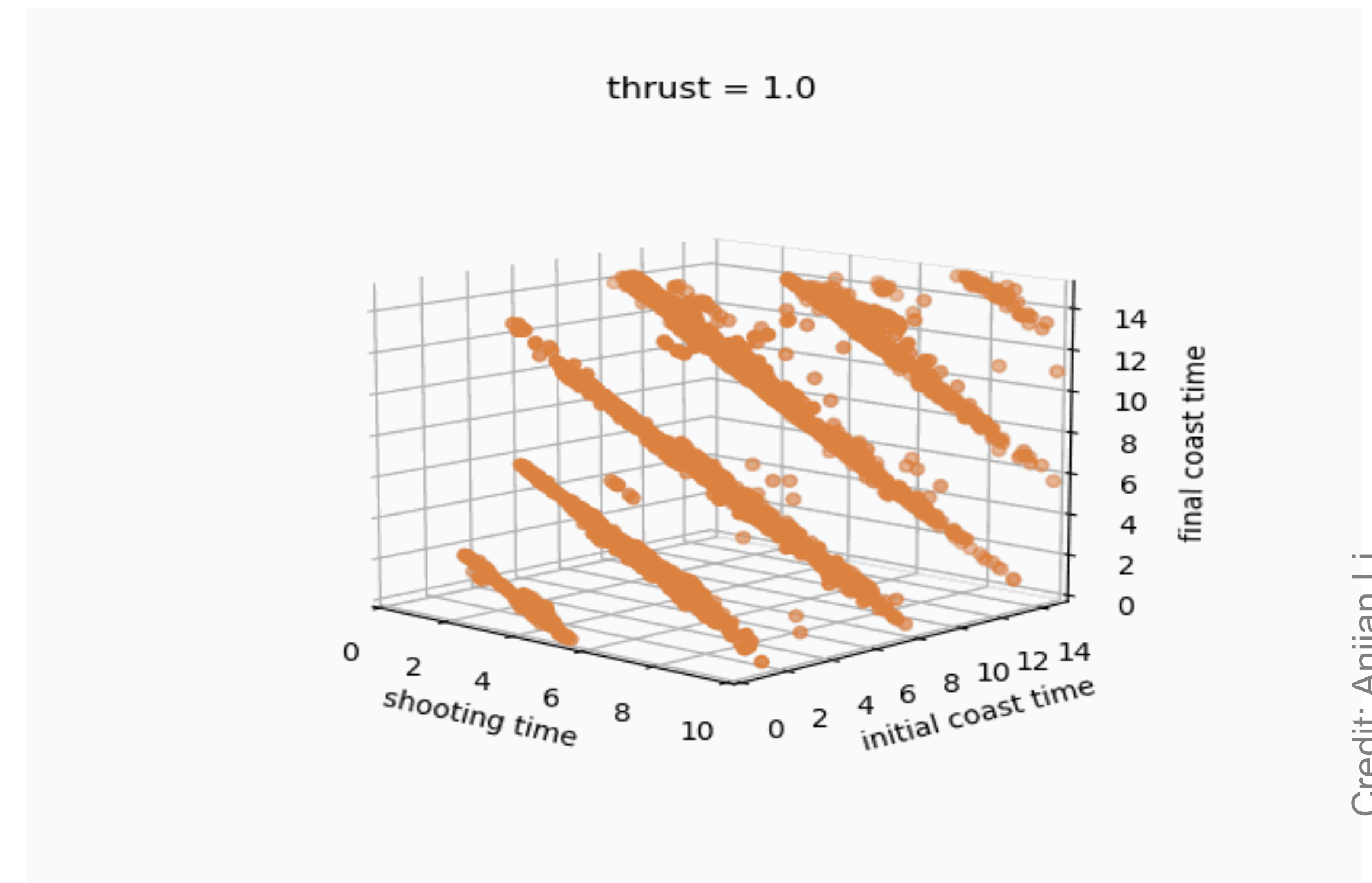
Credit: Anjian Li

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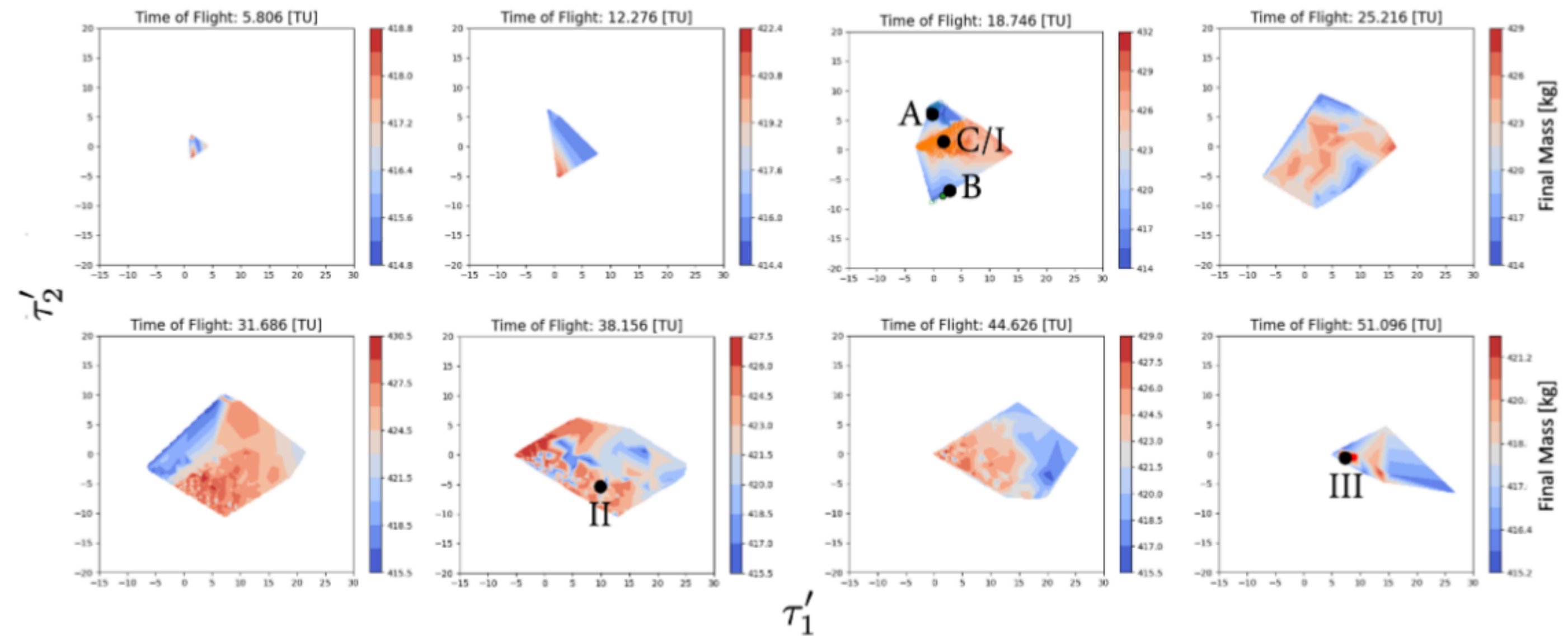
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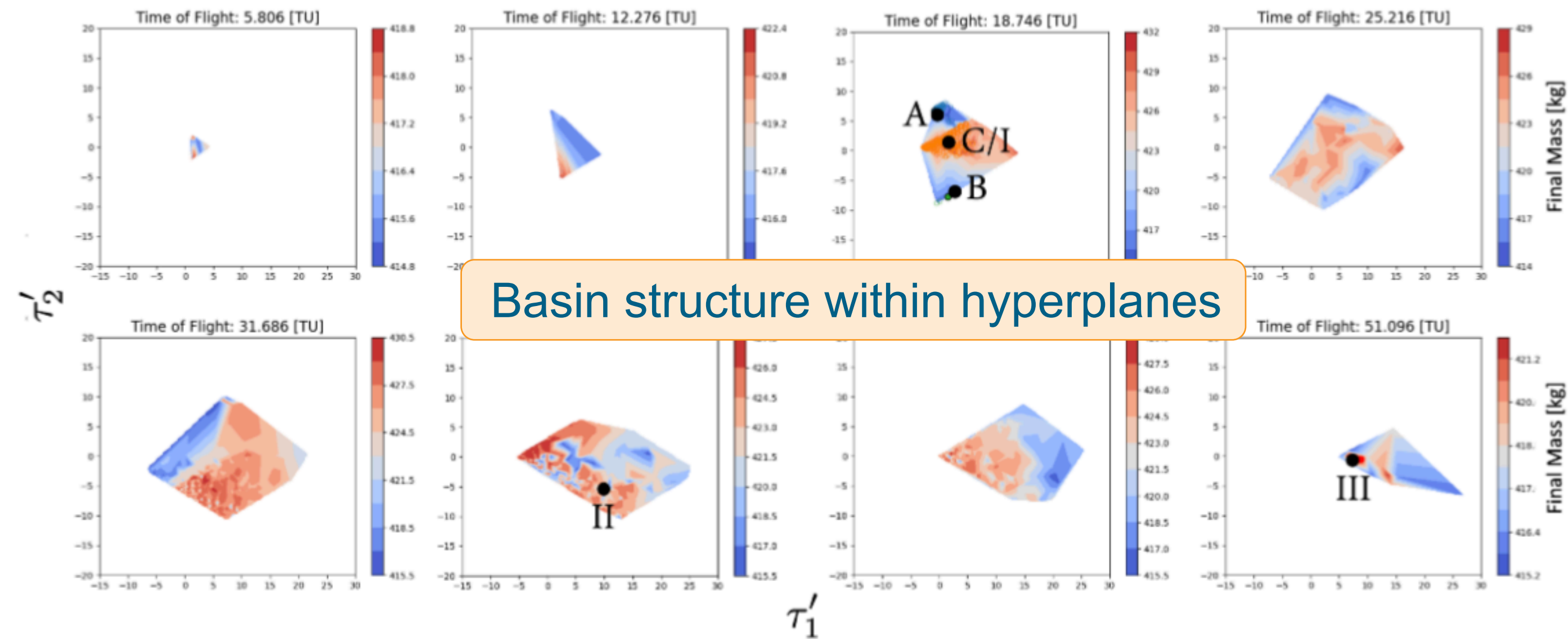
Credit: Anjian Li

The solution landscape for low-thrust problems has macro and micro structures across and within hyperplanes



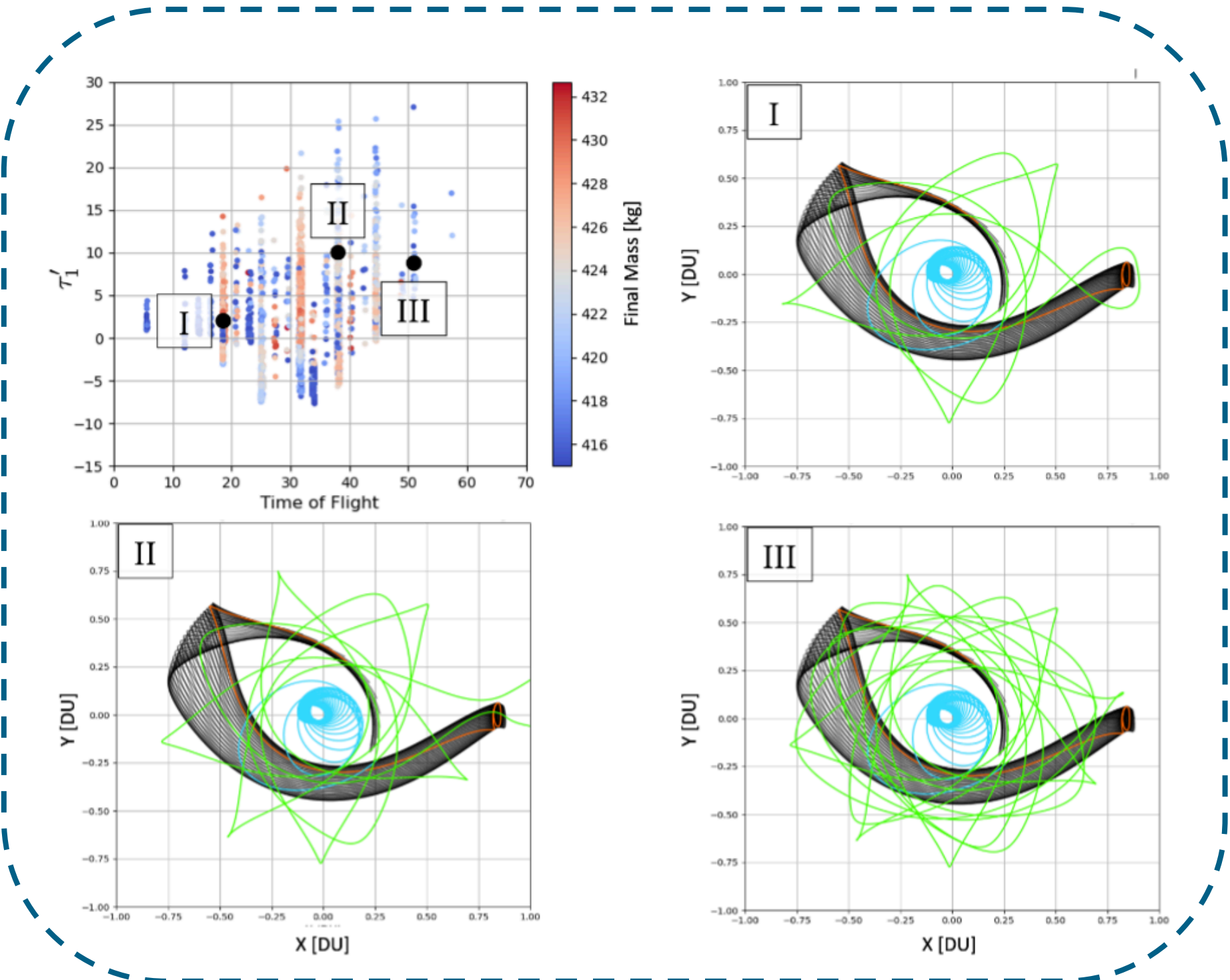
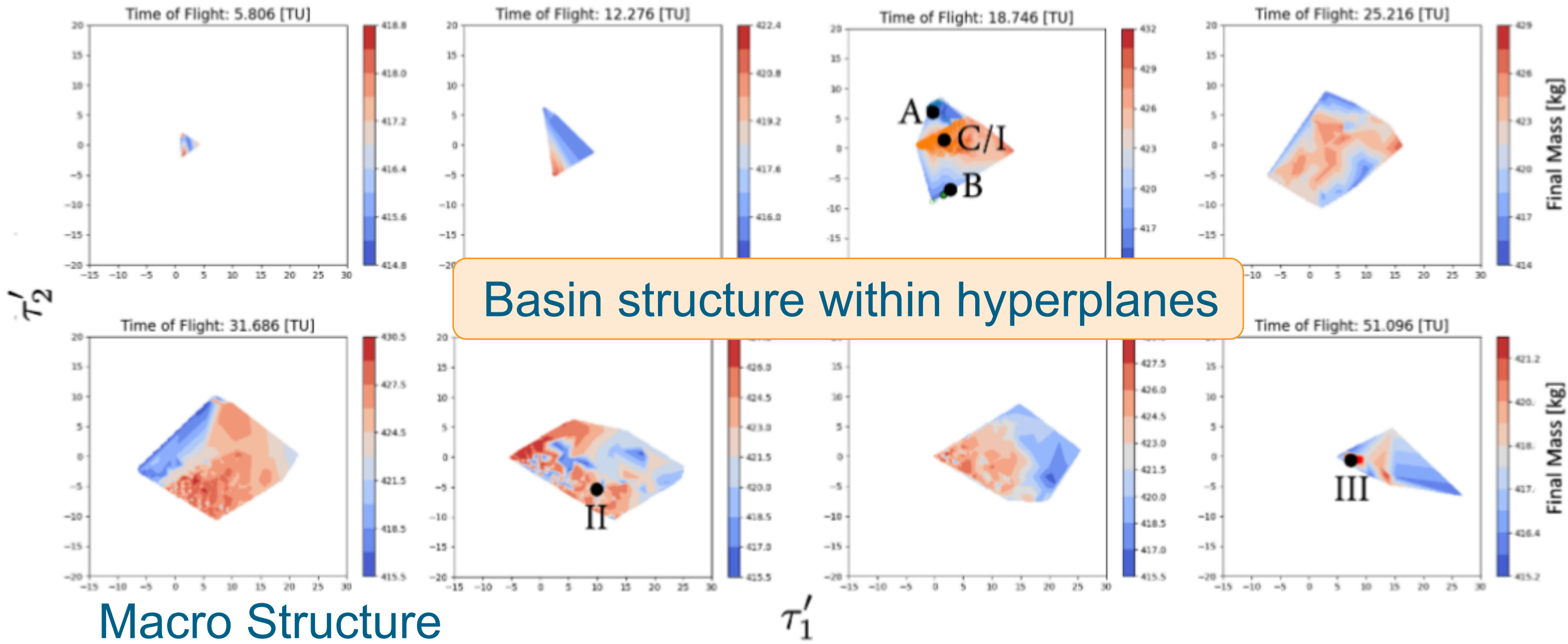
GEO to Earth-Moon L1 Halo Low-Thrust Transfer

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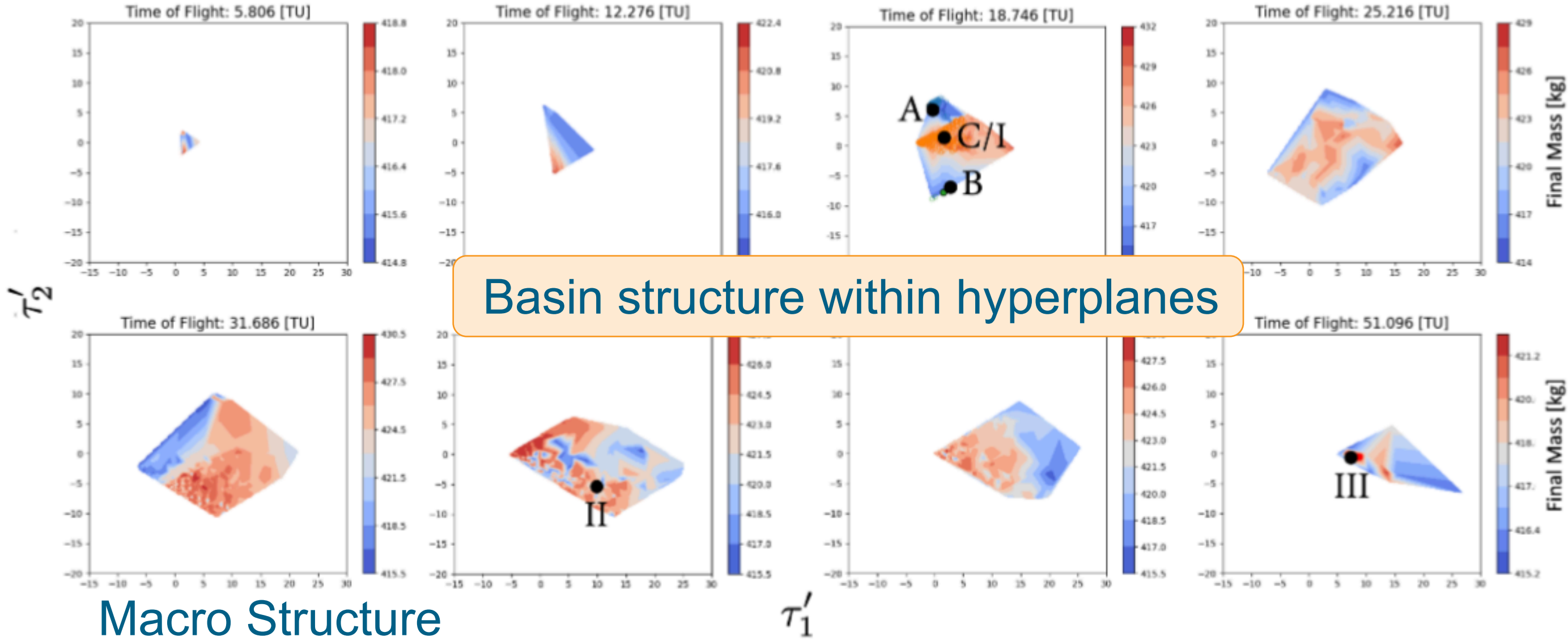
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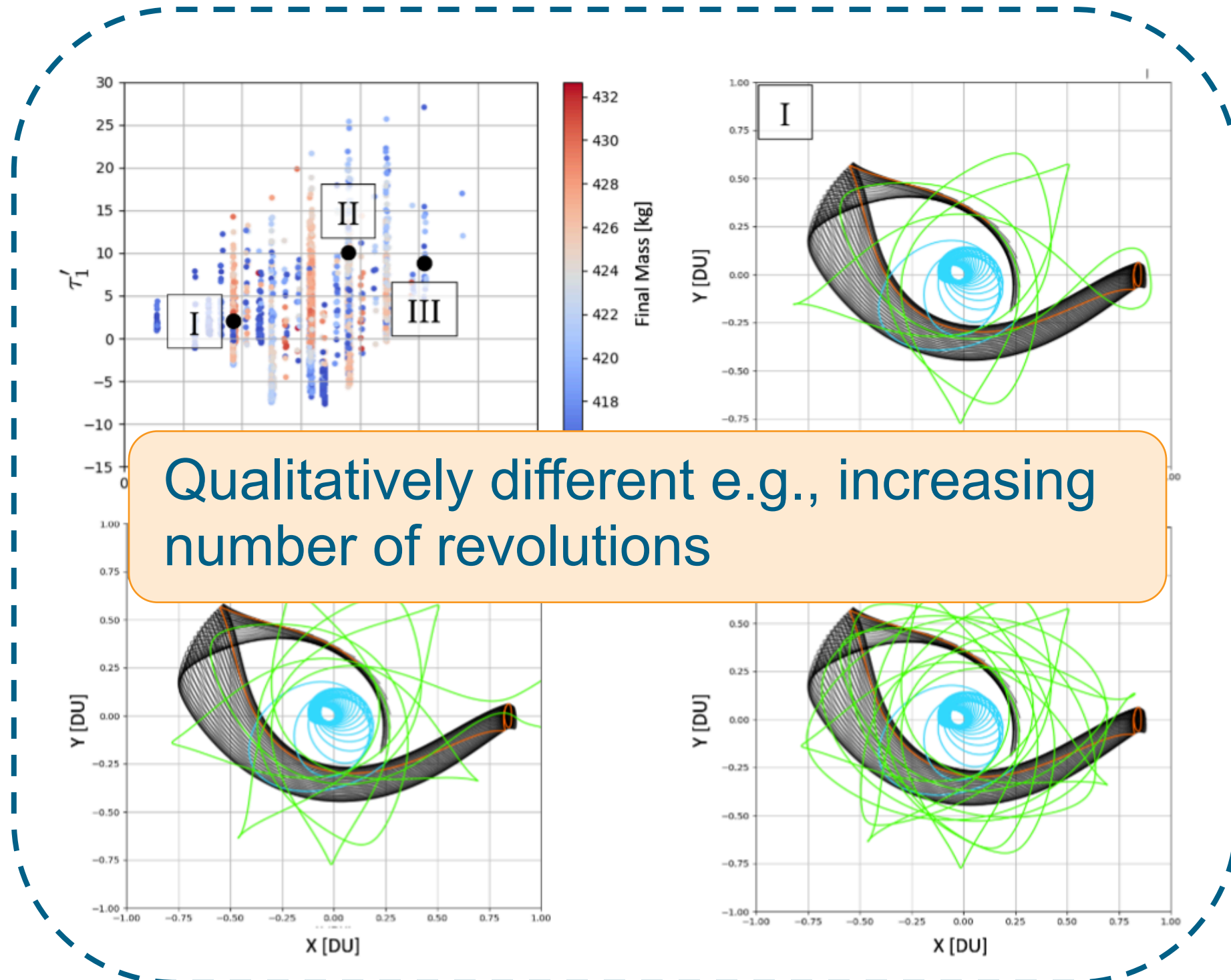


GEO to Earth-Moon L1 Halo Low-Thrust Transfer

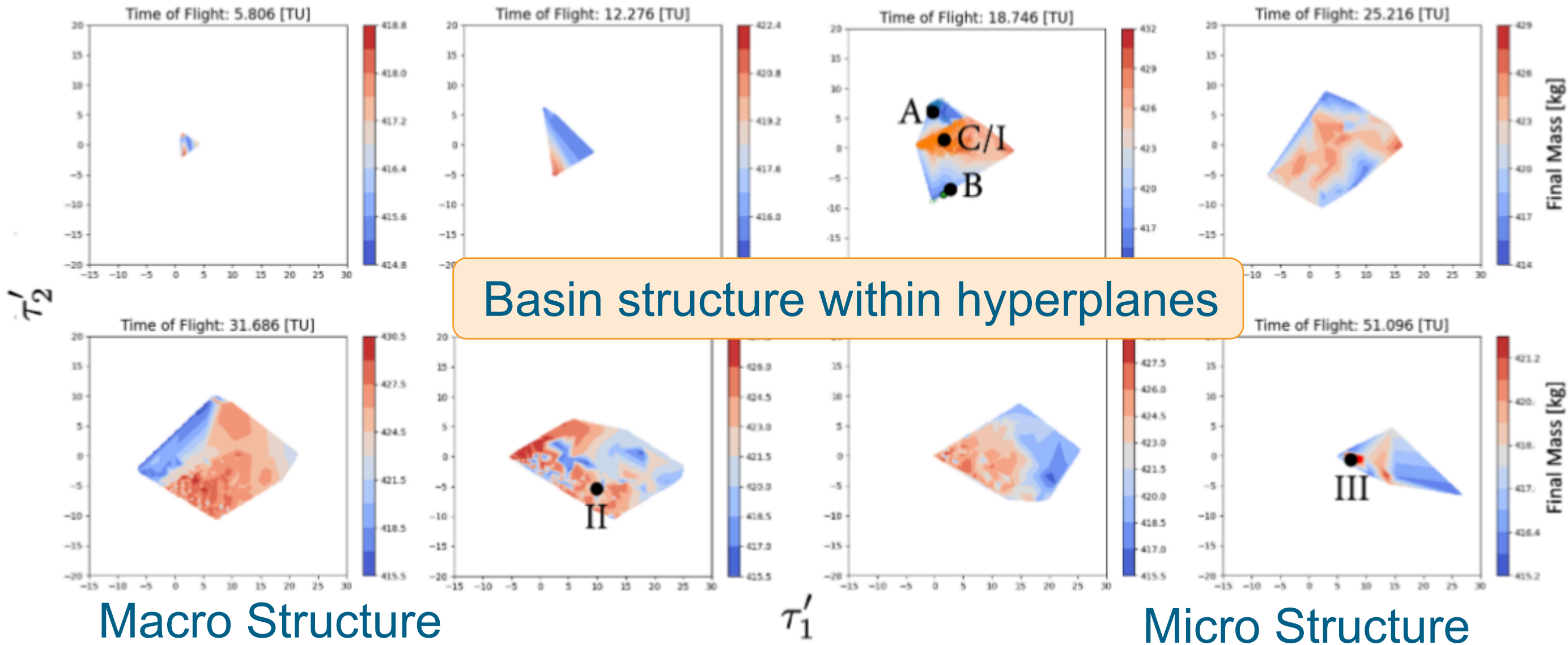
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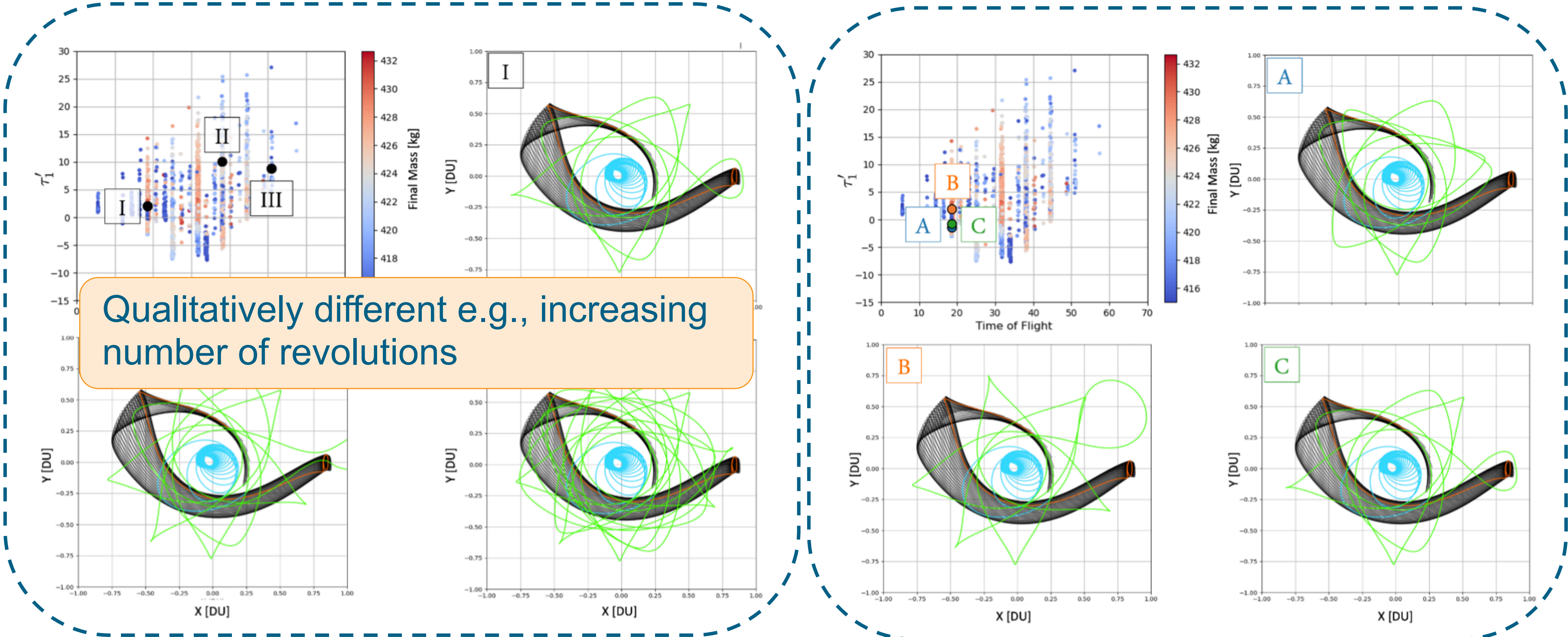


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Macro Structure

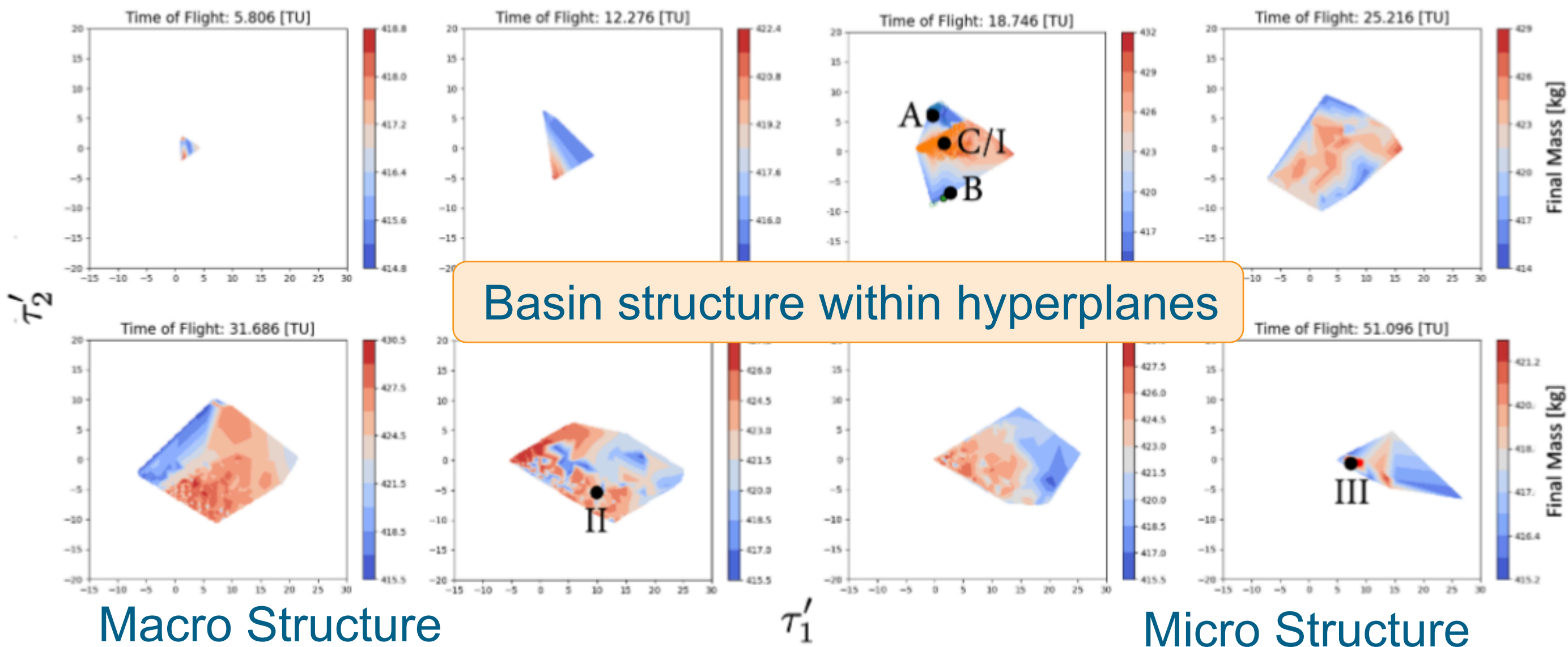
Micro Structure



GEO to Earth-Moon L1 Halo Low-Thrust Transfer

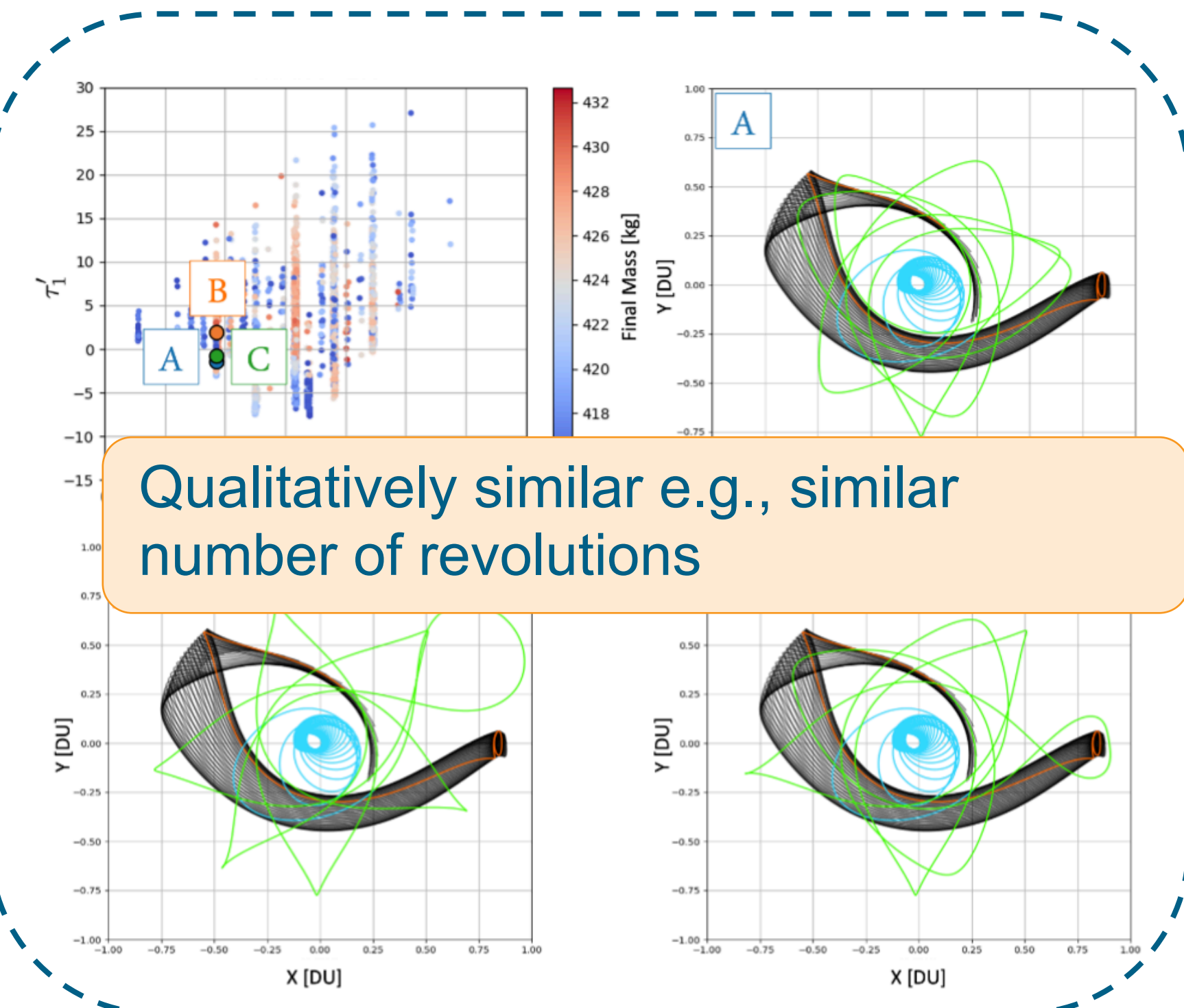
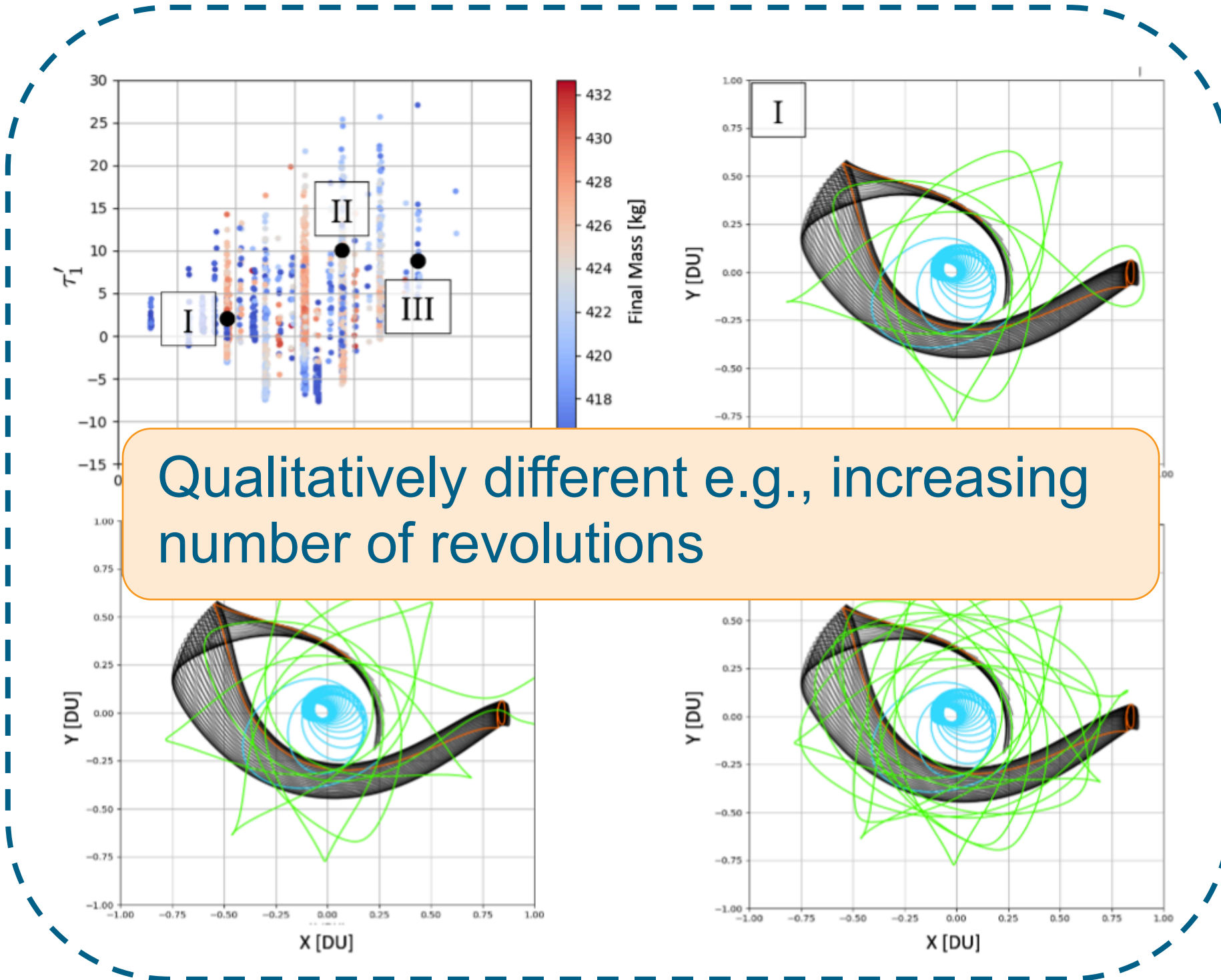
Beeson et al. (2024)

The solution landscape for low-thrust problems has macro and micro structures across and within hyperplanes



Macro Structure

Micro Structure



GEO to Earth-Moon L1 Halo Low-Thrust Transfer

A conditional global search seeks to exploit this structure to generate good quality initial guesses for robust problems

We explore two global search strategies: non-conditional vs. conditional

- Let P_k denote the robust problem with k realizations
 - $k = 0$ denotes the non-robust problem
- A **non-conditional** strategy samples x directly from a **fixed distribution** $\mathbb{P}(X_k)$ with global support e.g., Uniform
- A **conditional** strategy maps solutions from a simpler problem $P_{k'}$ into the search space for P_k , where $k' < k$ using a **conditional distribution** $\mathbb{P}(X_k | X_{k'})$

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$$\mathcal{S}(k | k') \equiv \pi \circ \mathcal{M}_{k'}^k$$

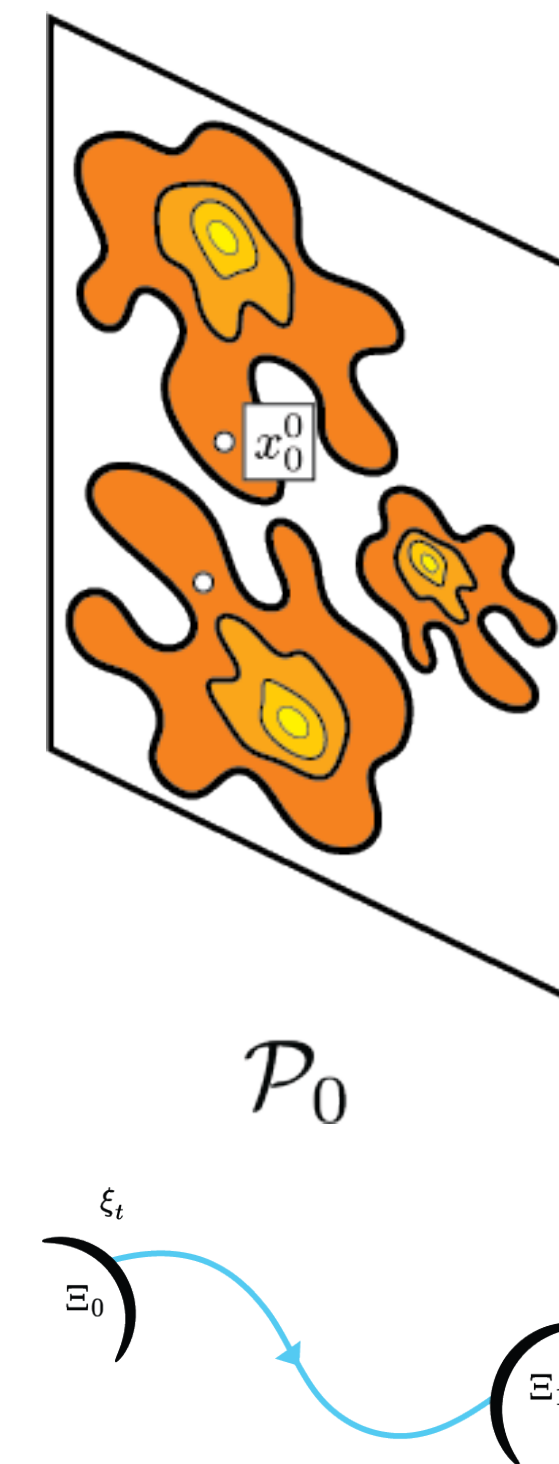
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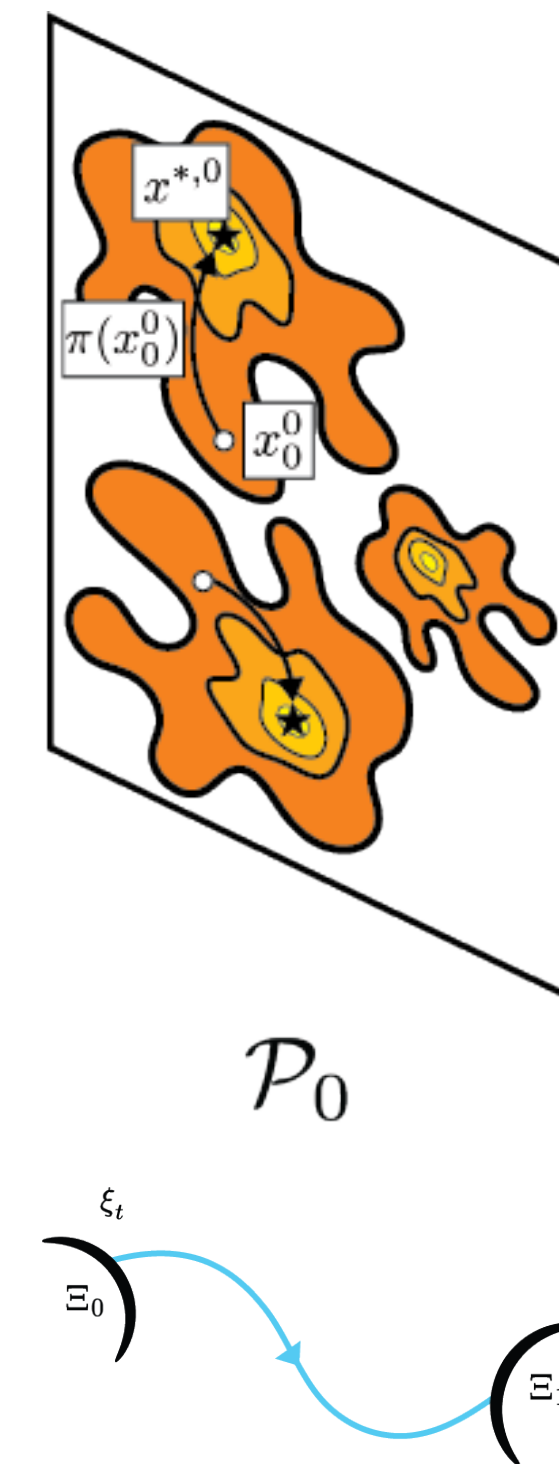
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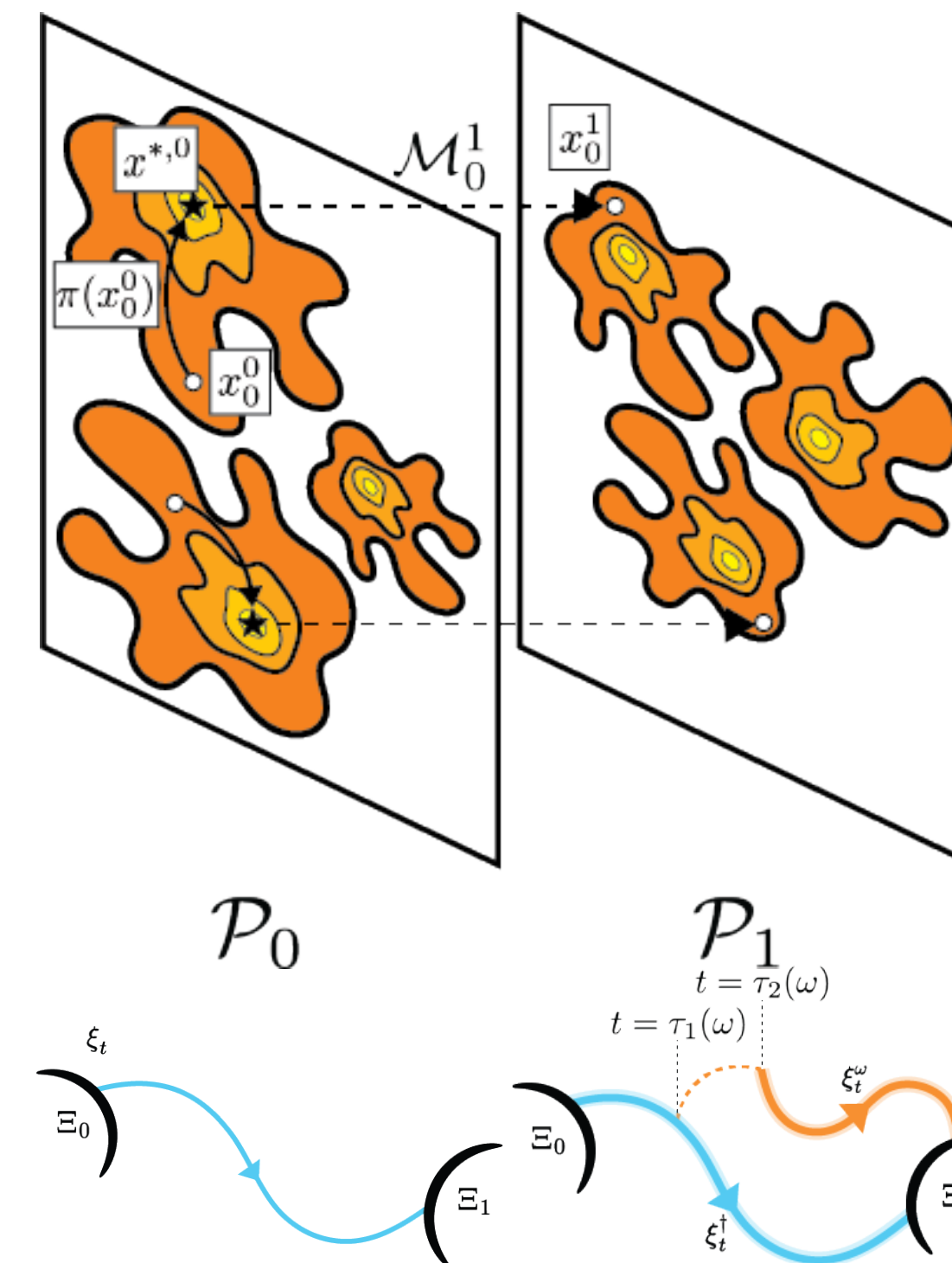
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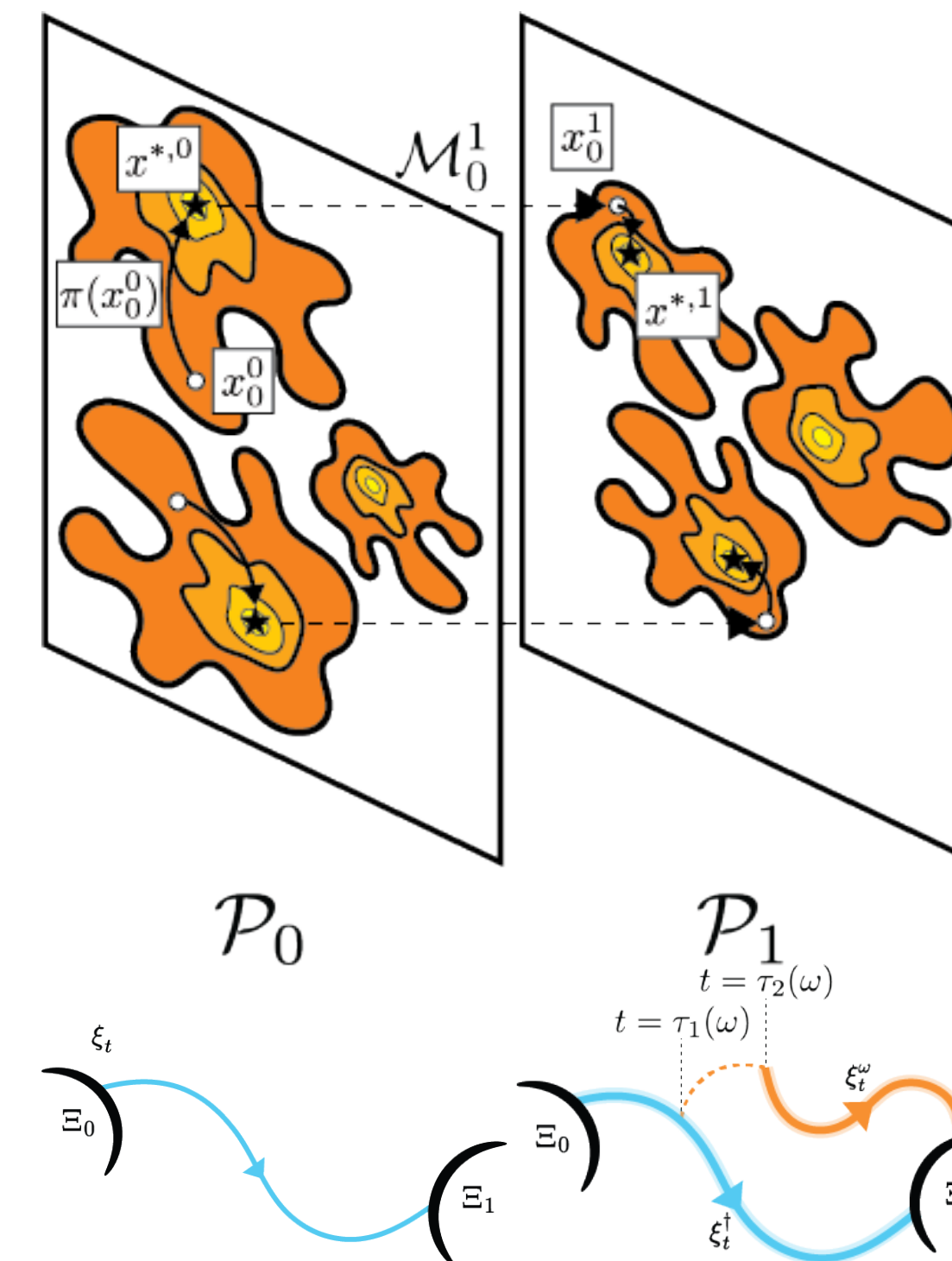
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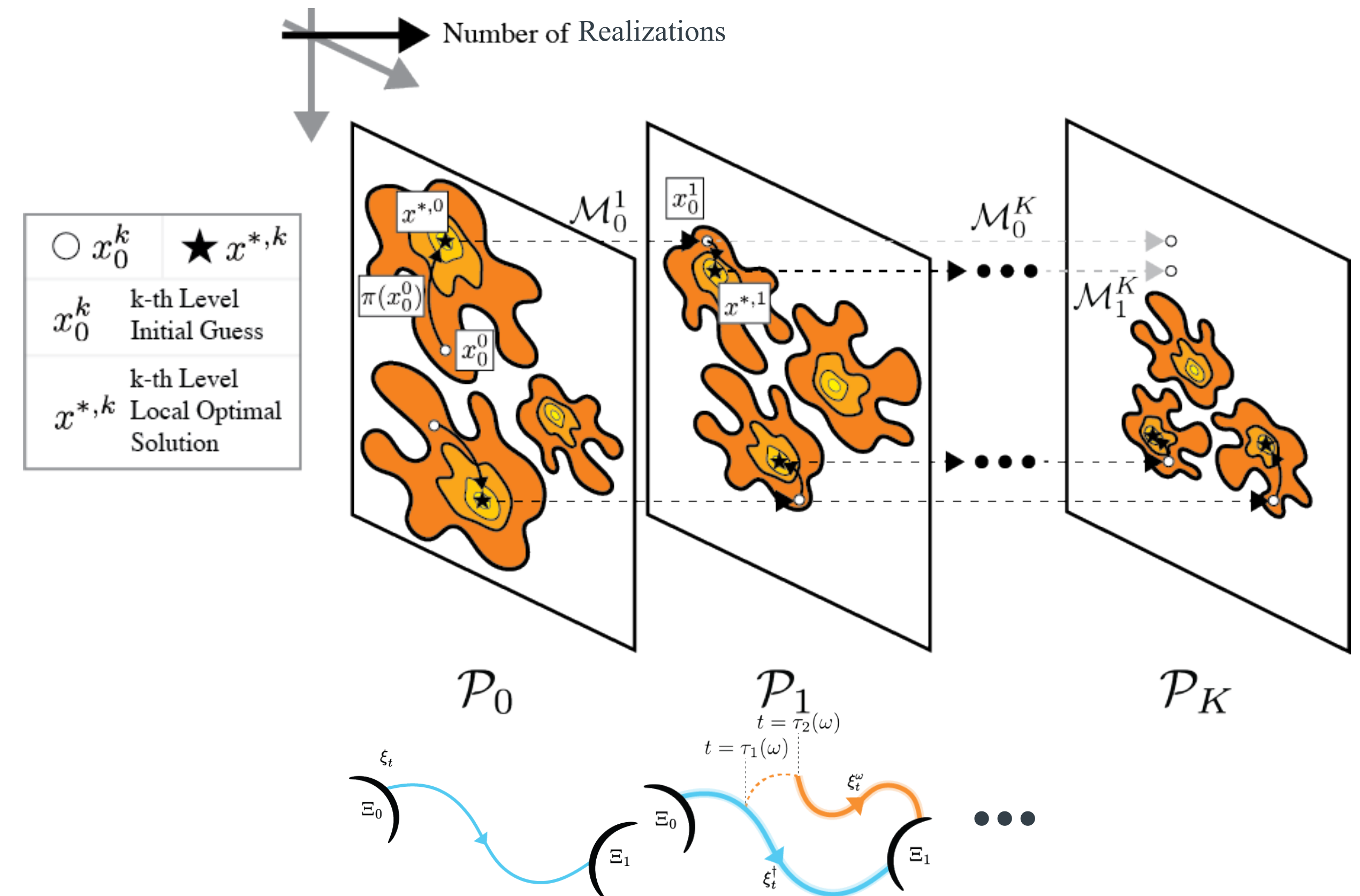
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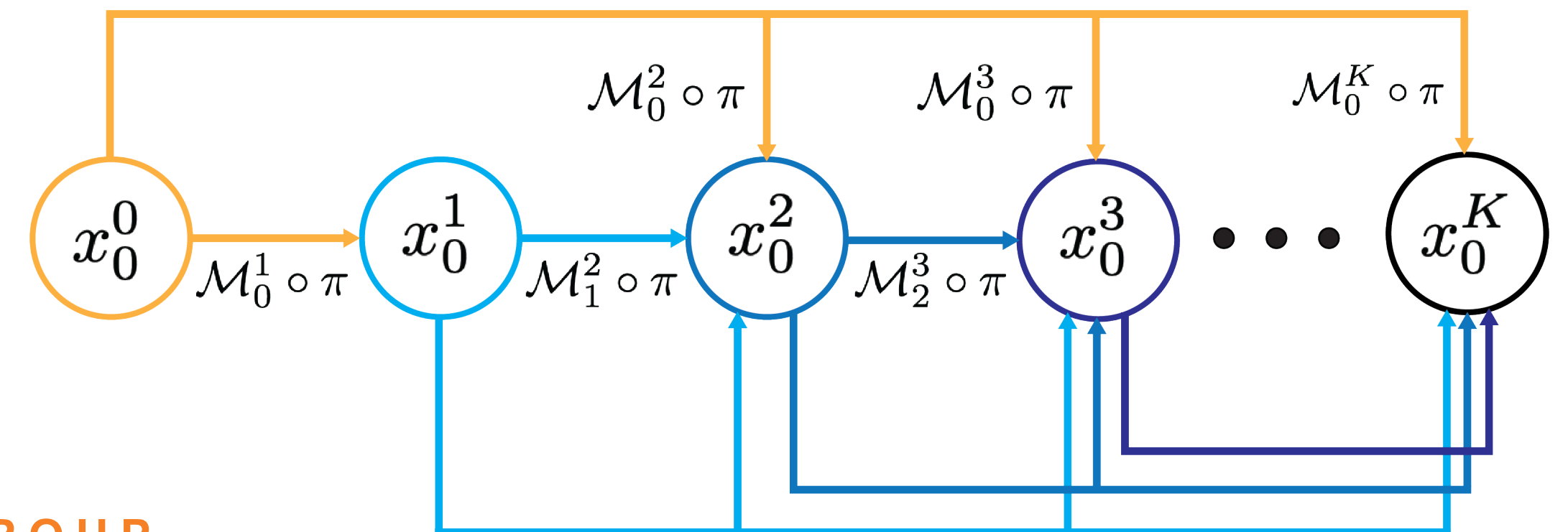
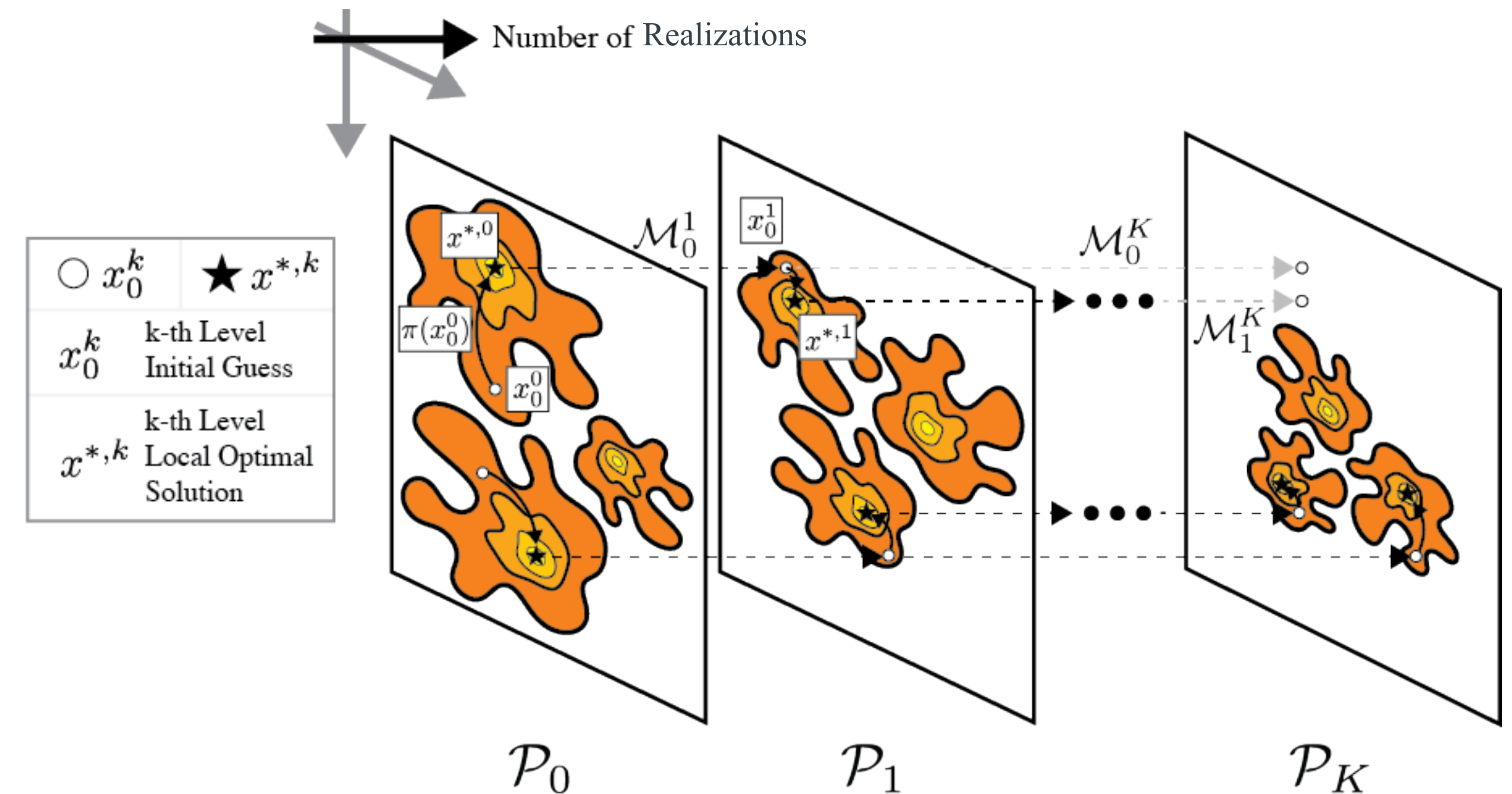
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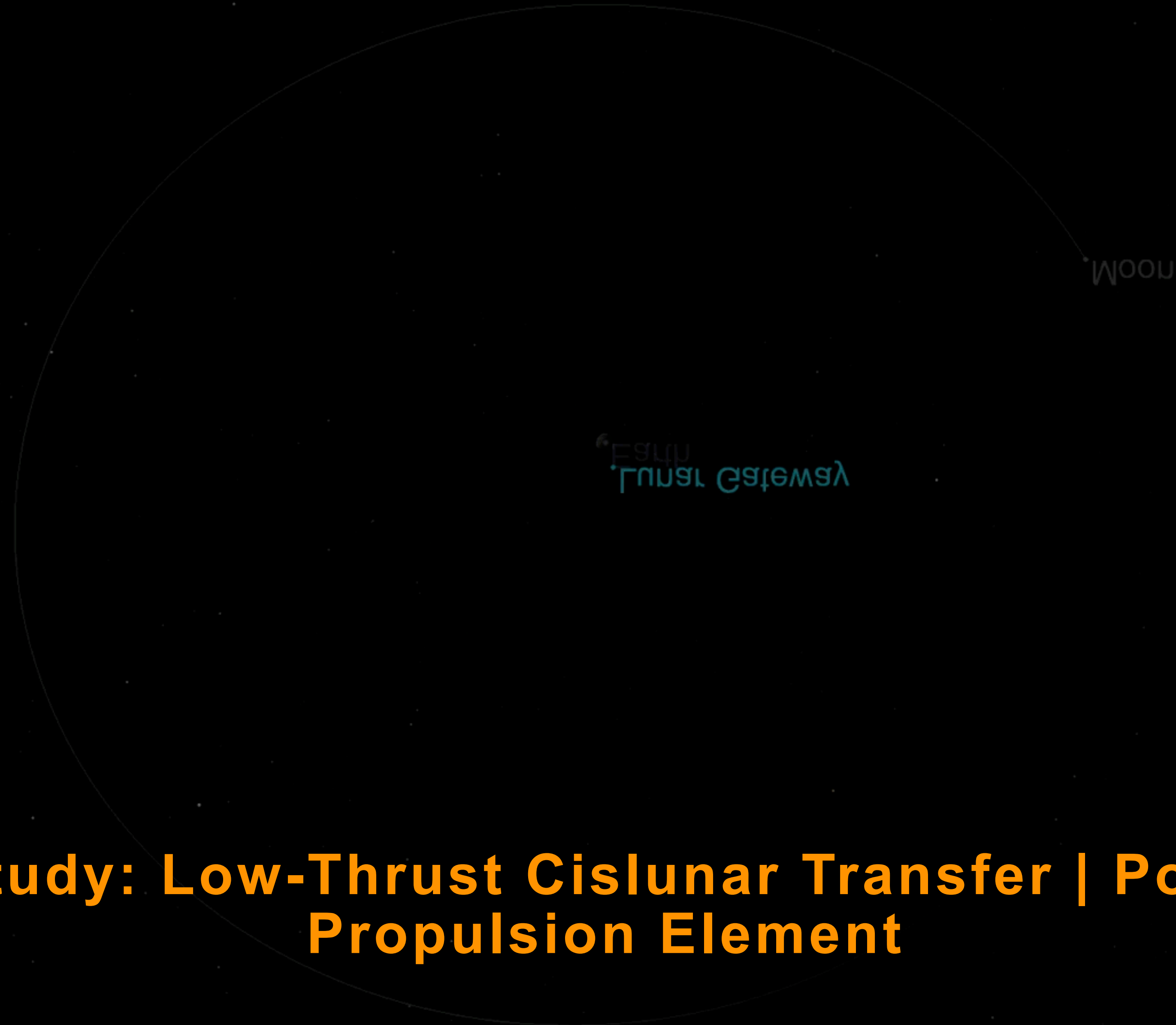
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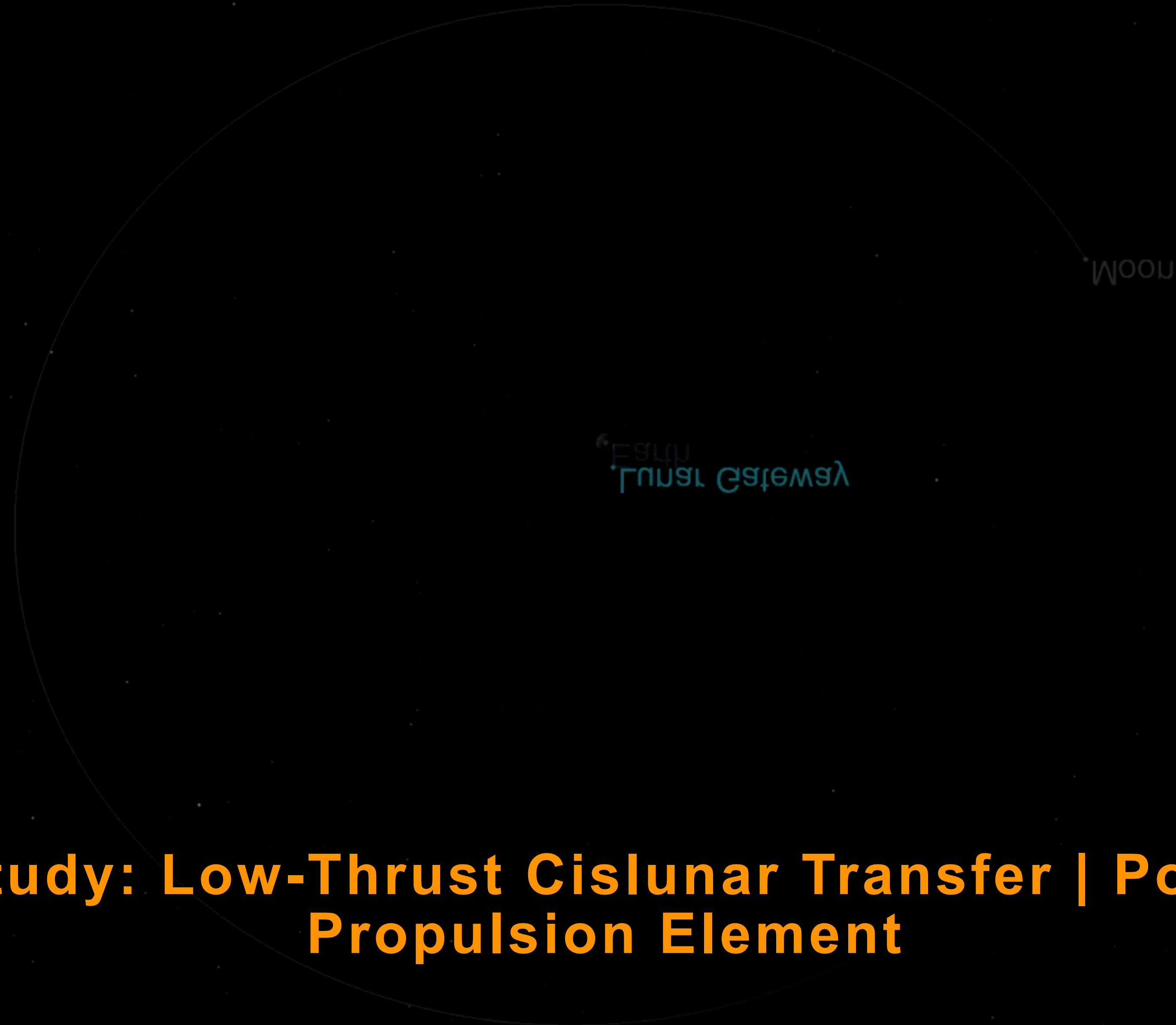
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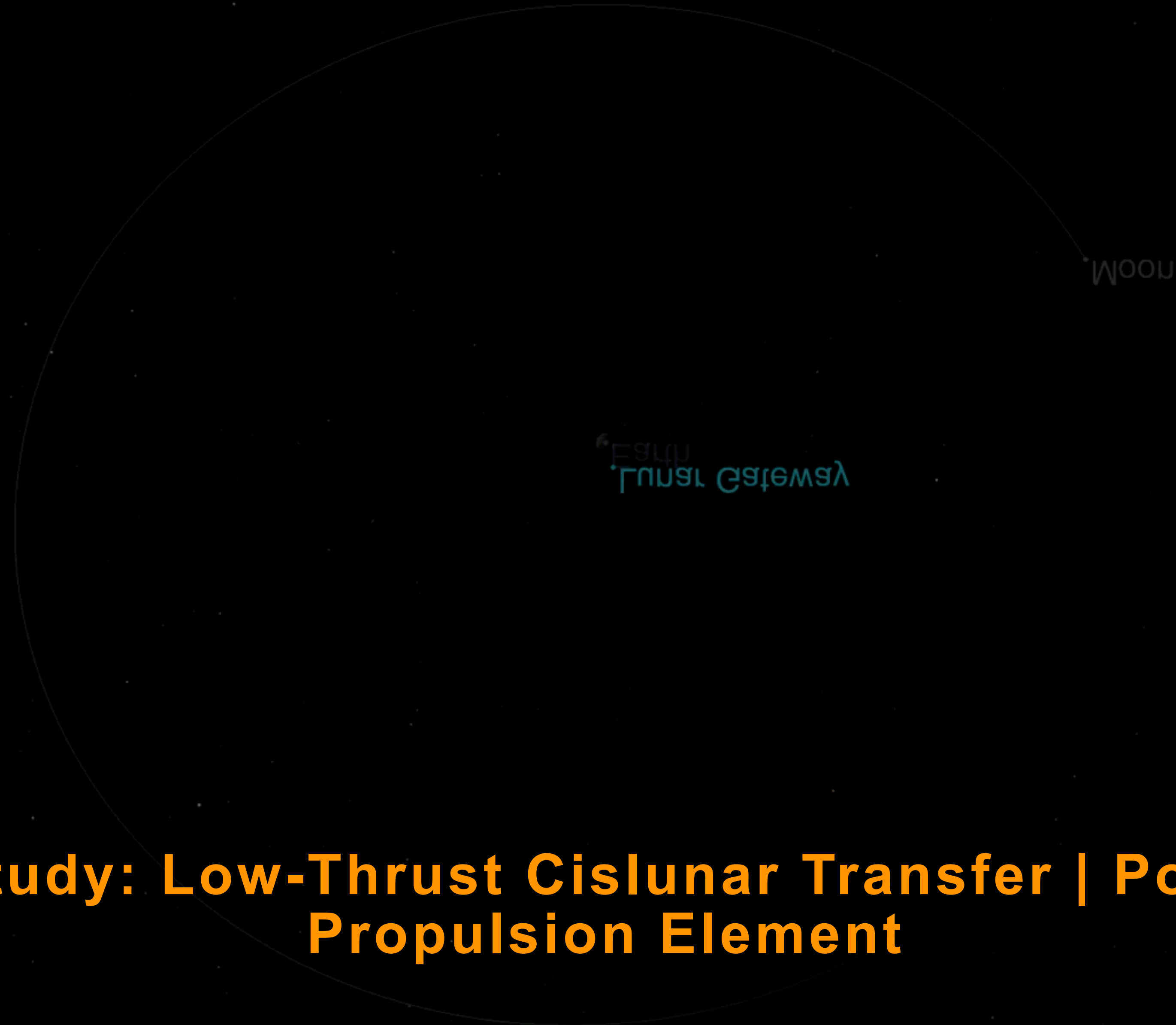




Case study: Low-Thrust Cislunar Transfer | Power and Propulsion Element



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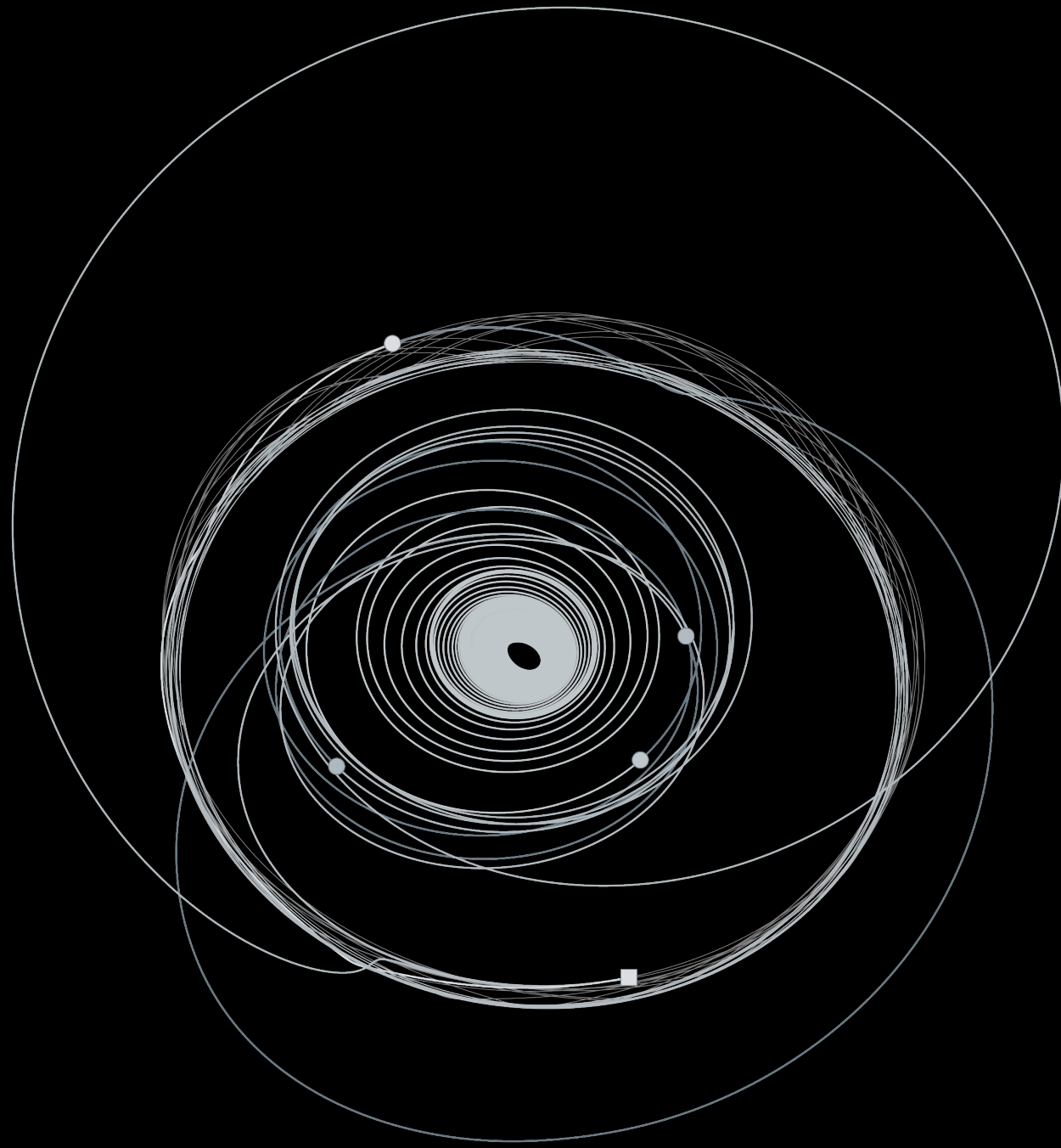


Case study: Low-Thrust Cislunar Transfer | Power and Propulsion Element

Increasing outage duration →

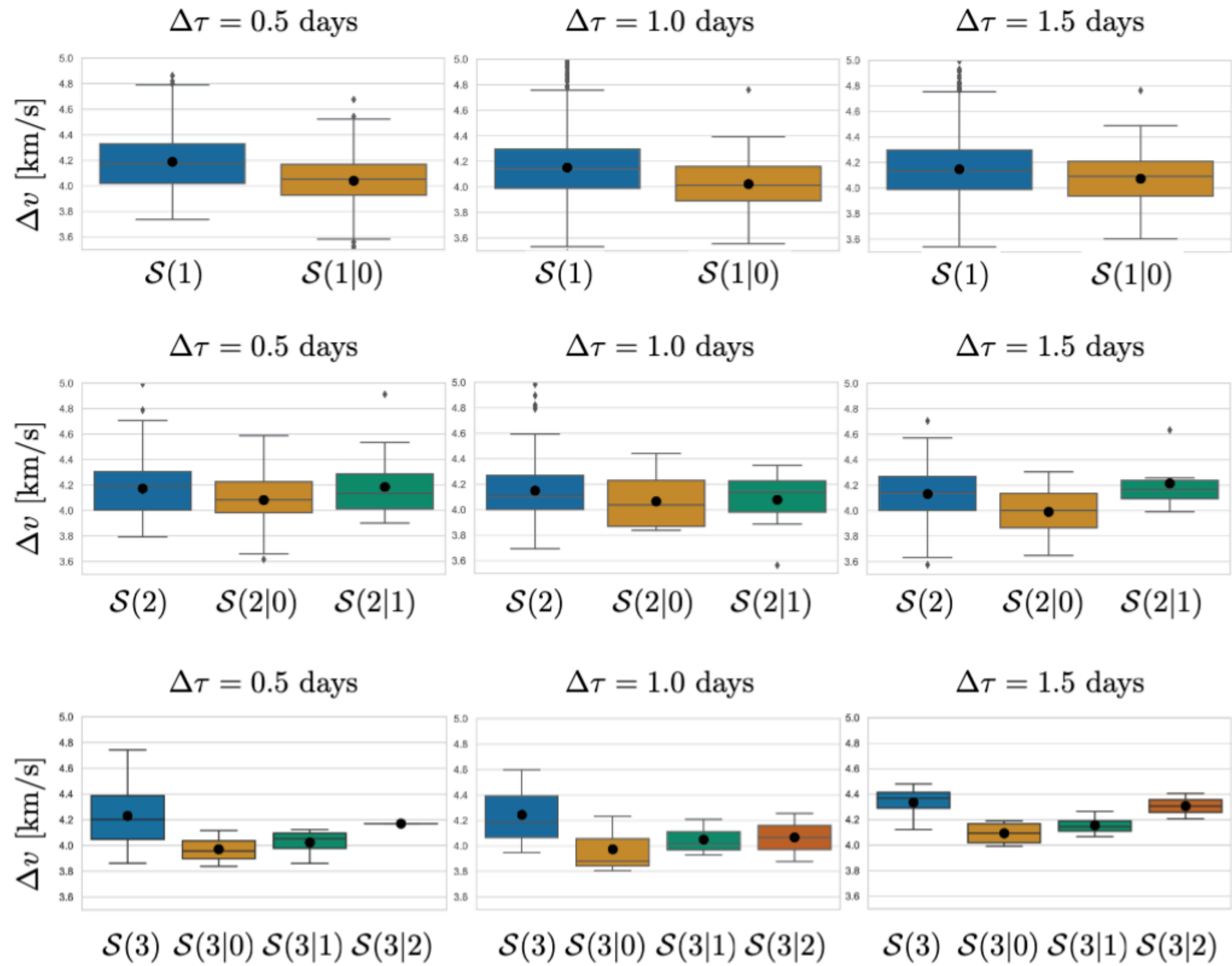
The study evaluates each strategy using three metrics:

- Solution quality Δv
- Feasibility ratio
- Time-to-solve



GEO to Earth-Moon L2 NRHO Low-Thrust Transfer

Increasing number of realizations ↓



$\mathcal{S}(k)$ = Non-Conditional $\mathcal{S}(k|0)$ = Conditional using non-robust (0 realizations)

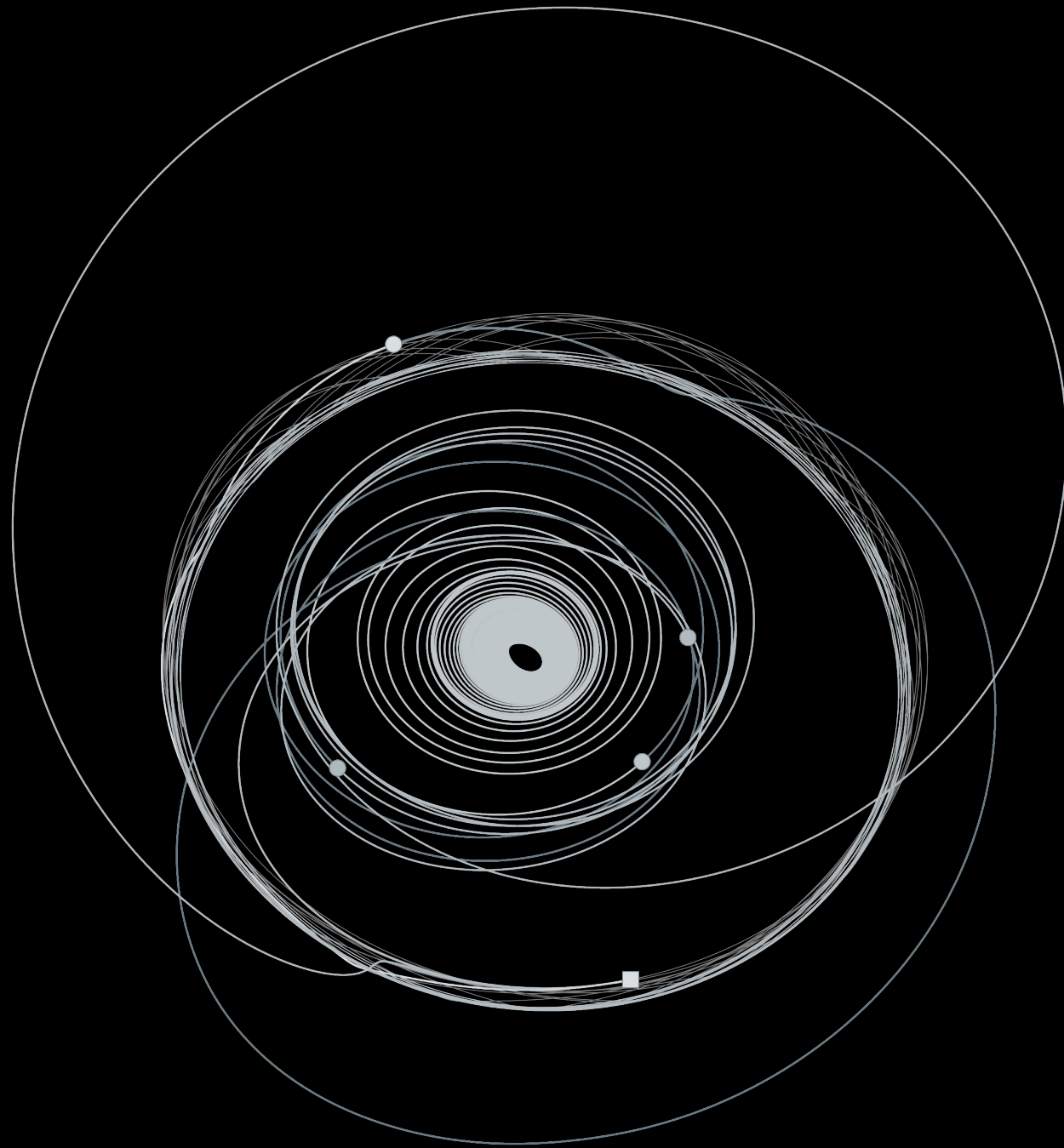
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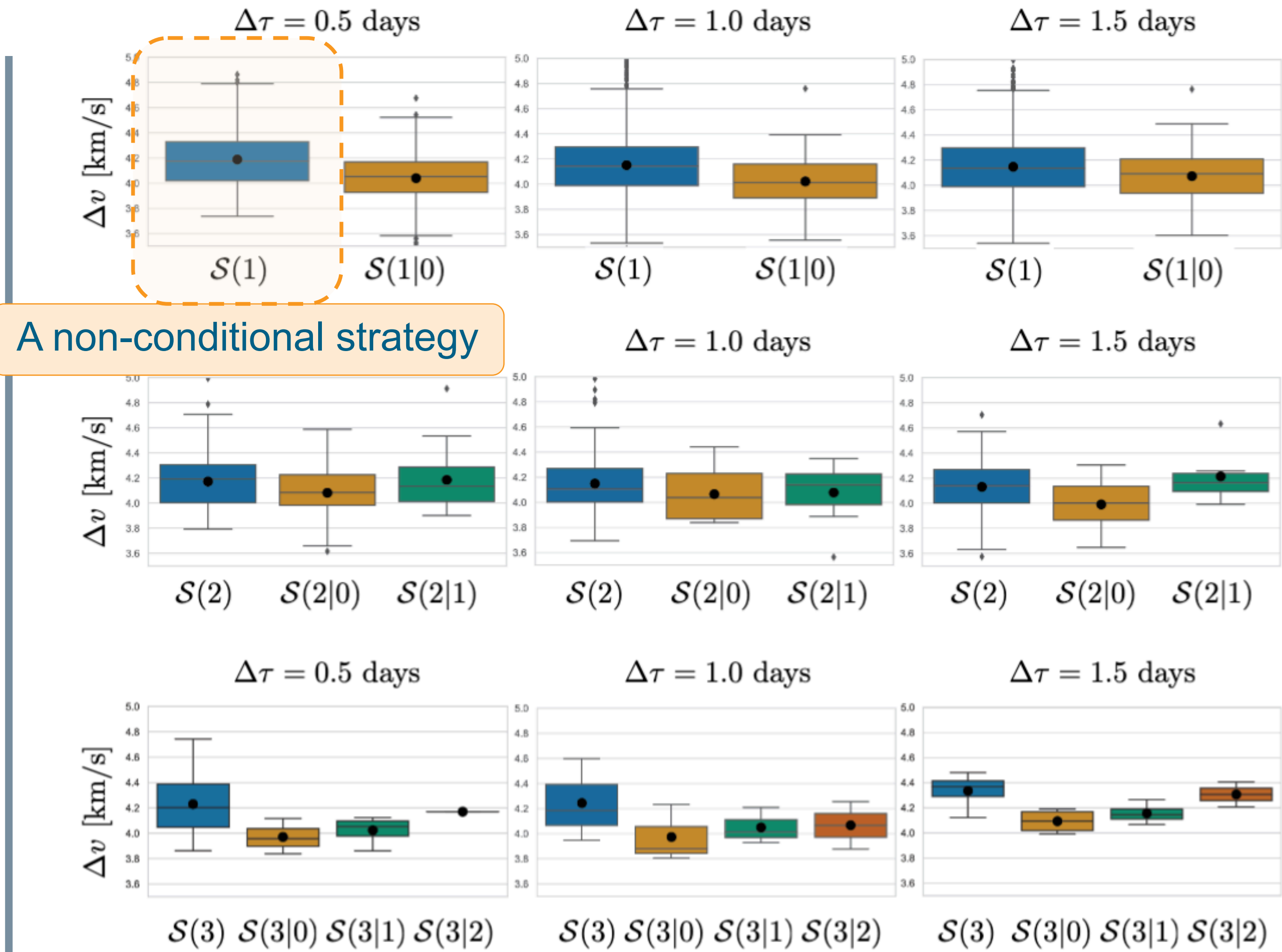
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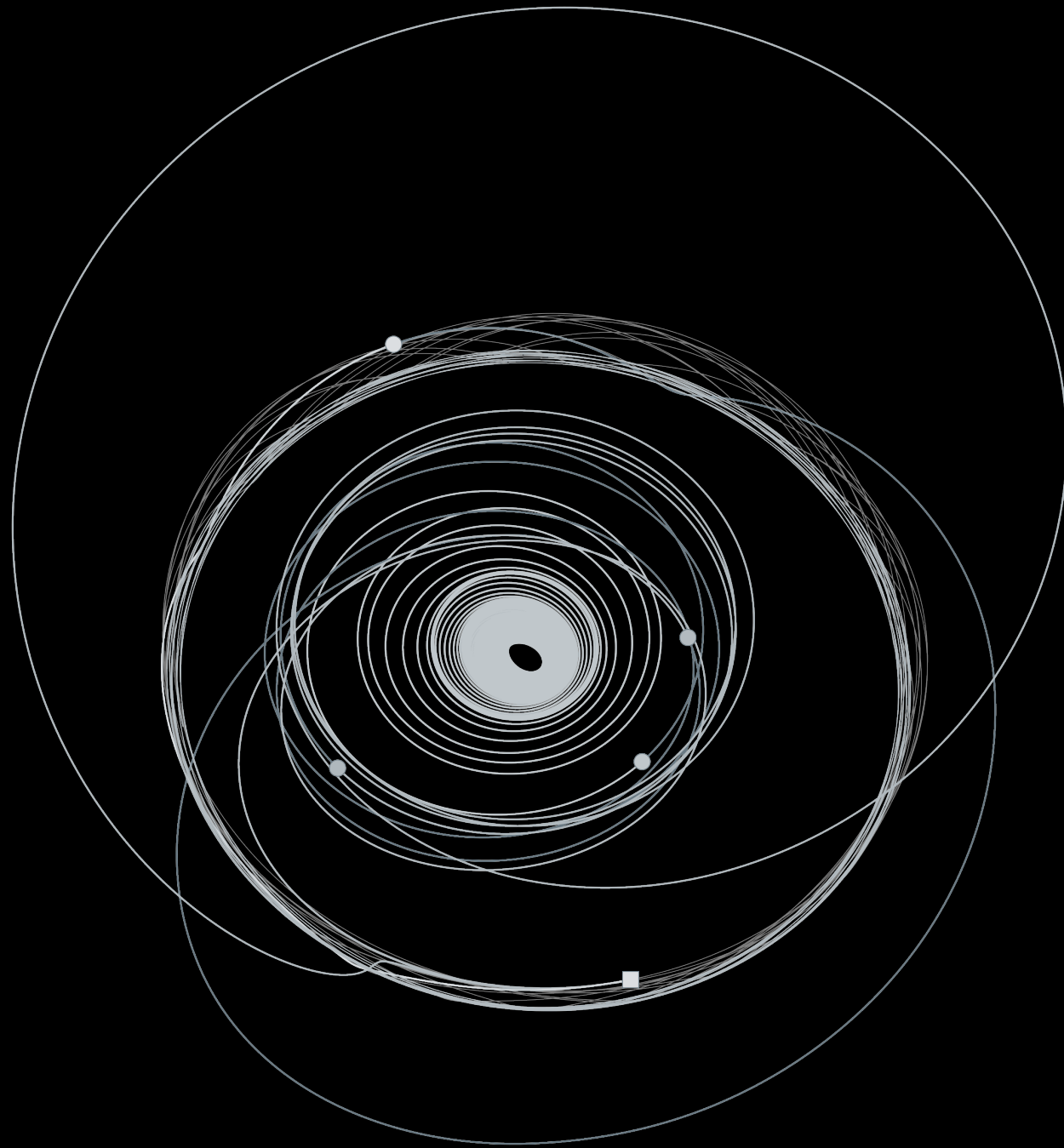


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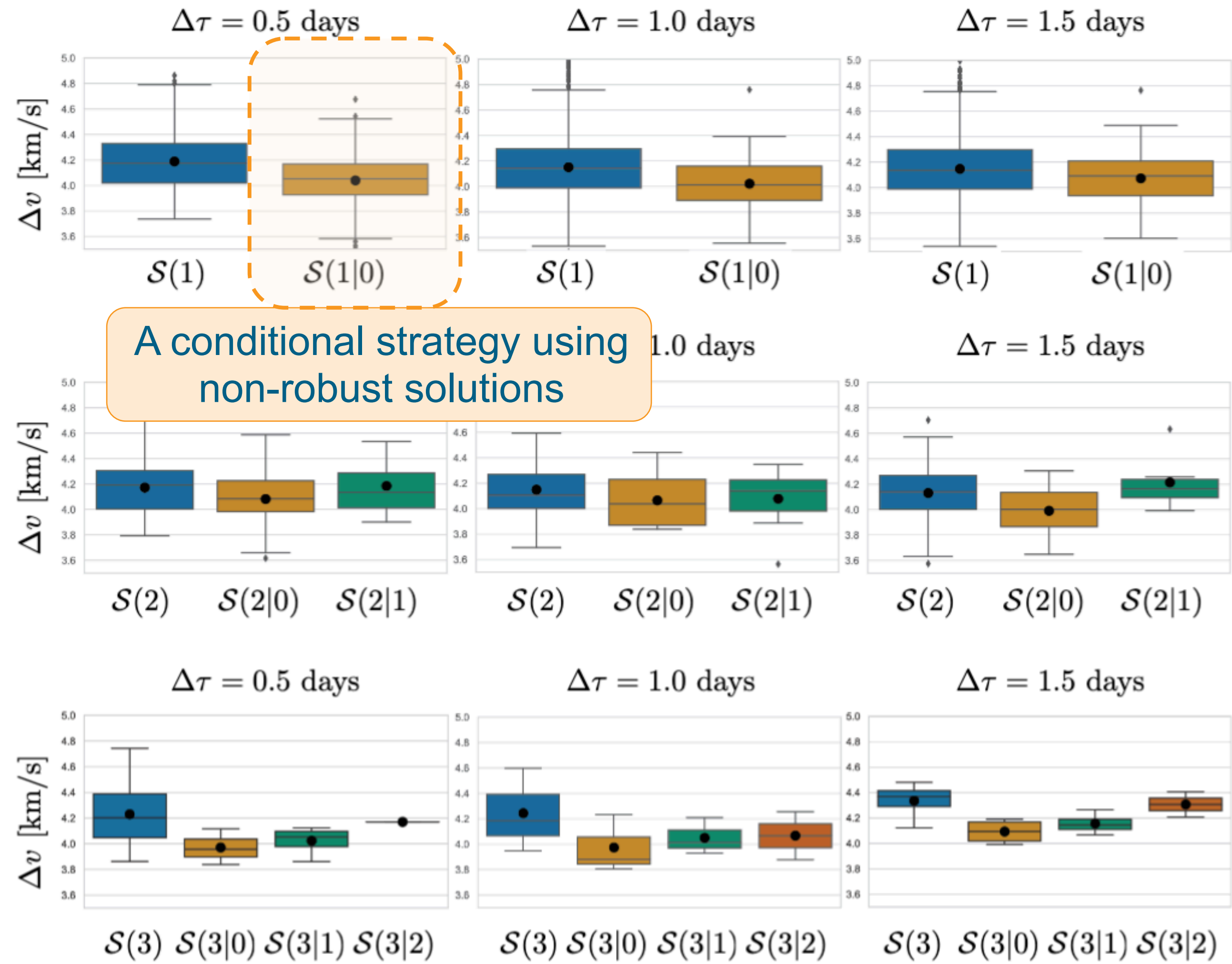
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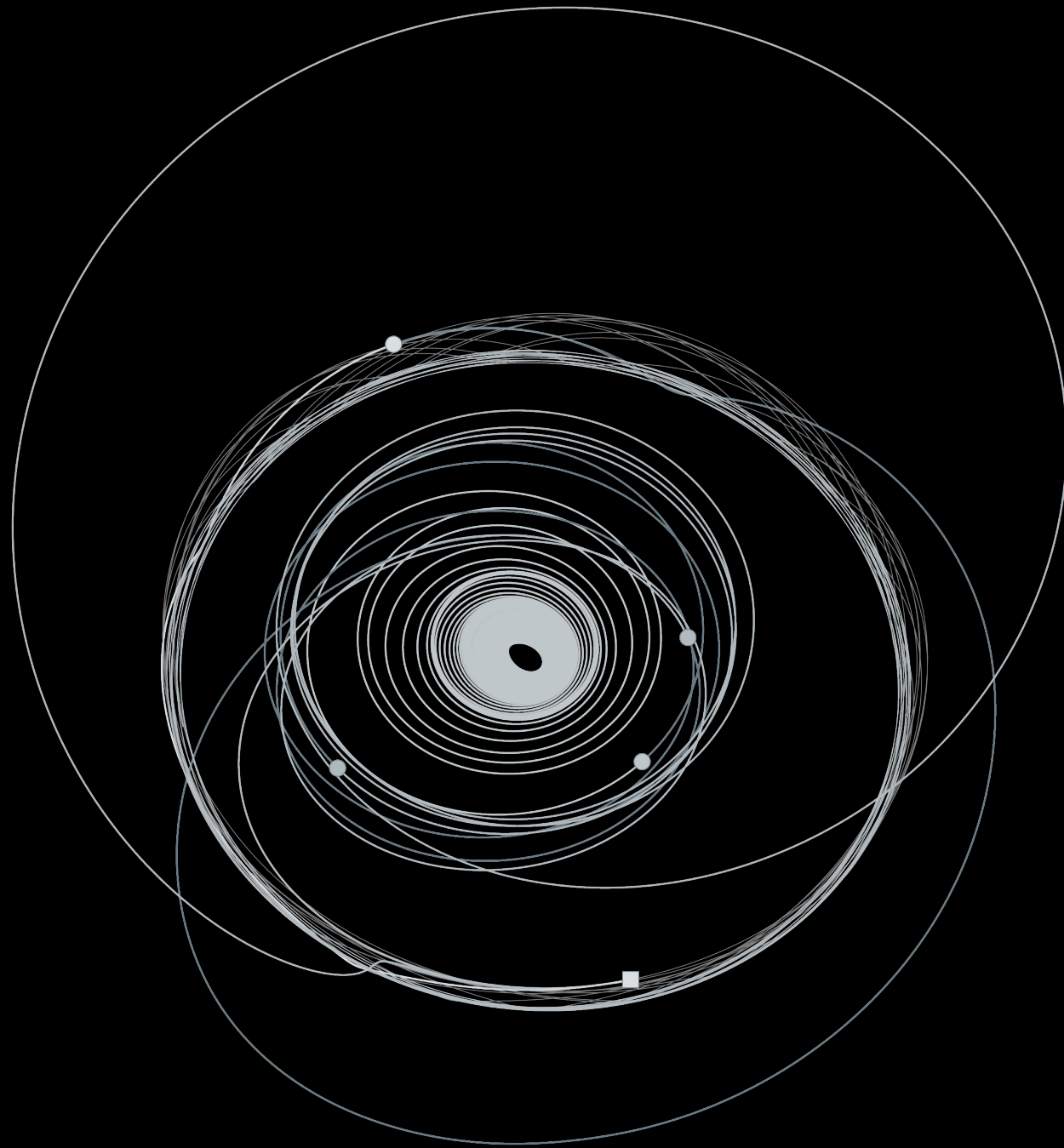
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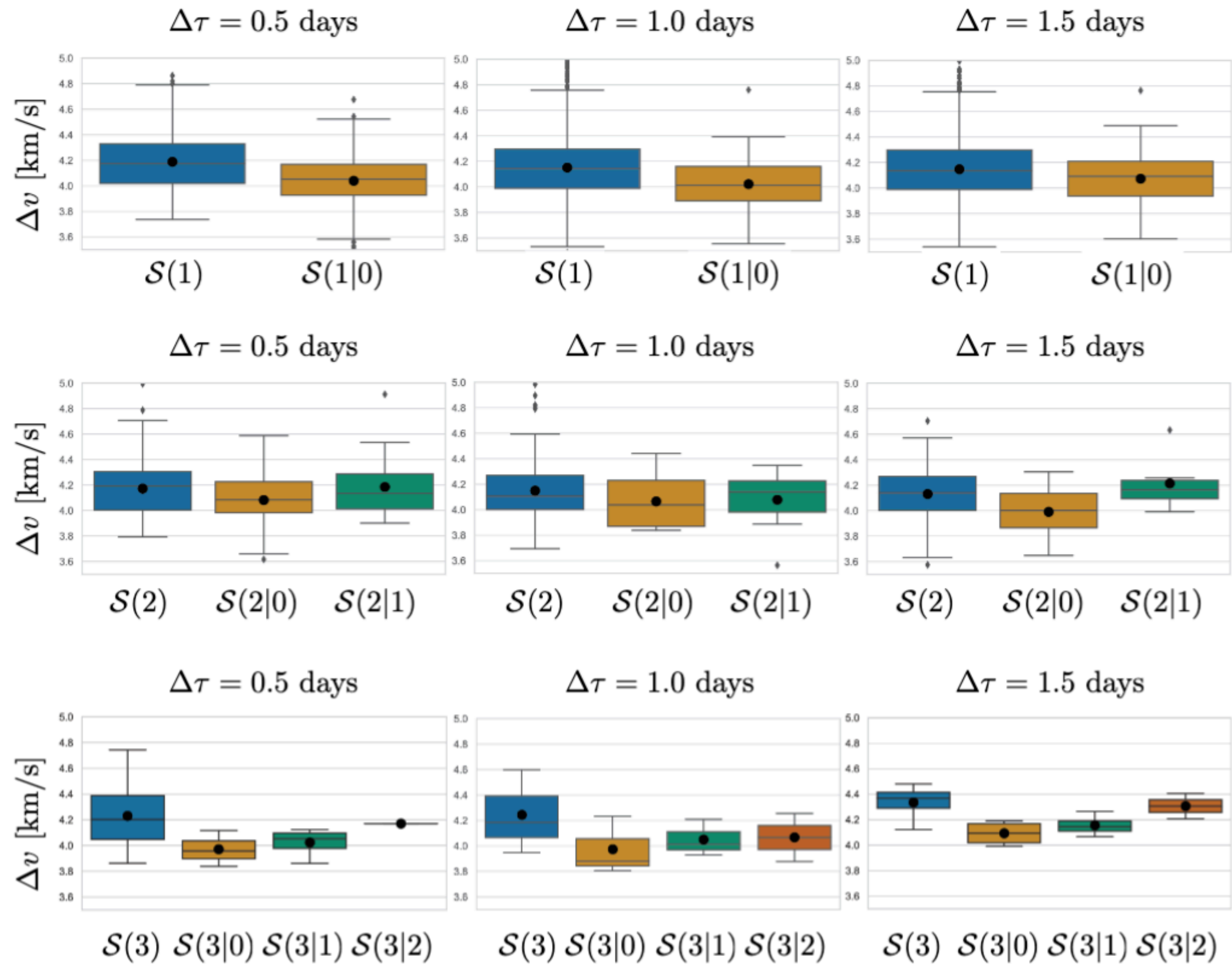
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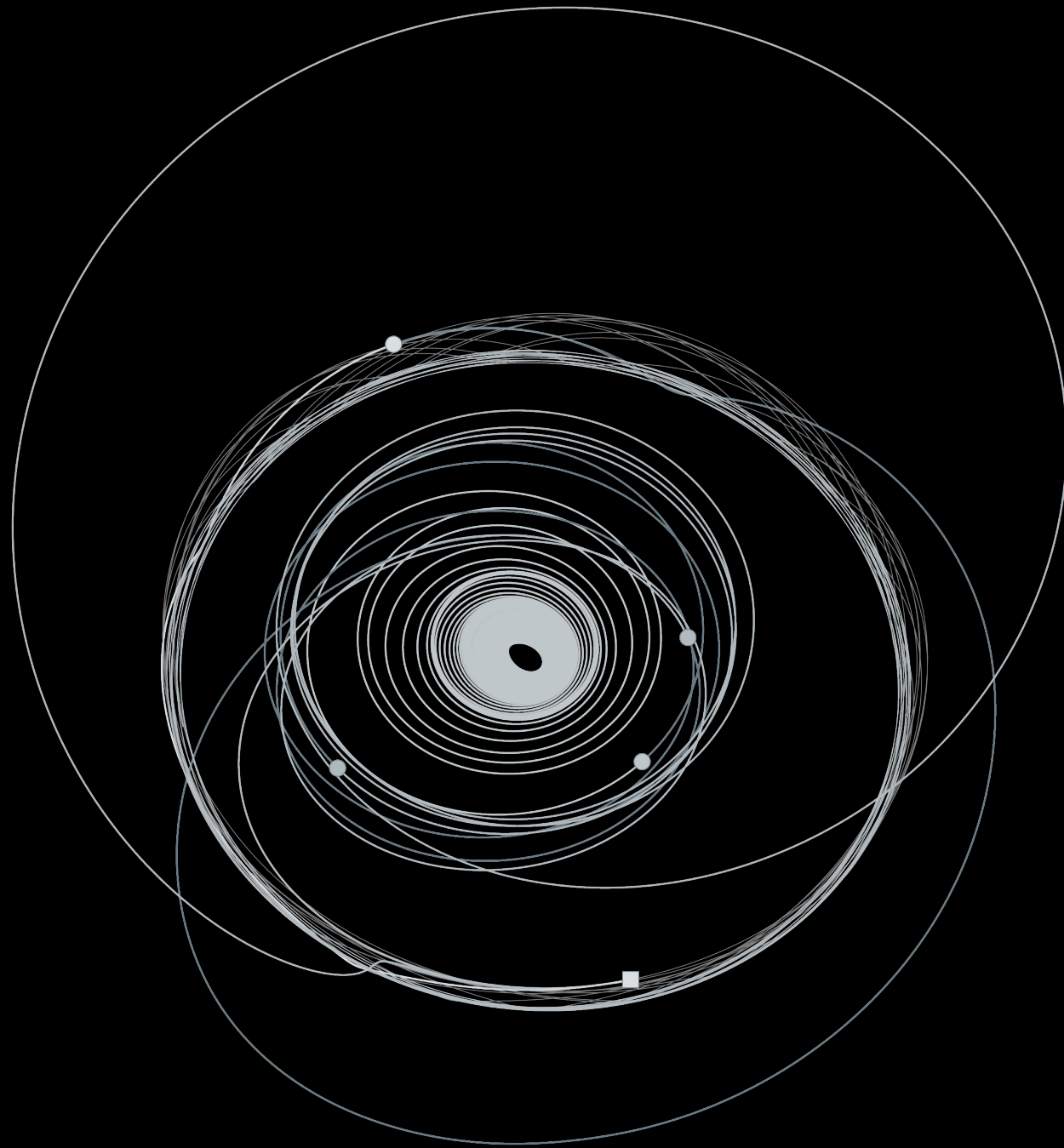


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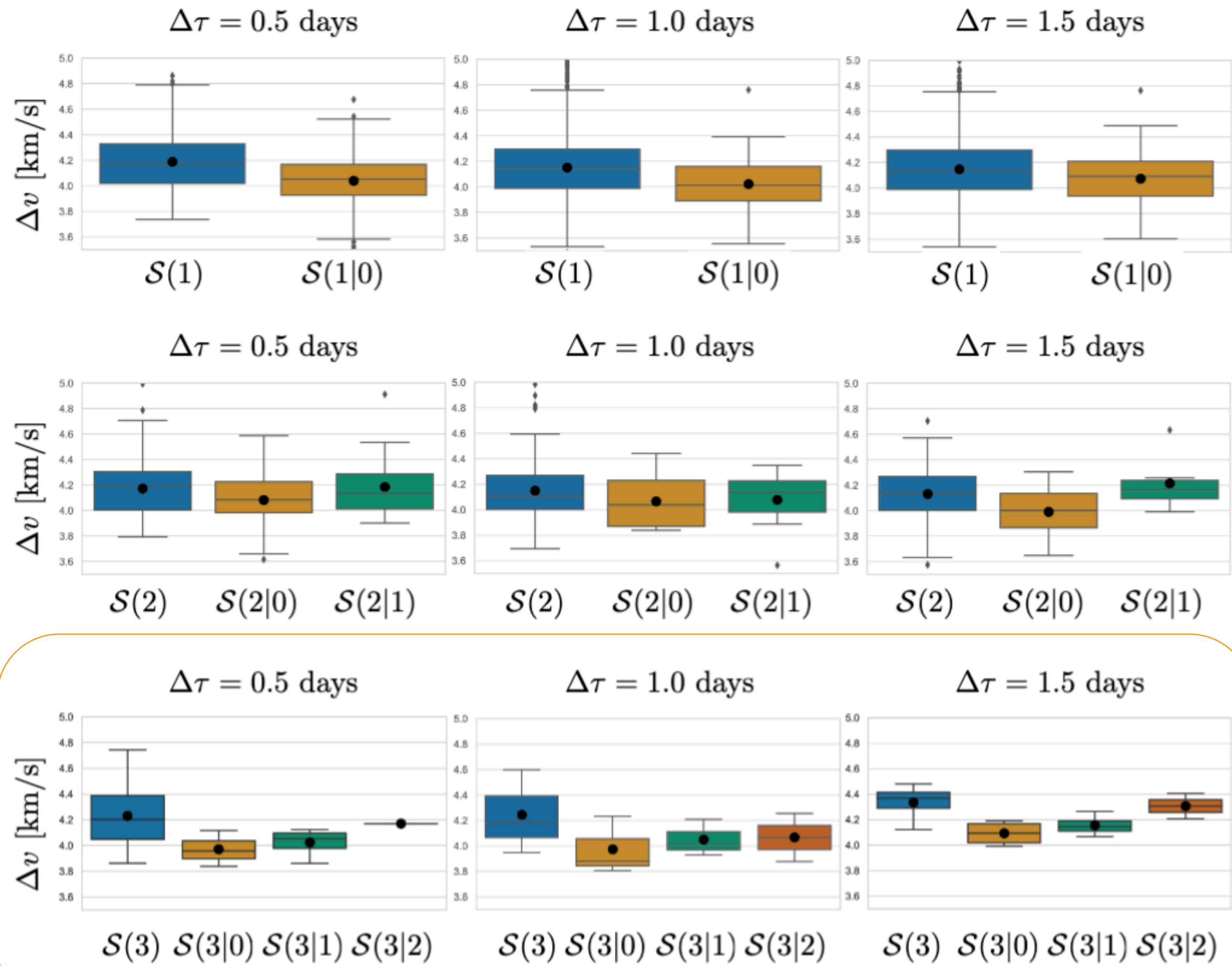
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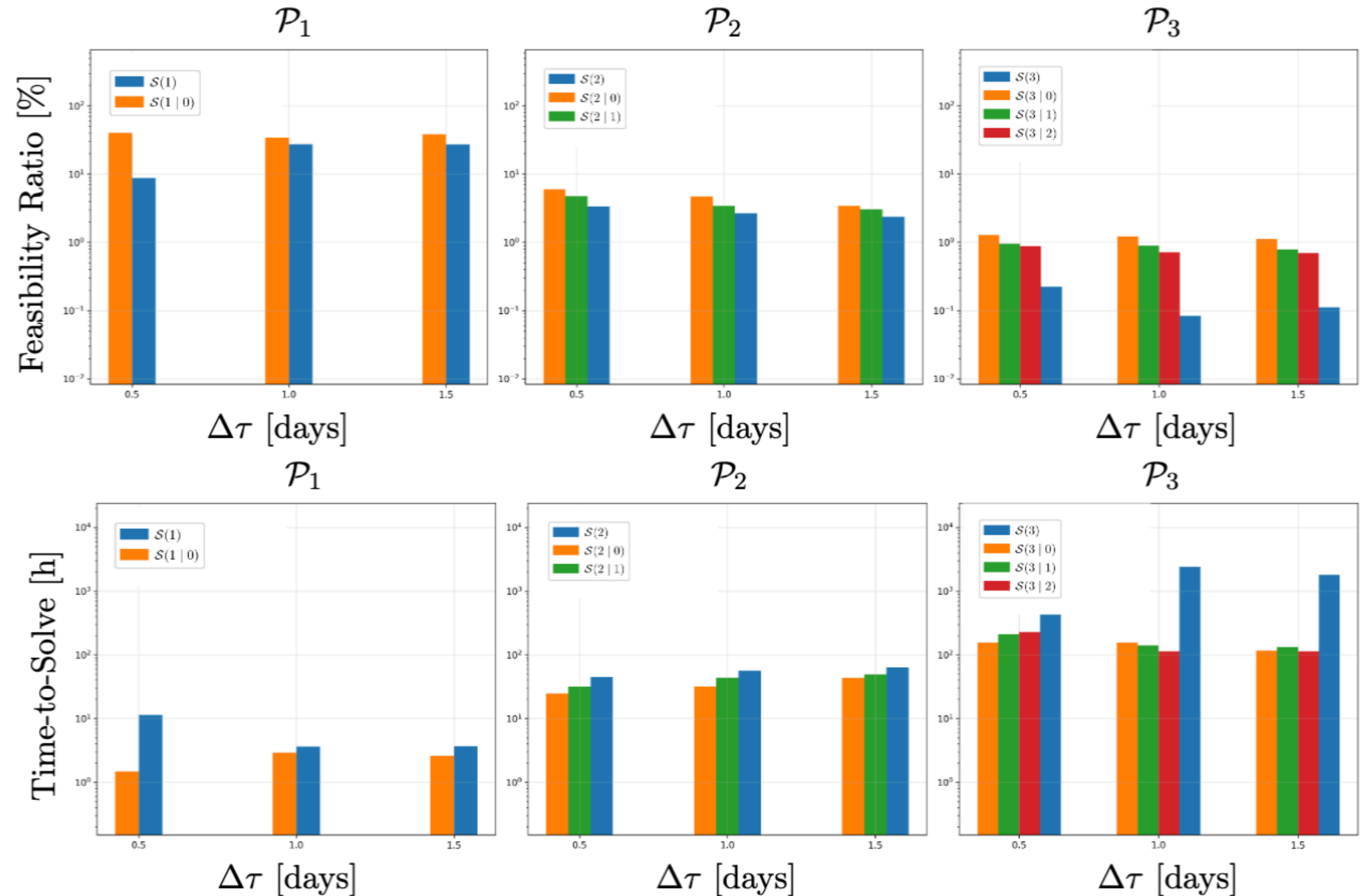
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$\mathcal{S}(k|0)$ wins as k grows

Across the *conditional strategies*, warm-starting with non-robust solutions $\mathcal{S}(k | 0)$ improves both solution quality and numerical performance.



For more details:



Sinha and Beeson (JGCD, 2025)

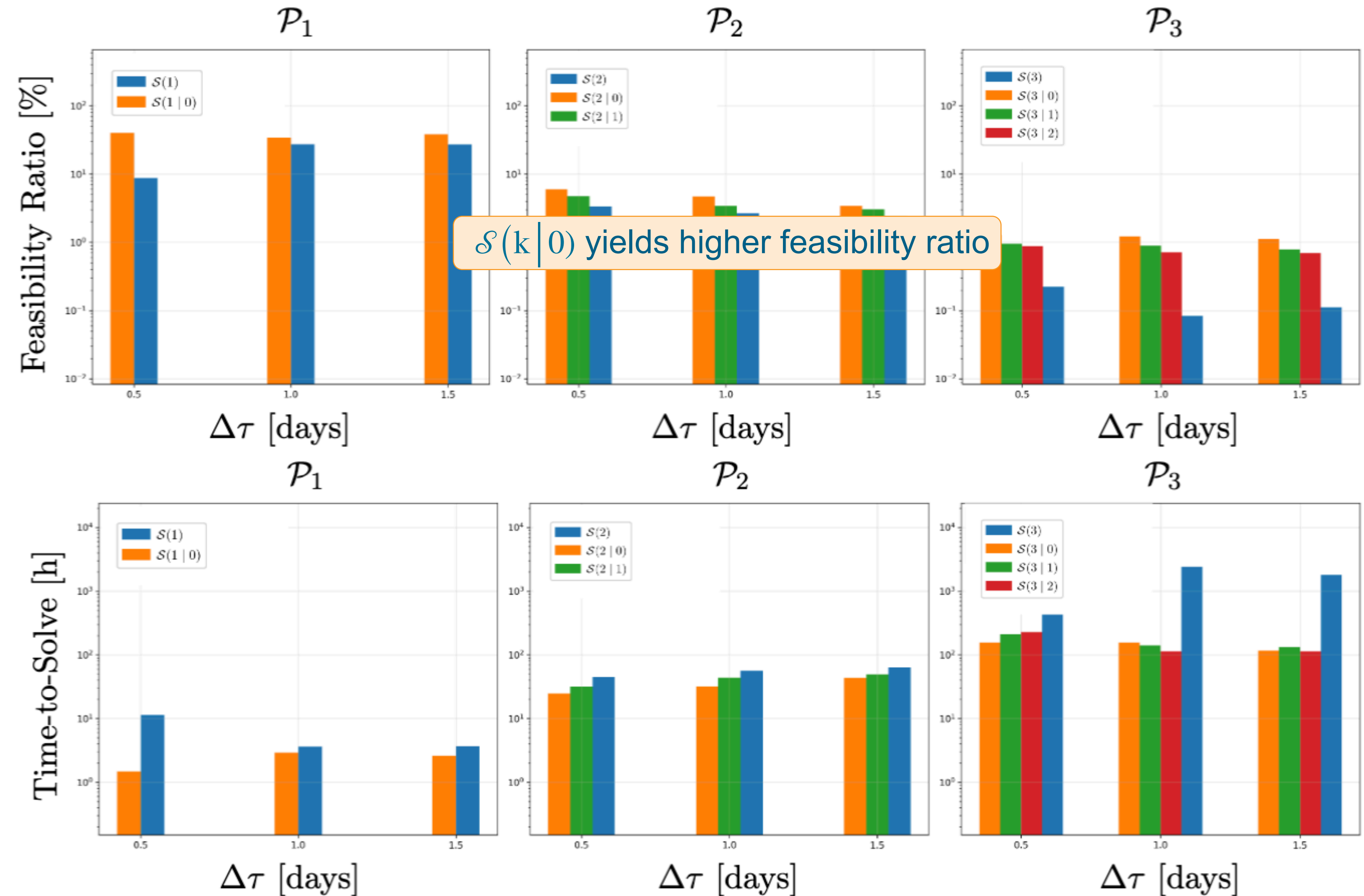
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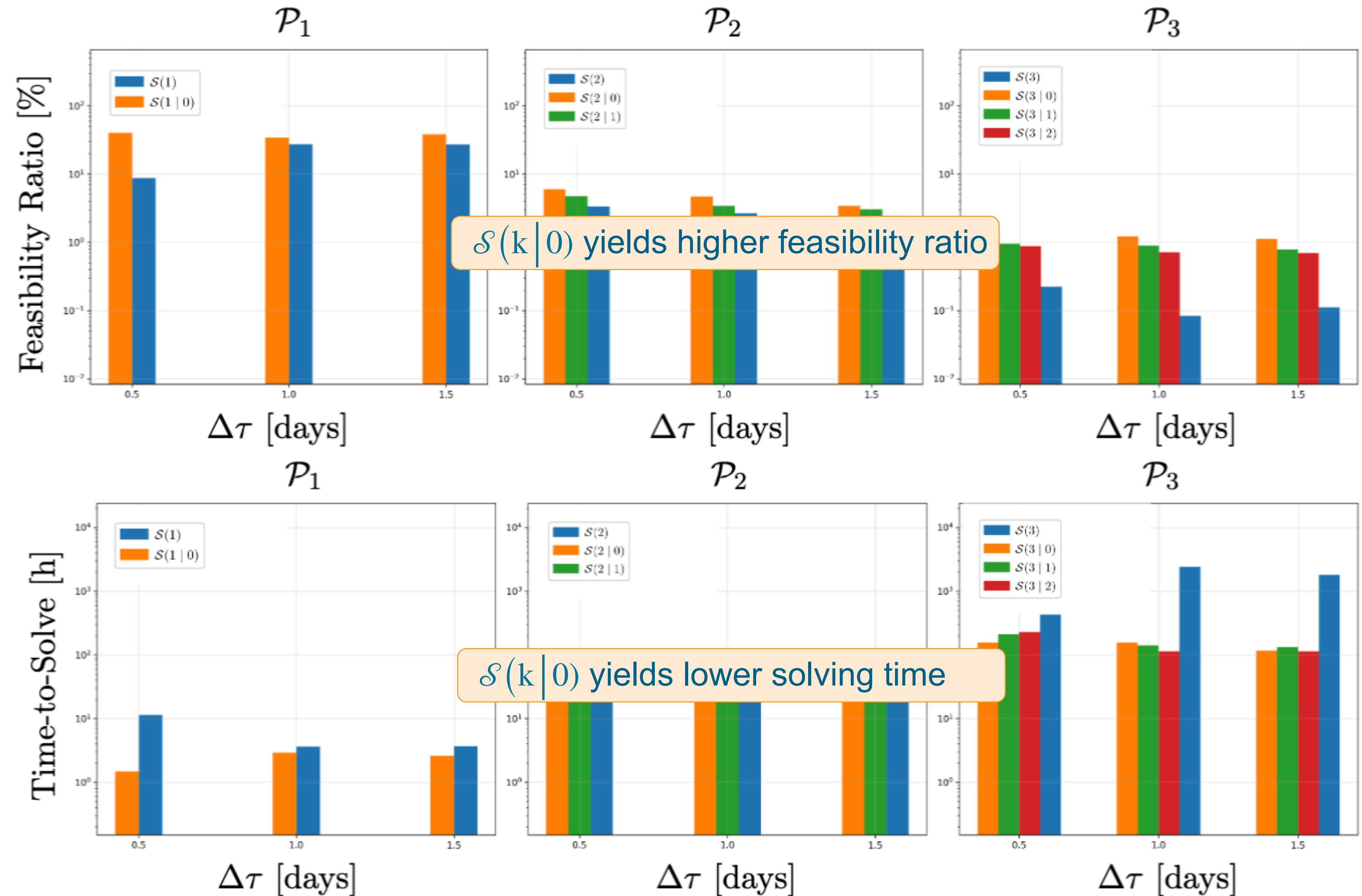
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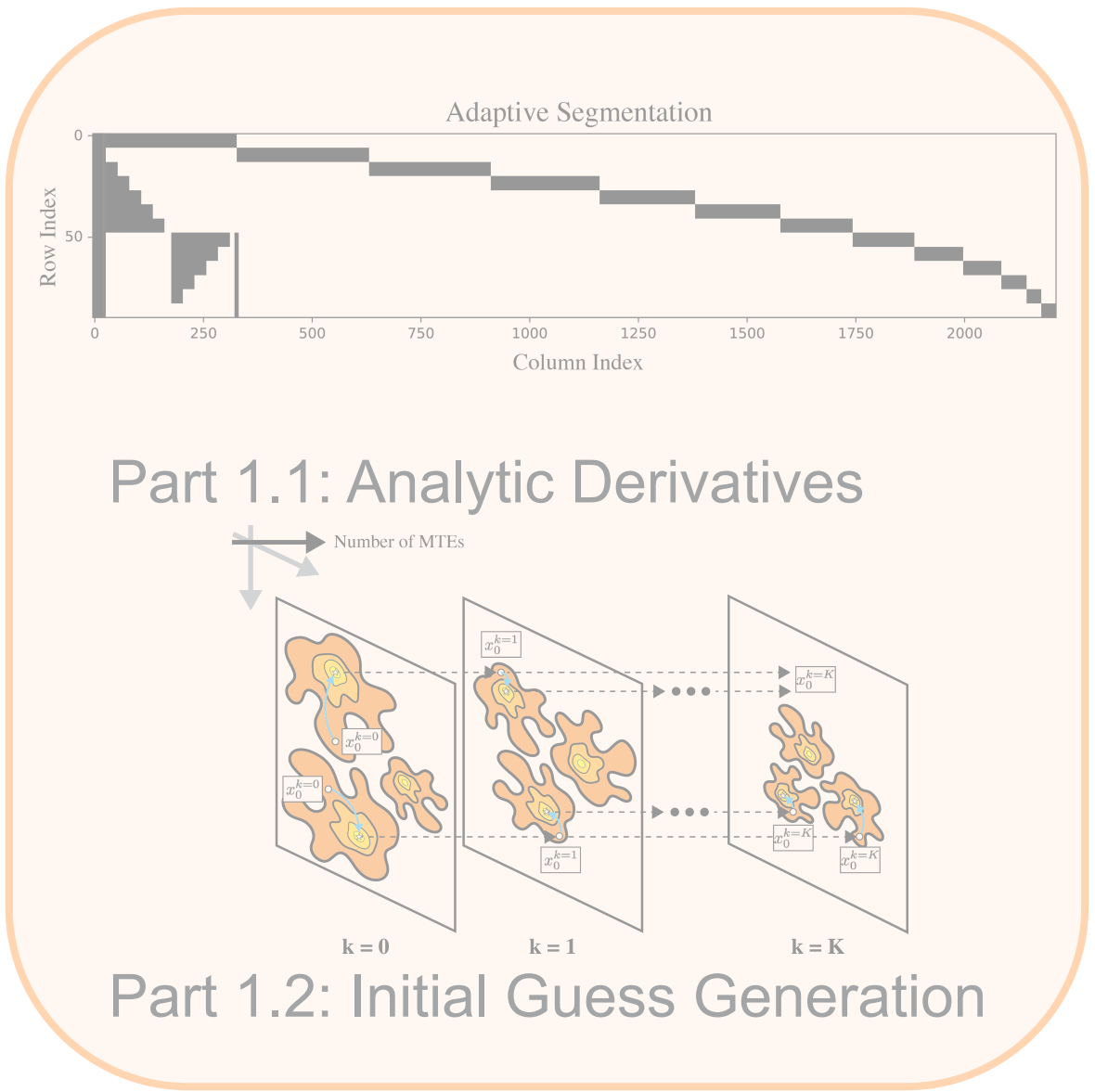
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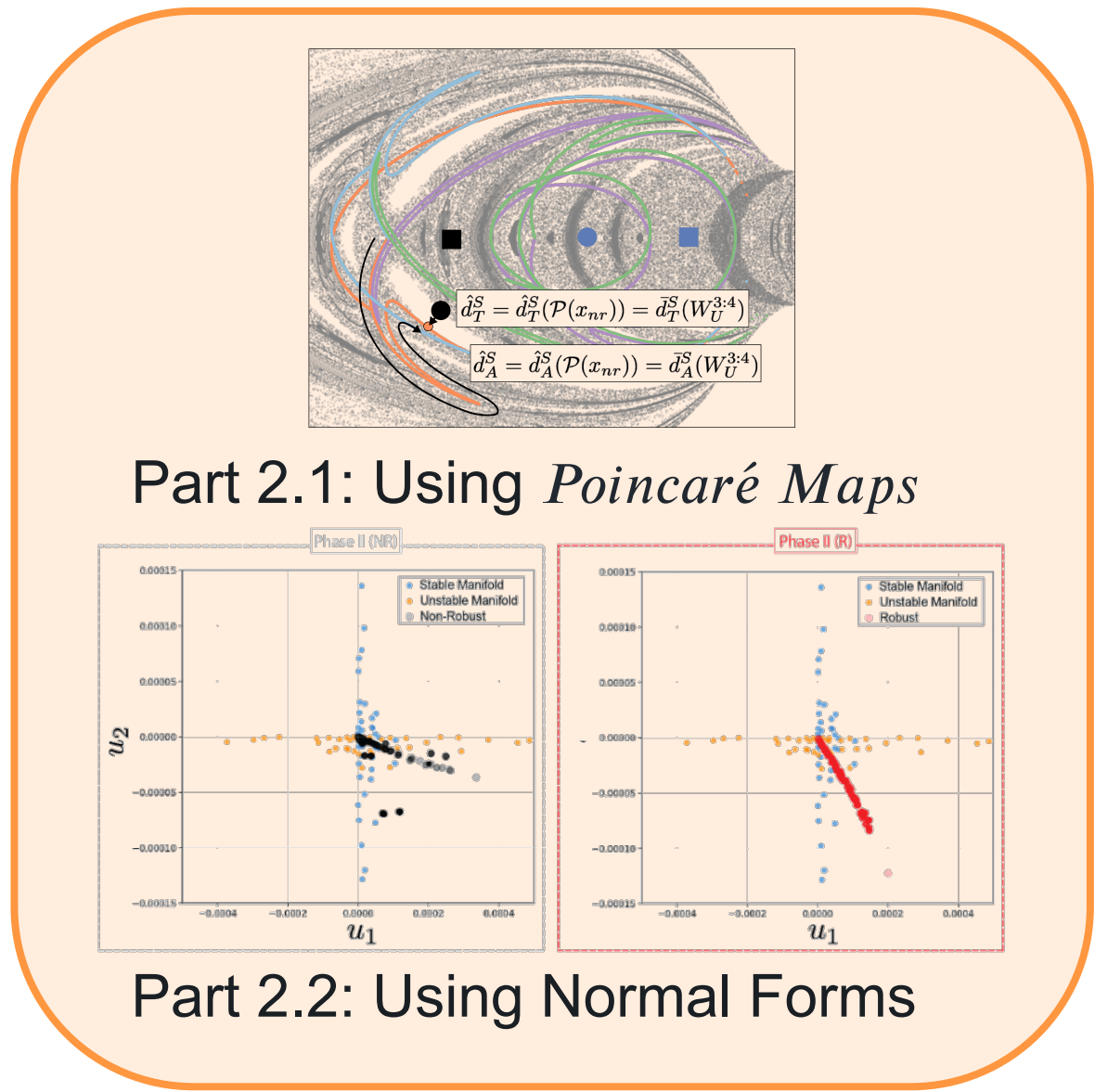


Part 2: Dynamical Systems Analysis

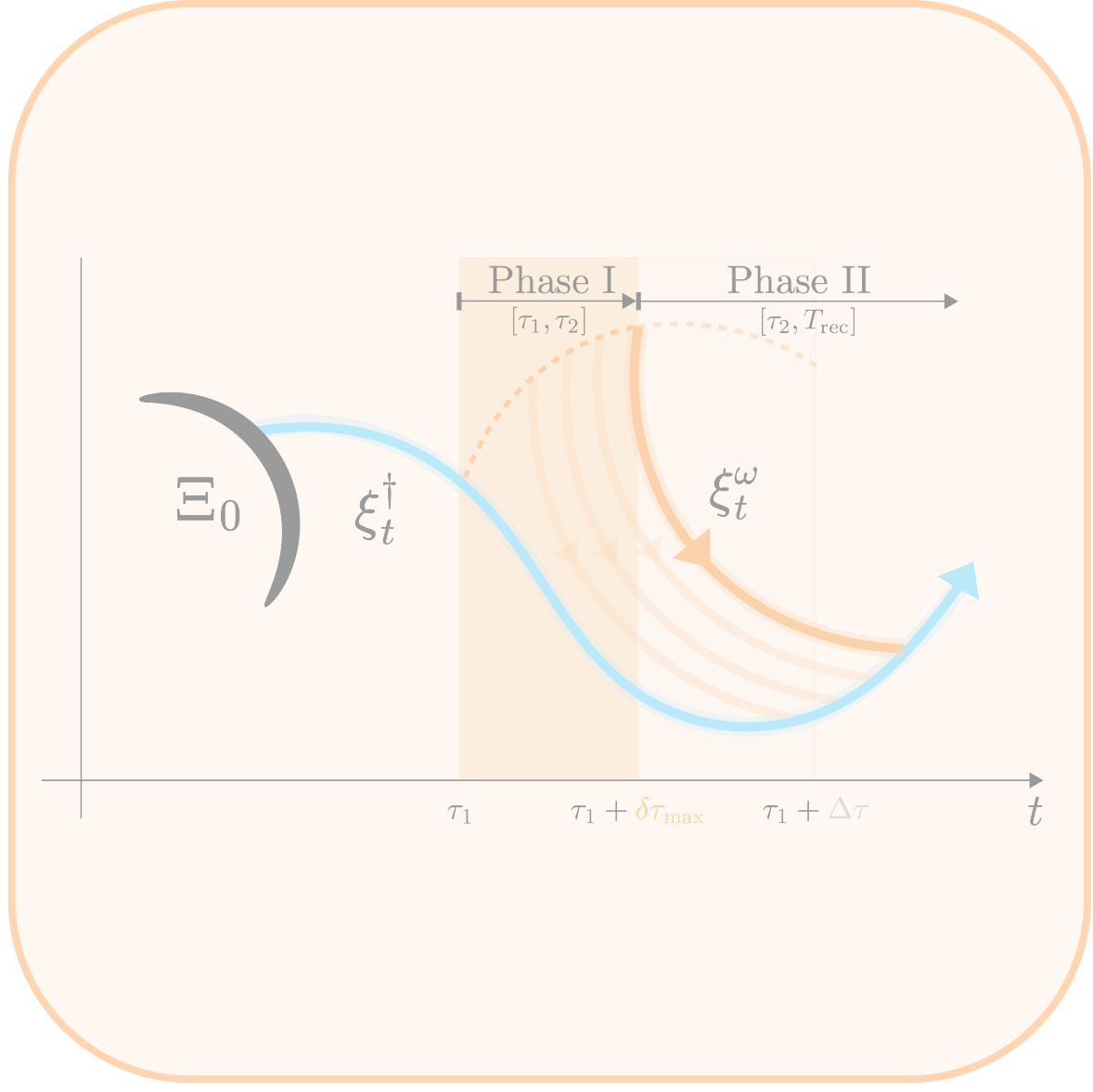
Part 1: Scalable Computational Methods



Part 2: Dynamical Systems Analysis



Part 3: Bilevel Optimal Control Framework



Part 0: Missed Thrust Design Problem Formulation



Part 2.1: Using Poincaré Maps For Understanding The Role Of Dynamical Structures On Missed Thrust Design

Amlan Sinha and Ryne Beeson, “Statistical Analysis of the Role of Invariant Manifolds on Robust Trajectories”, Journal of Guidance, Control, and Dynamics, Vol. 48, No. 8 (2025), pp. 1818-1839, doi: doi/abs/10.2514/1.G008818.

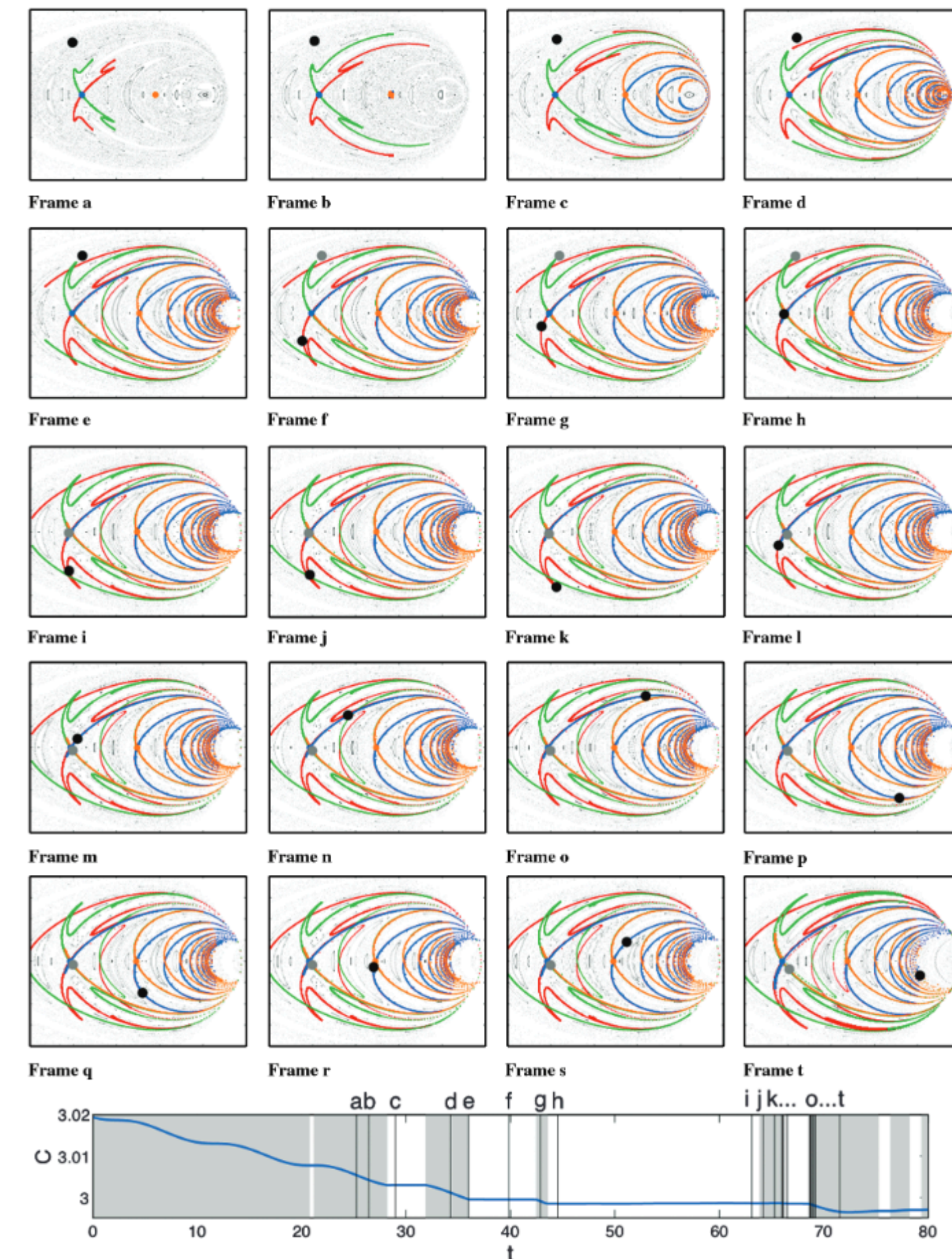


Non-robust optimal LT solutions leverage dynamical structures in multibody systems

LT trajectory design in multibody systems is non-intuitive

- Global search problem needs to be solved iteratively on a **high design cadence**
- Prior knowledge about the relationship between **low-thrust solutions and dynamical structures** can help speed expedite global search
 - ✓ Better initial guesses
 - ✓ Dynamically meaningful constraints
- Non-robust optimal solutions remain close to these **dynamical structures**

Anderson and Lo. JGCD. 2009



Anderson and Lo. JGCD. 2009

The simplest model for representing multibody dynamics is the Circular Restricted Three-Body Problem (CR3BP)

We consider the motion of a spacecraft motion in the CR3BP

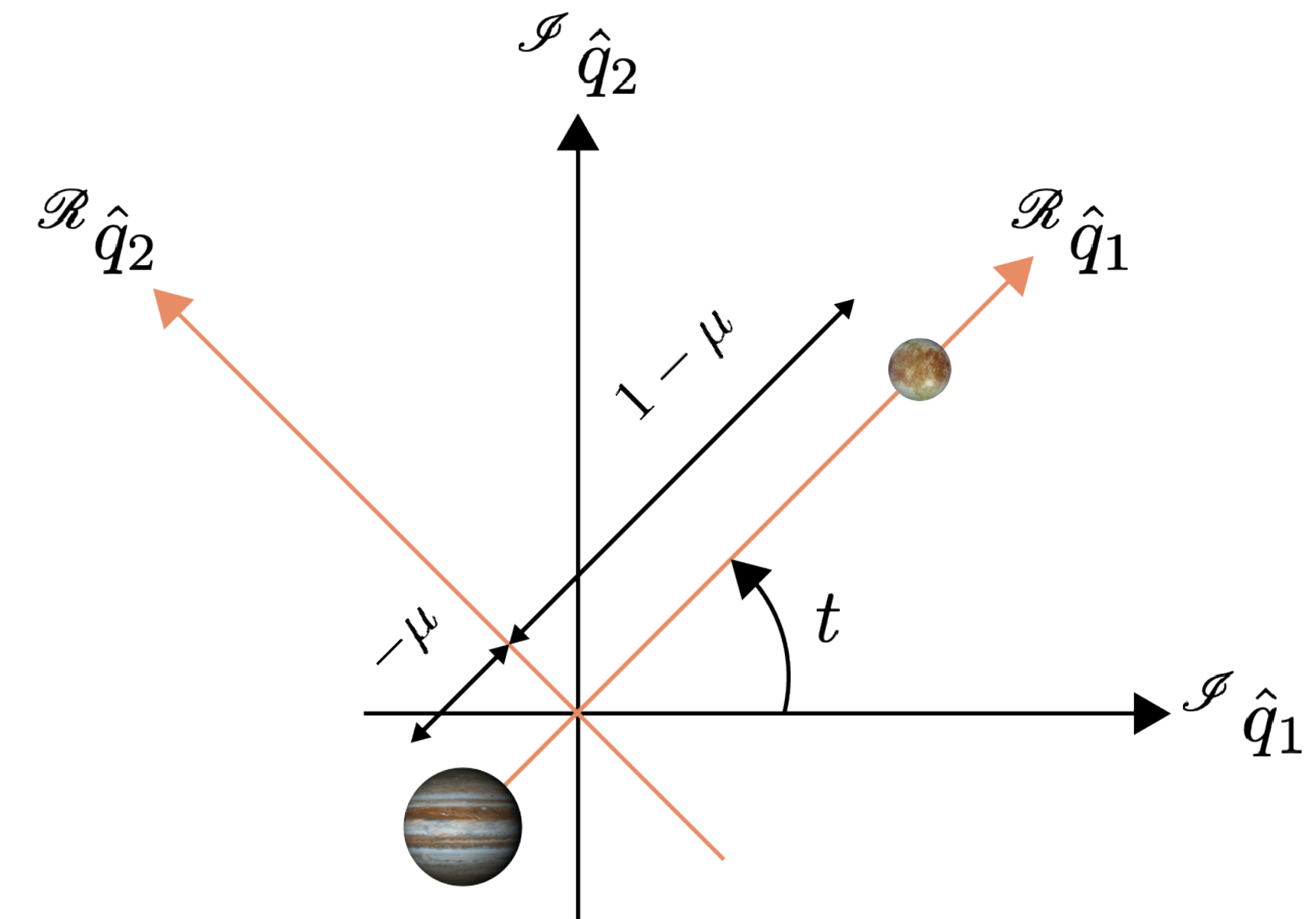
$$\mathbf{f}(t, \boldsymbol{\xi}; \alpha) = \begin{bmatrix} \mathbf{v} \\ \mathbf{a}_{\text{nat}}(t, \boldsymbol{\xi}; \alpha) \\ 0 \end{bmatrix},$$

$$\mathbf{a}_{\text{nat}}(\boldsymbol{\xi}) = -2\hat{\mathbf{q}}_3 \times {}^{\mathcal{R}}\dot{\mathbf{q}} + \nabla \bar{U}(\mathbf{q})$$

$$\bar{U}(q_1, q_2, q_3) = \frac{1}{2}(q_1^2 + q_2^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$

$$r_1(q_1, q_2, q_3) := \sqrt{(q_1 + \mu)^2 + q_2^2 + q_3^2},$$

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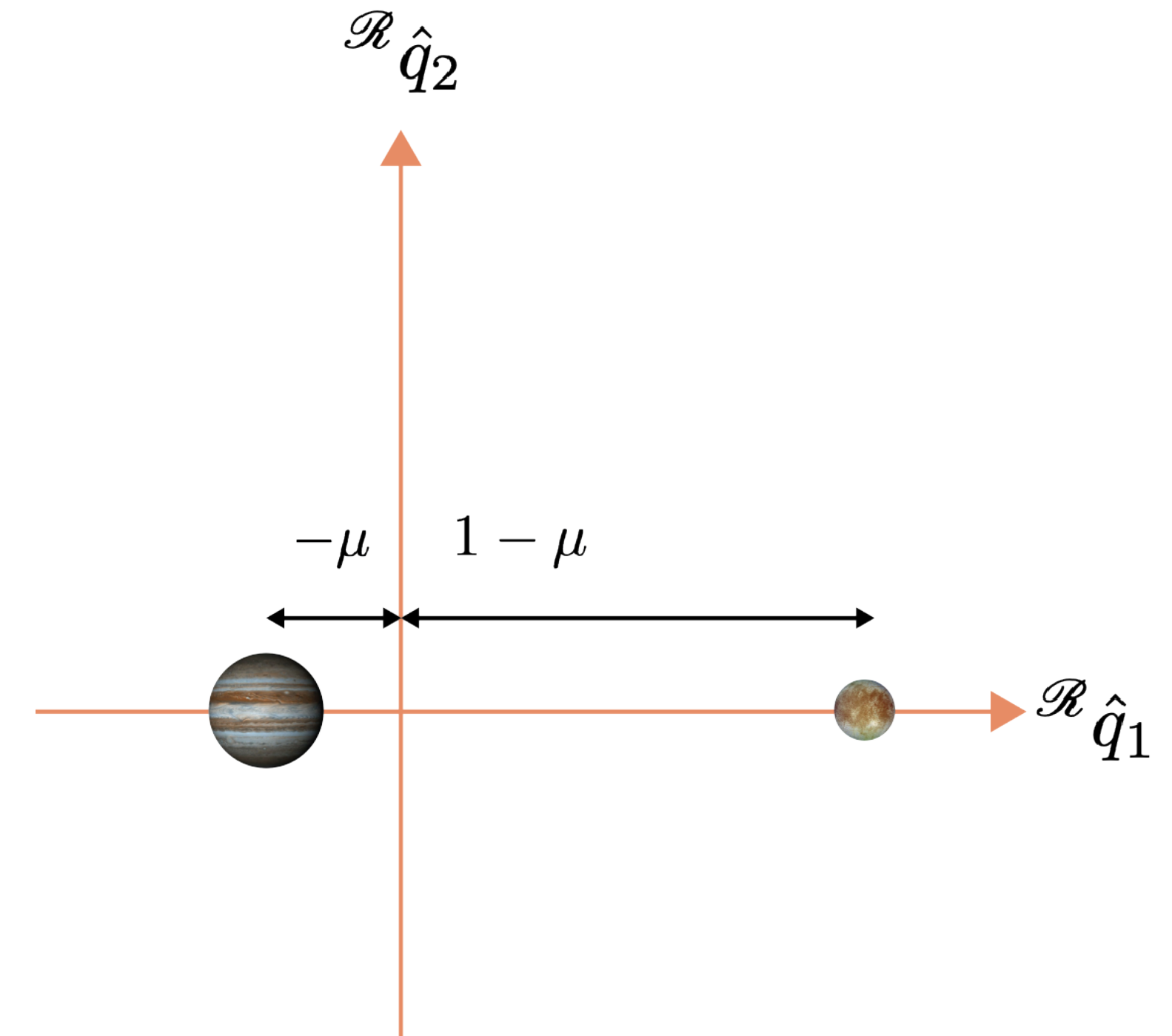
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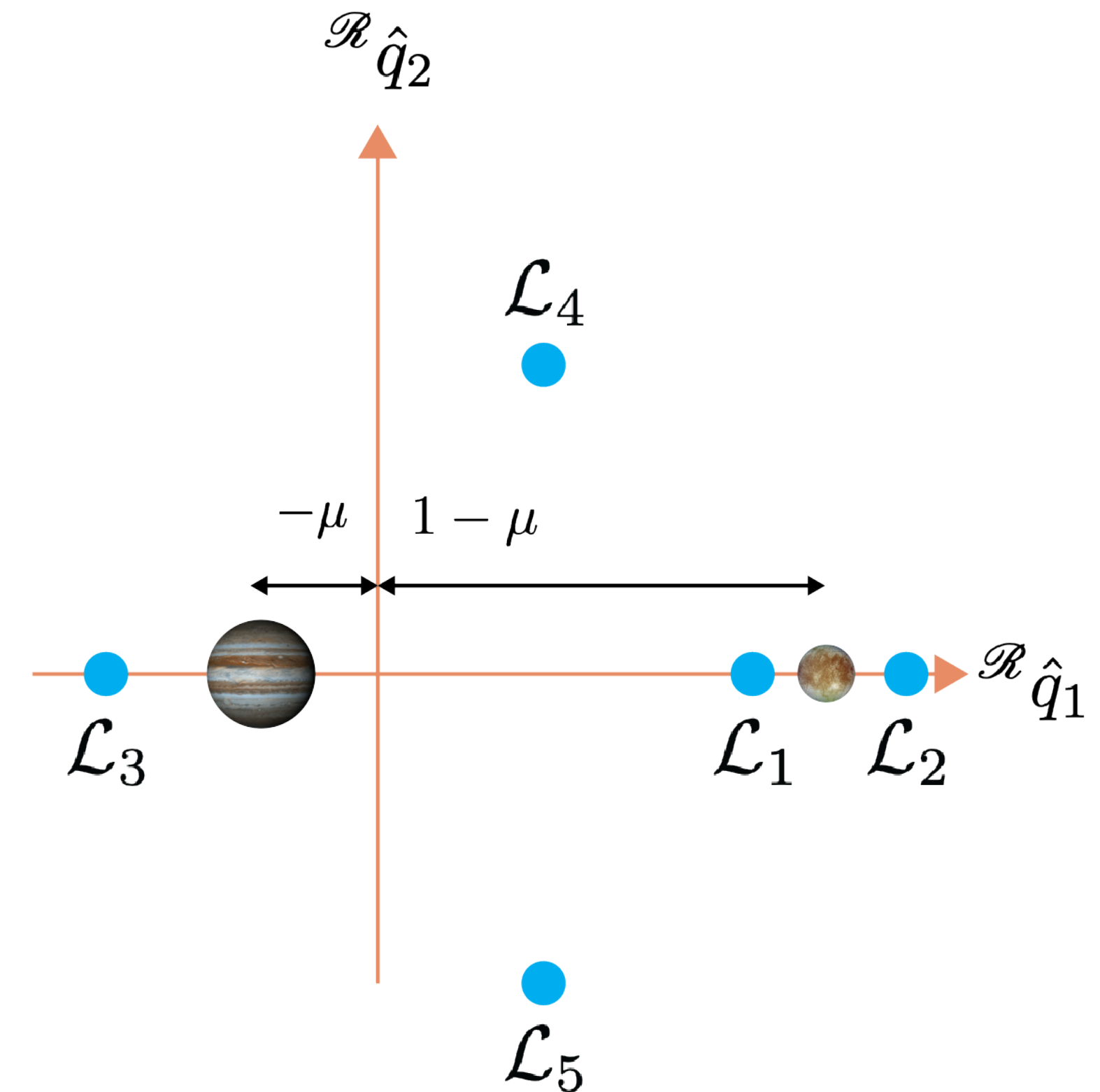
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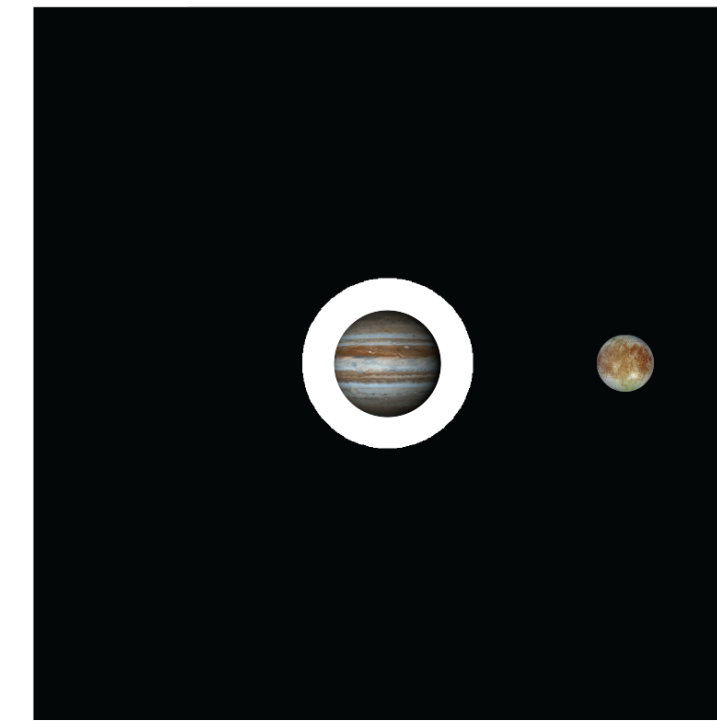
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$$\bar{U}(q_1, q_2, q_3) = \frac{1}{2}(q_1^2 + q_2^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$

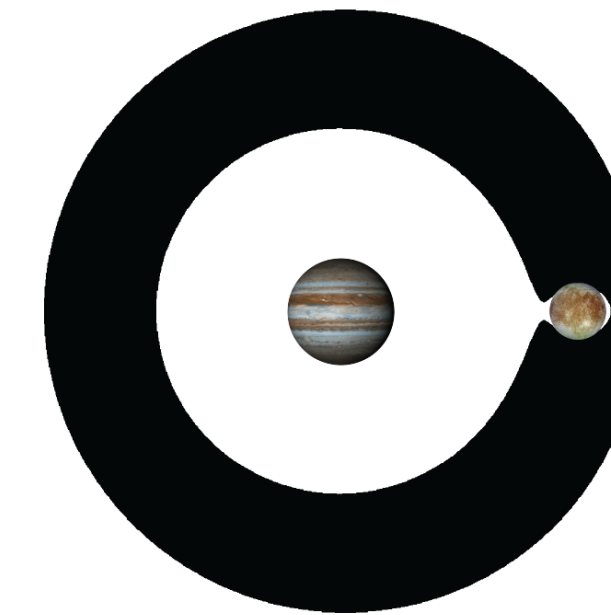
$$r_1(q_1, q_2, q_3) := \sqrt{(q_1 + \mu)^2 + q_2^2 + q_3^2},$$

$$r_2(q_1, q_2, q_3) := \sqrt{(q_1 - (1 - \mu))^2 + q_2^2 + q_3^2}$$

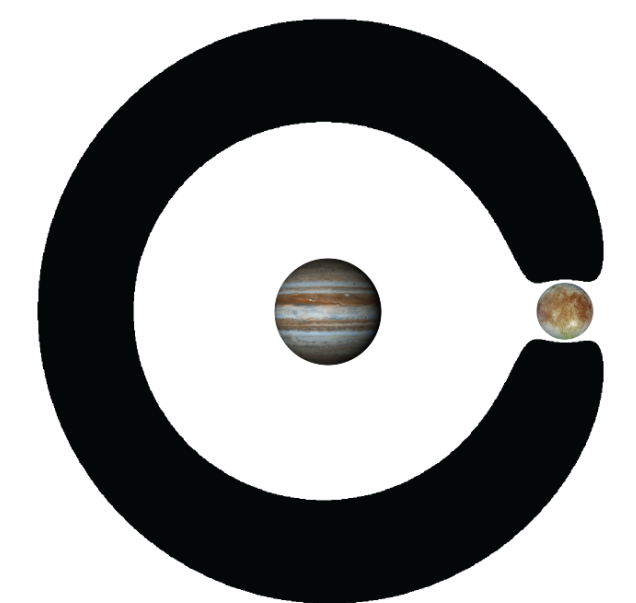
$$E < E_{\mathcal{L}_1}$$



$$E_{\mathcal{L}_1} < E < E_{\mathcal{L}_2}$$



$$E_{\mathcal{L}_2} < E < E_{\mathcal{L}_3}$$



$$E_{\mathcal{L}_3} < E < E_{\mathcal{L}_4} = E_{\mathcal{L}_5} \quad E_{\mathcal{L}_4} = E_{\mathcal{L}_5} < E$$



$$-2E = C = -2\bar{U}(q_1, q_2, q_3) - \|\dot{q}\|^2$$

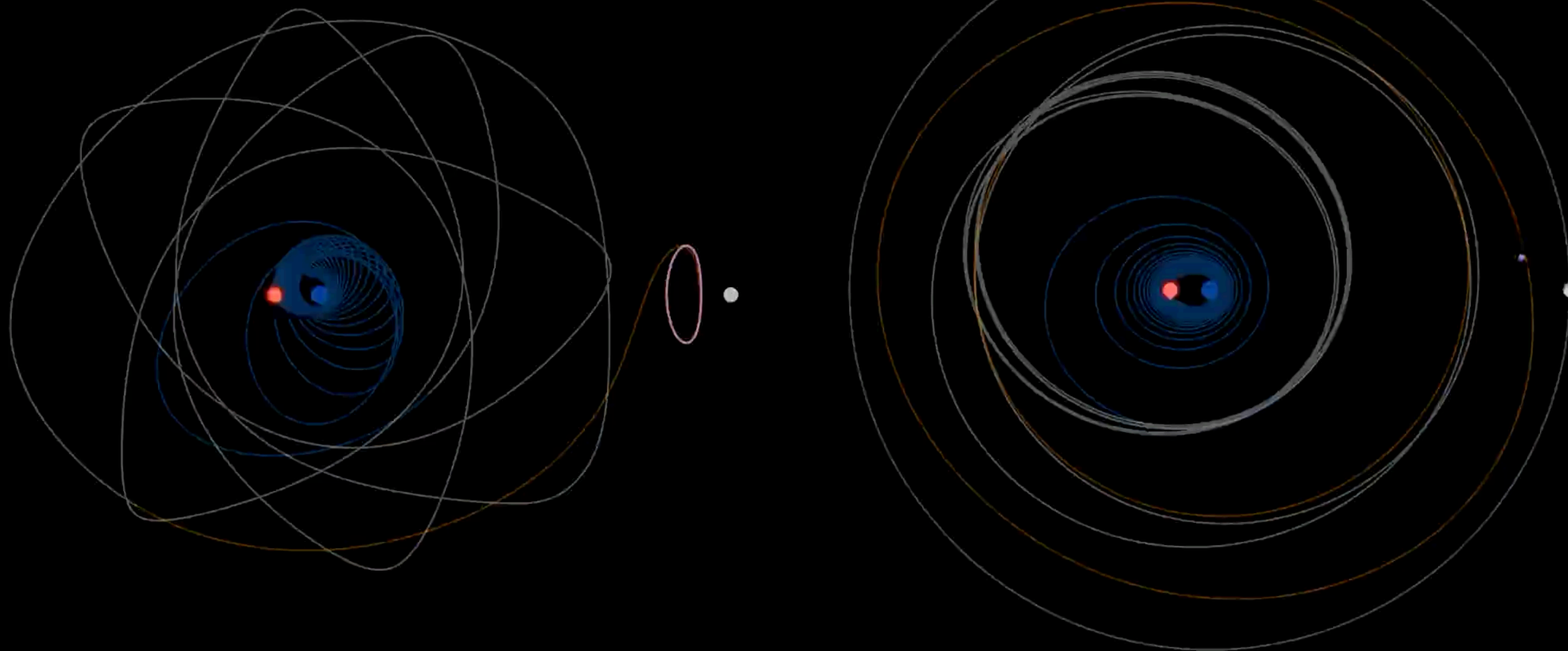
C = Jacobi Constant
governs allowable motion

An example low-thrust solution in the Earth-Moon system

Synodic frame

Inertial frame

0.00 days

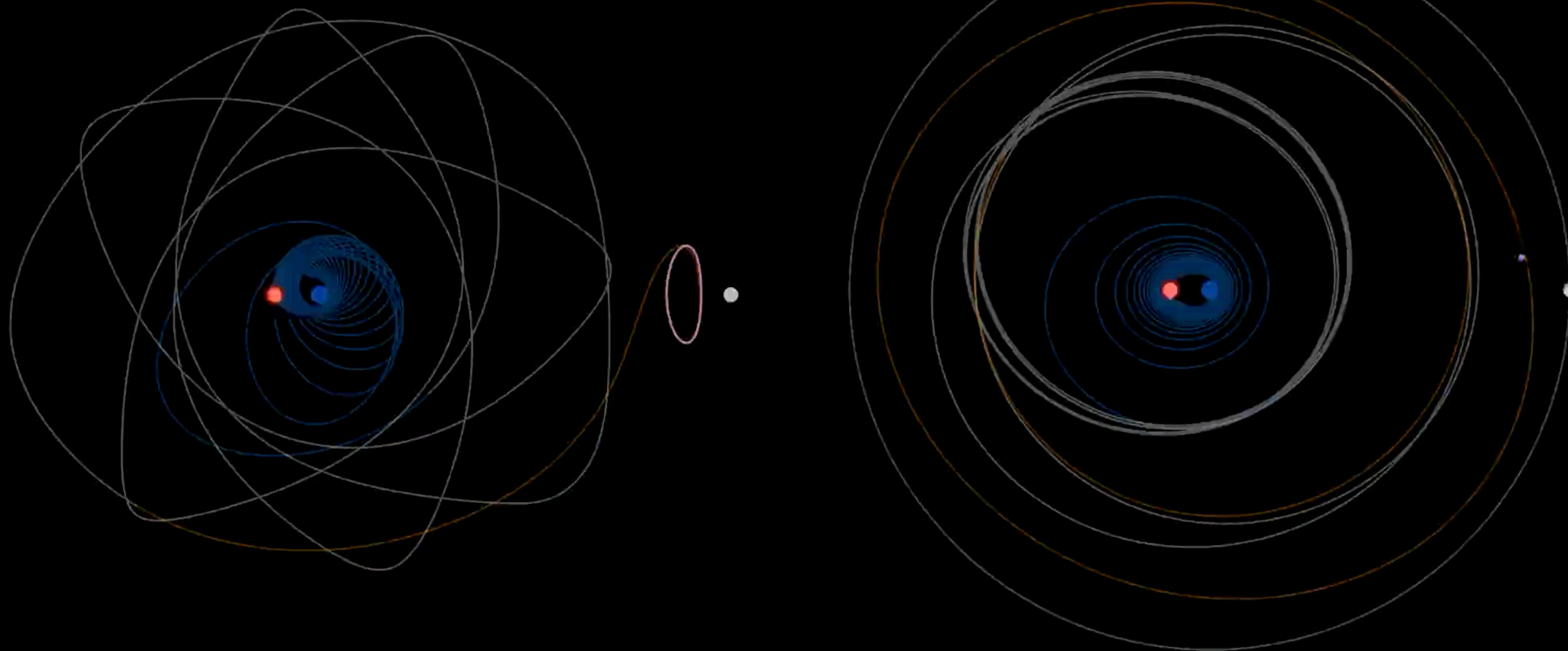


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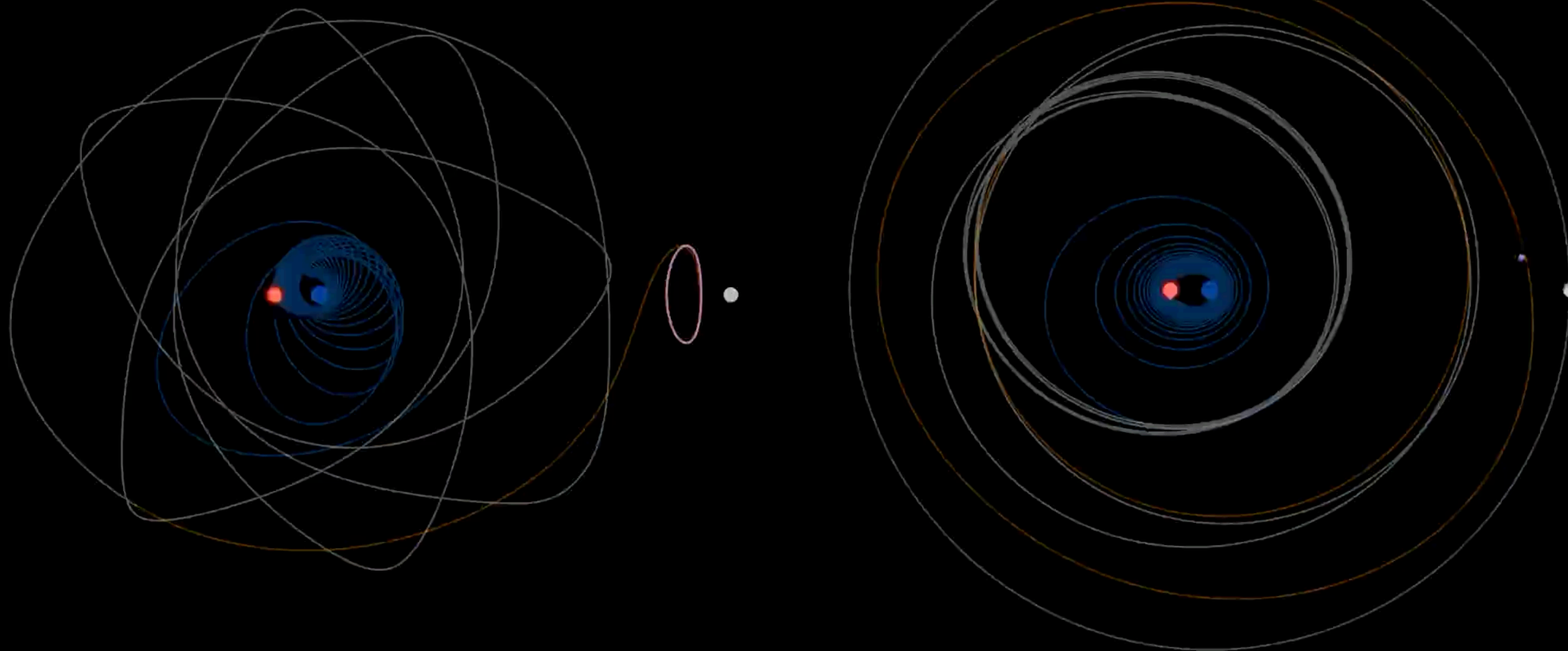


An example low-thrust solution in the Earth-Moon system

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The invariant manifolds can also be leveraged for robust LT trajectory design

CR3BP exhibits rich dynamical properties

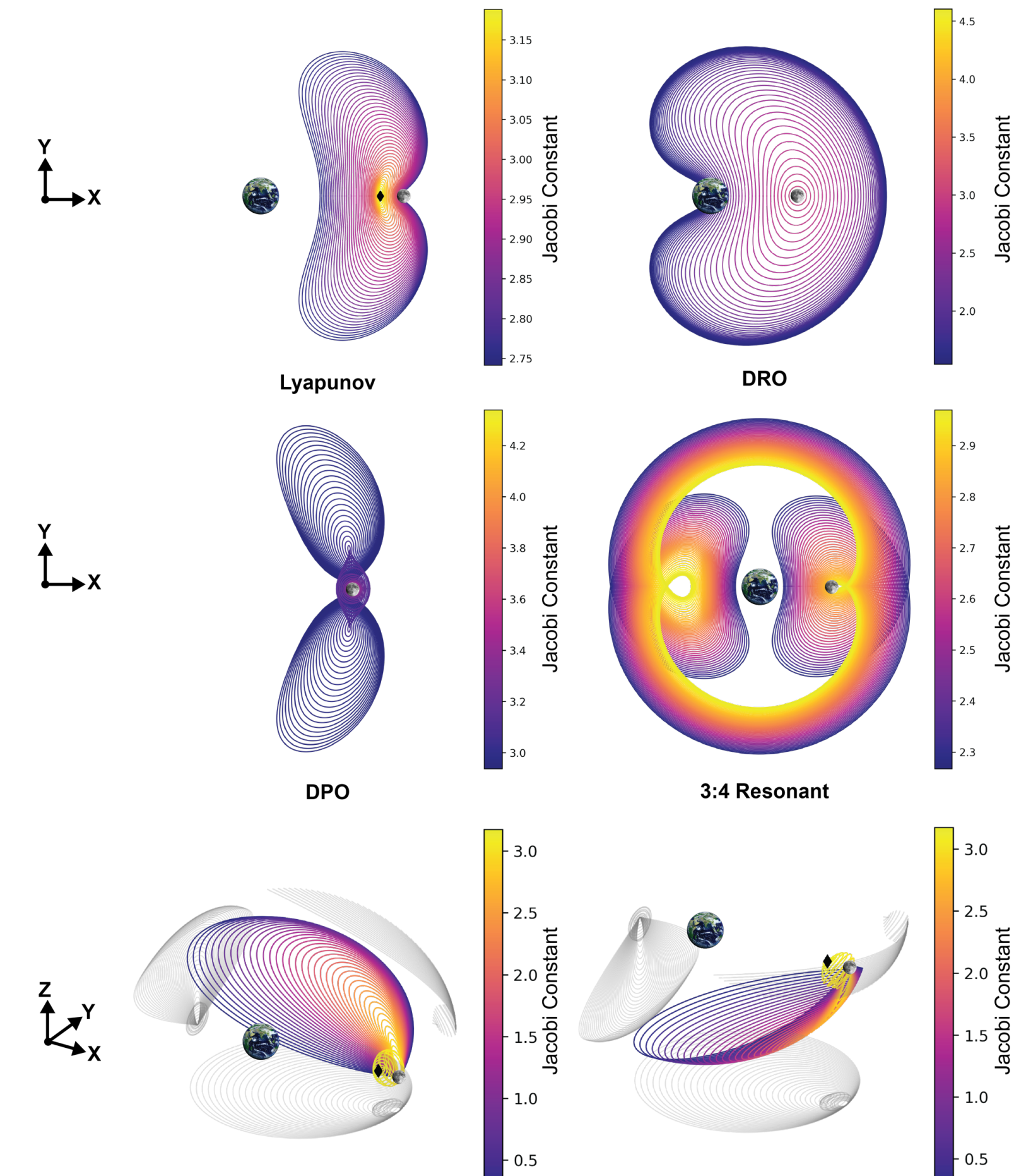
- The CR3BP admits many families of periodic orbits (e.g., Lyapunov, Halo, DRO, DPO, resonant families)
- Invariant manifolds act as interplanetary highways
- Hyperbolic periodic orbits possesses **stable** and **unstable** manifolds

Stable Manifold:

$$W_s^\gamma = \{\mathbf{z} \in \mathcal{M} : \varphi_t(\mathbf{z}) \rightarrow \gamma \text{ as } t \rightarrow +\infty\},$$

Unstable Manifold:

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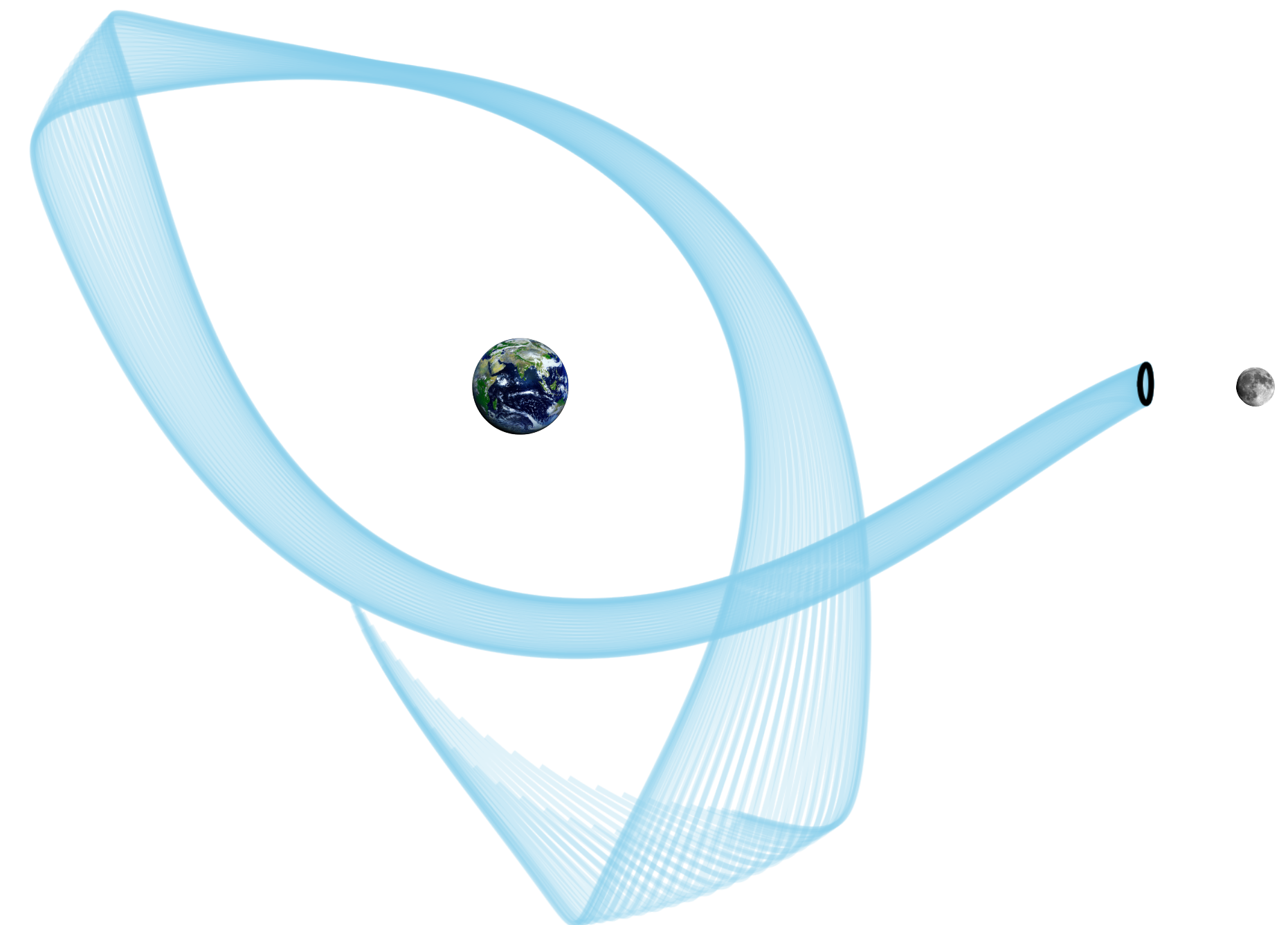
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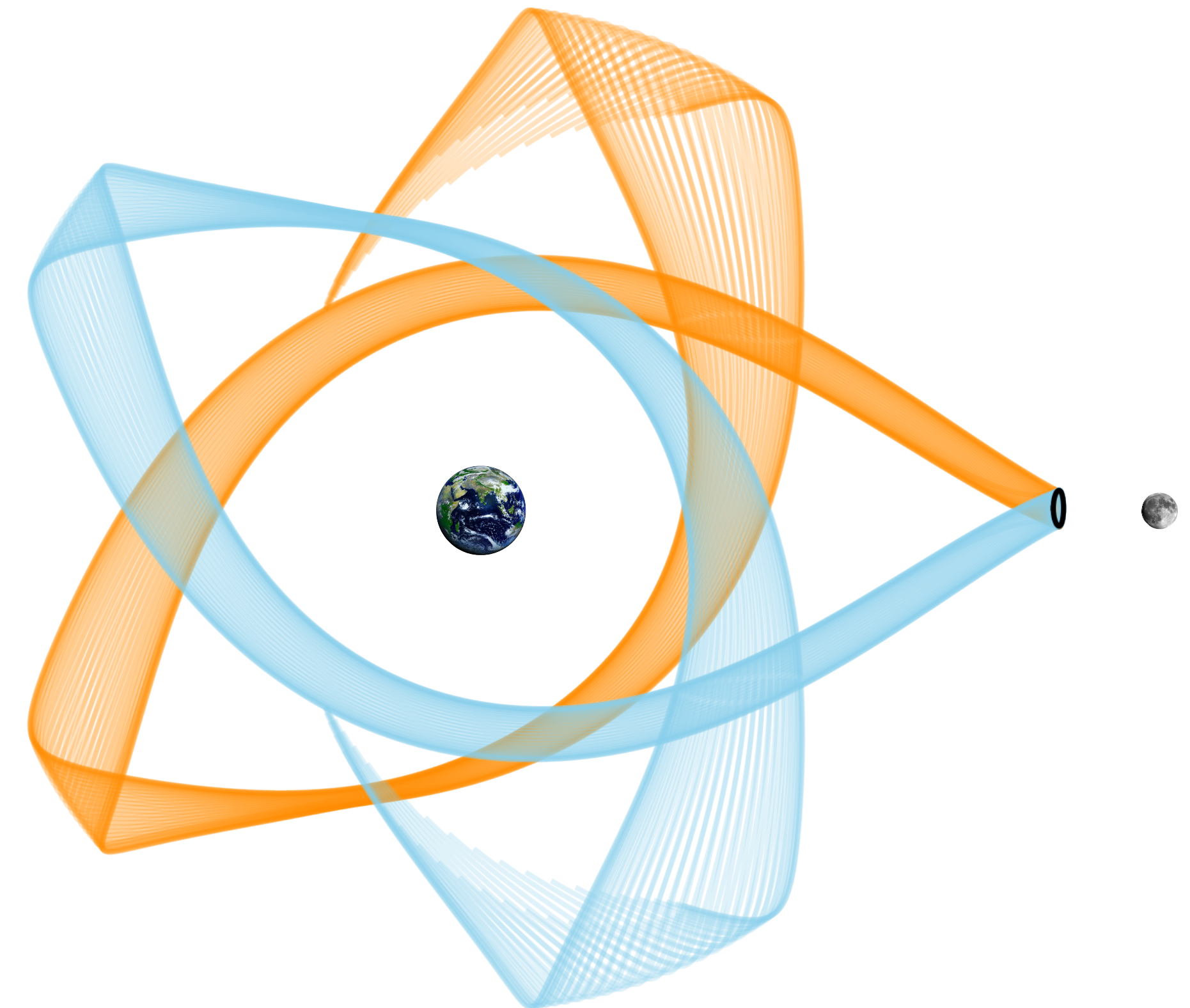
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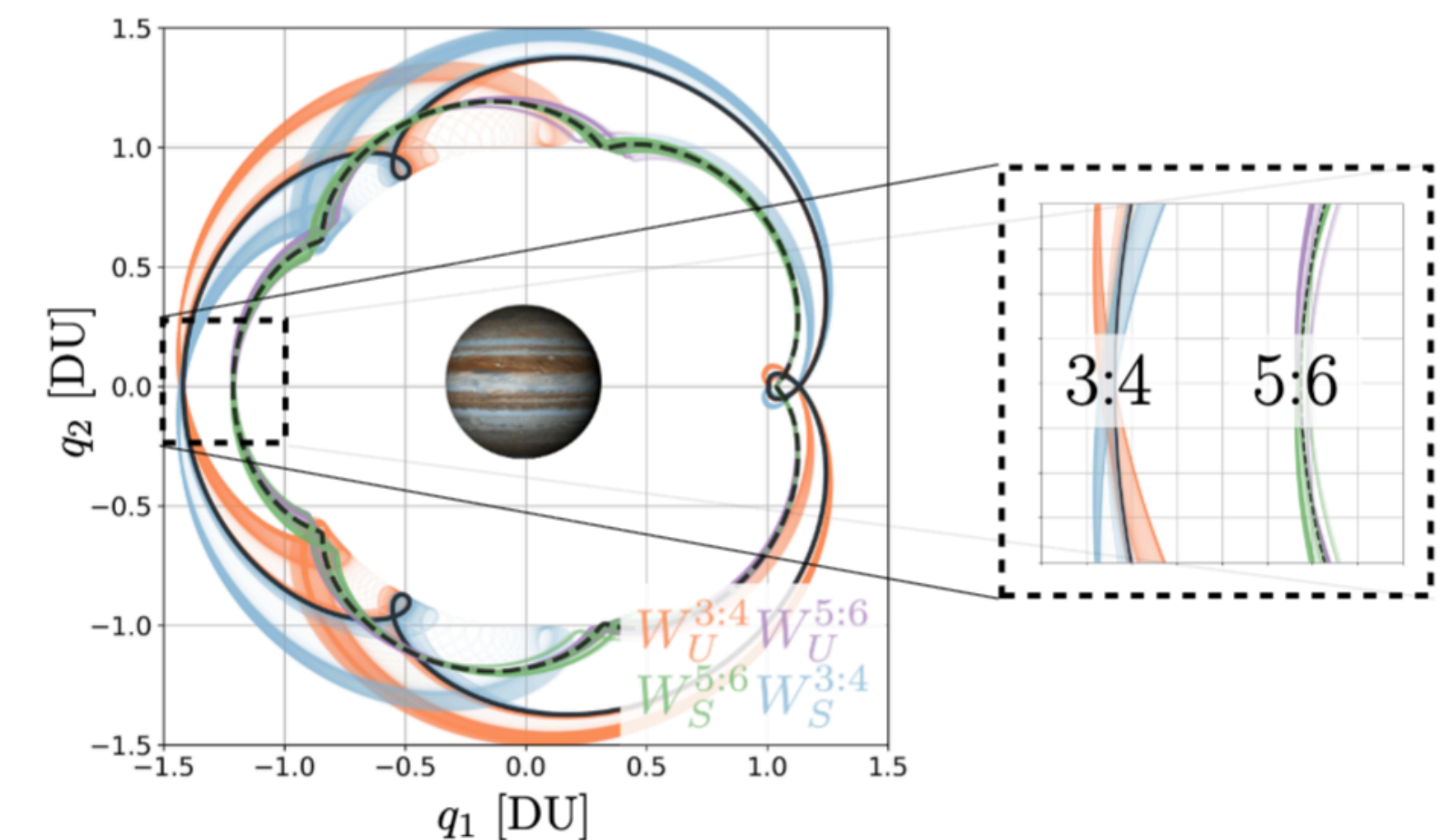
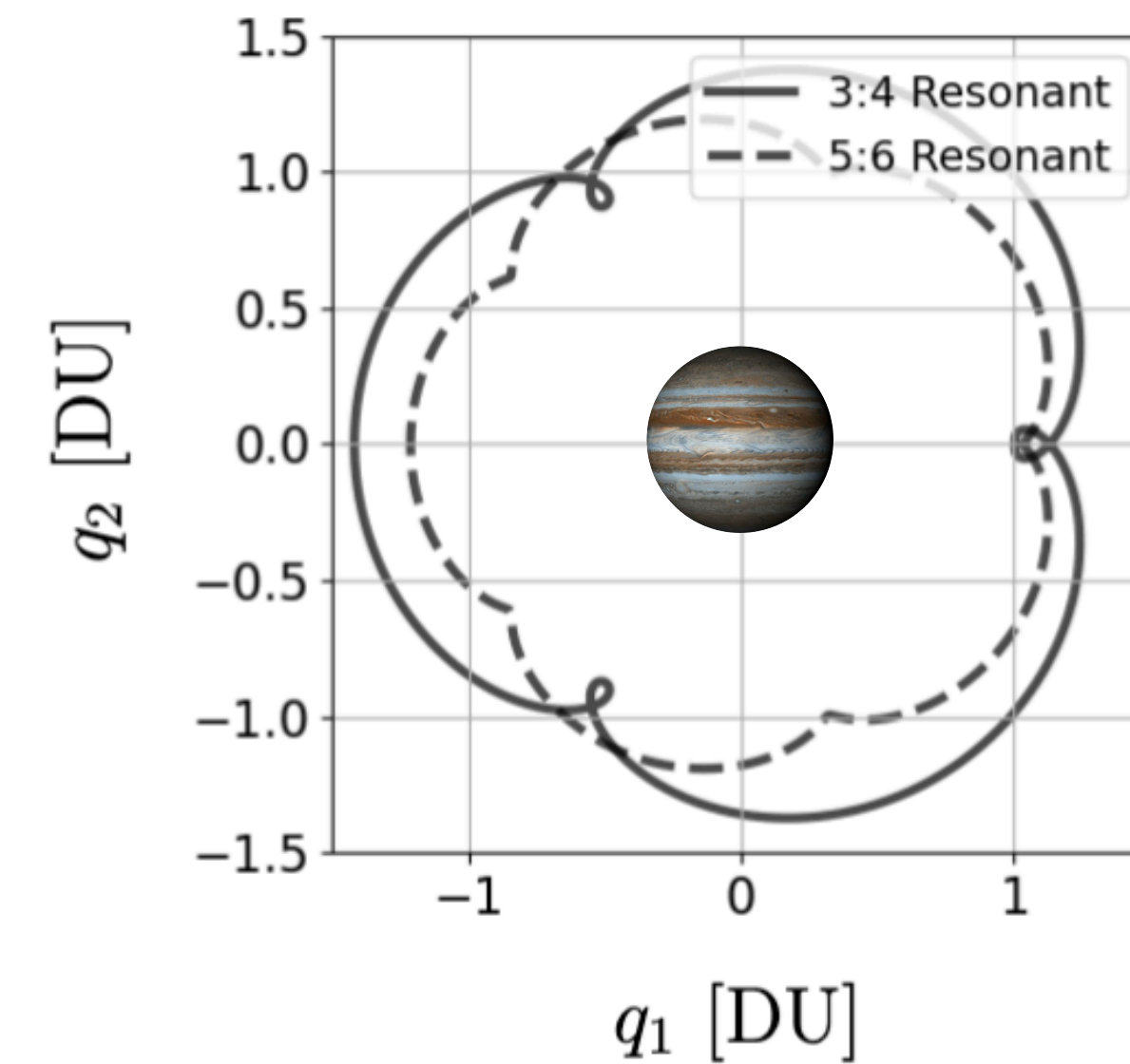


Case study: Low-thrust transfer between resonant orbits in the Jupiter-Europa (JE) CR3BP

The spacecraft motion is modeled in the JE CR3BP

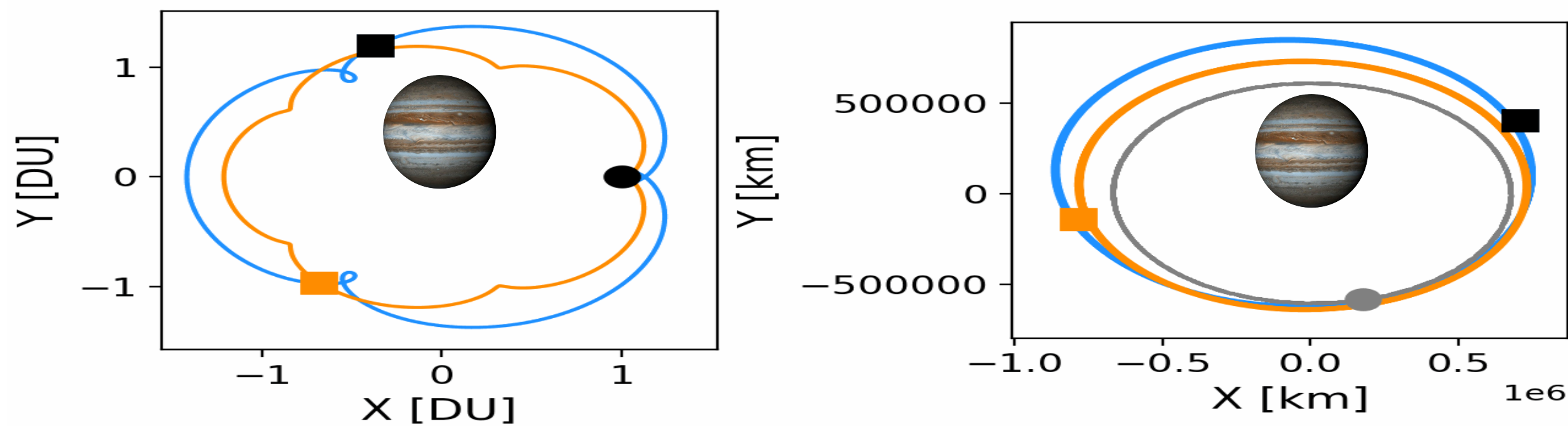
- The spacecraft begins on a 3:4 resonant orbit ($C_{3:4} = 2.995$) and ends on a 5:6 resonant orbit ($C_{5:6} = 3.005$).
- Our goal is to examine how the non-robust and robust solutions relate to the underlying invariant manifolds in this problem

Parameter	Value
Number of Segments	50
I_{SP} [s]	1000.0
Thrust Acceleration [m/s^2]	0.001
Dry Mass [kg]	700.0
Fuel Mass [kg]	300.0



Poincaré maps provide an effective framework for understanding global dynamics

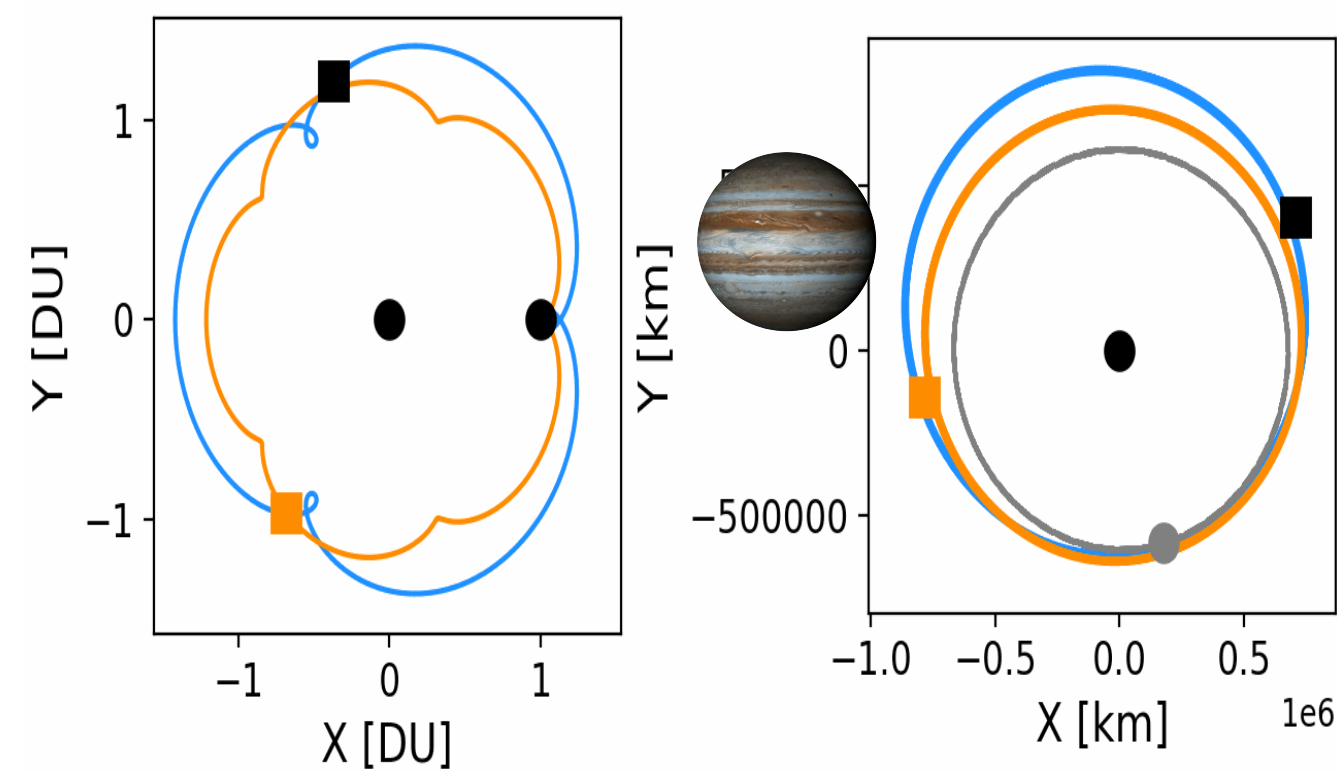
A Poincaré map $\mathcal{P}: \mathcal{S} \rightarrow \mathcal{S}$ reduces a continuous flow to a discrete map on a low dimensional section $\mathcal{S}$



Sinha and Beeson. JGCD. 2025

Poincaré maps provide an effective framework for understanding global dynamics

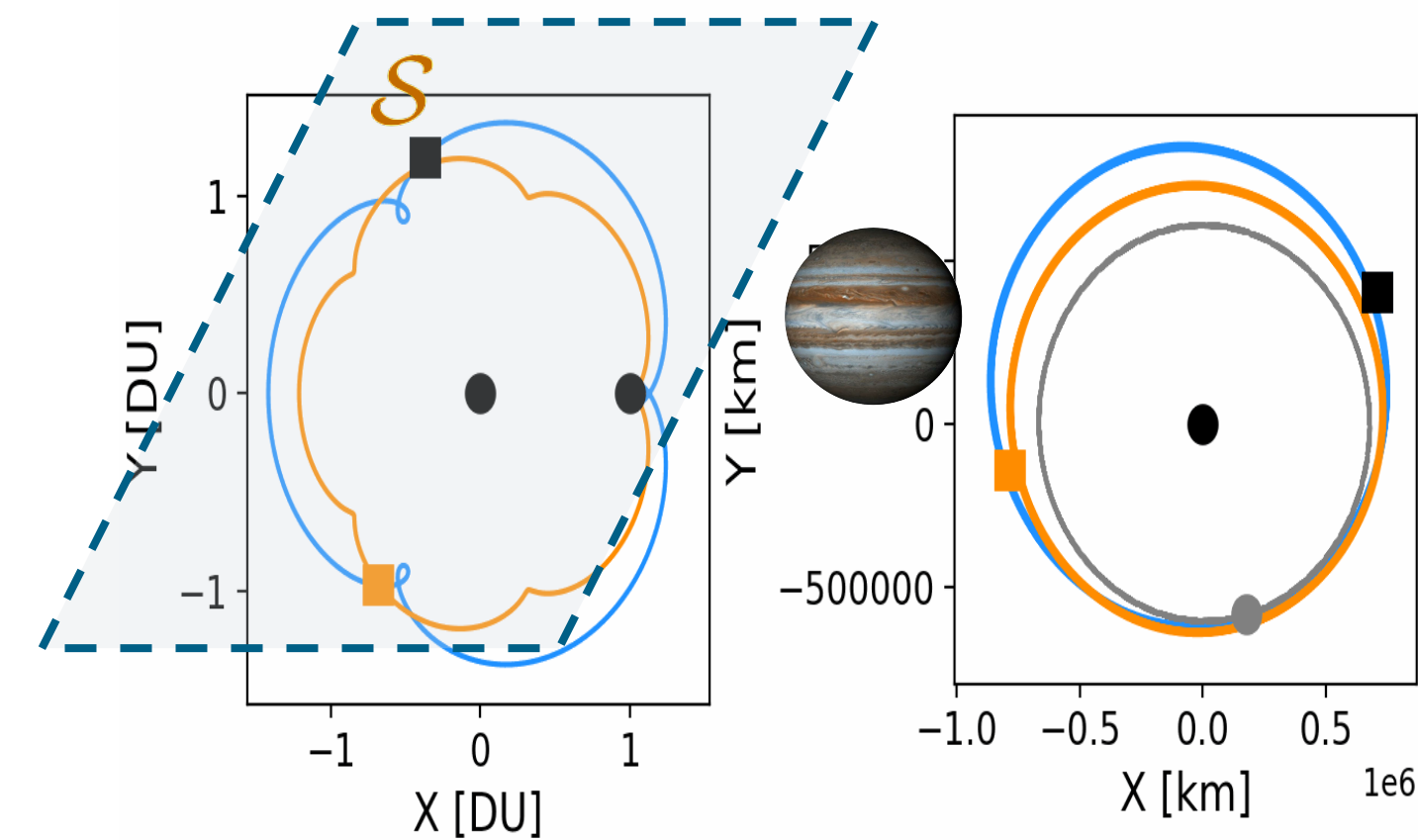
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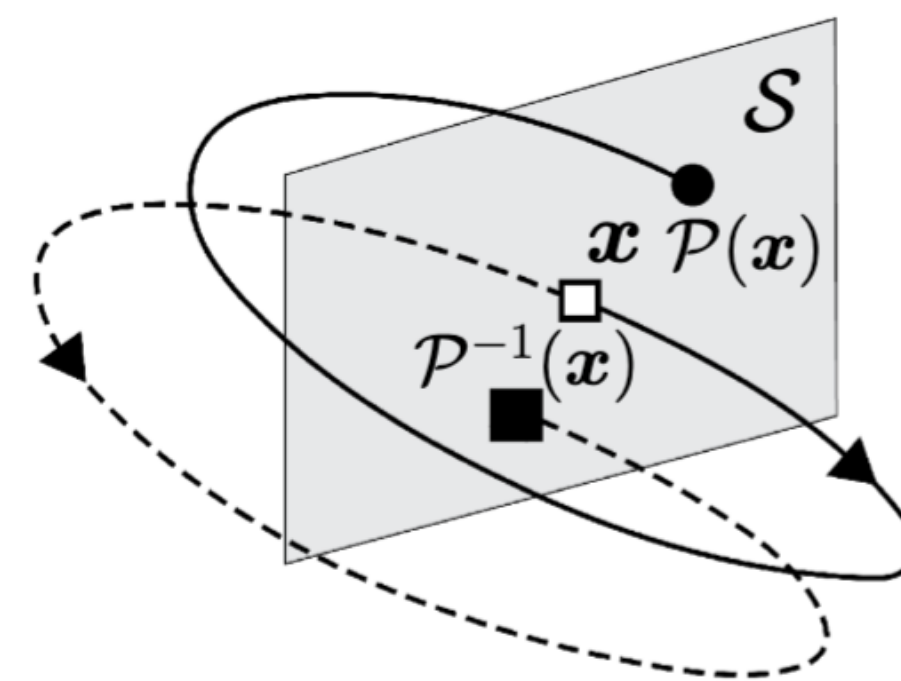
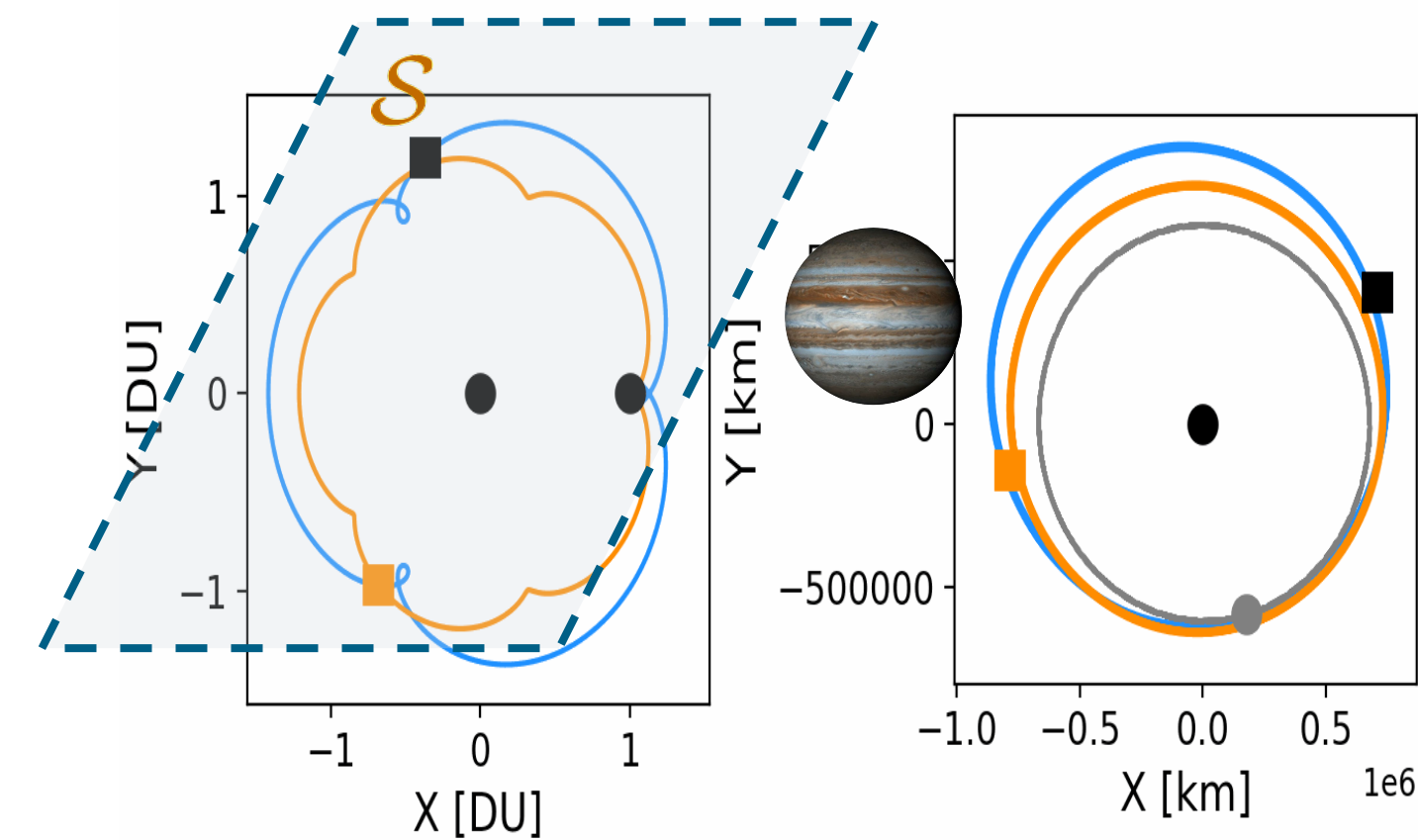
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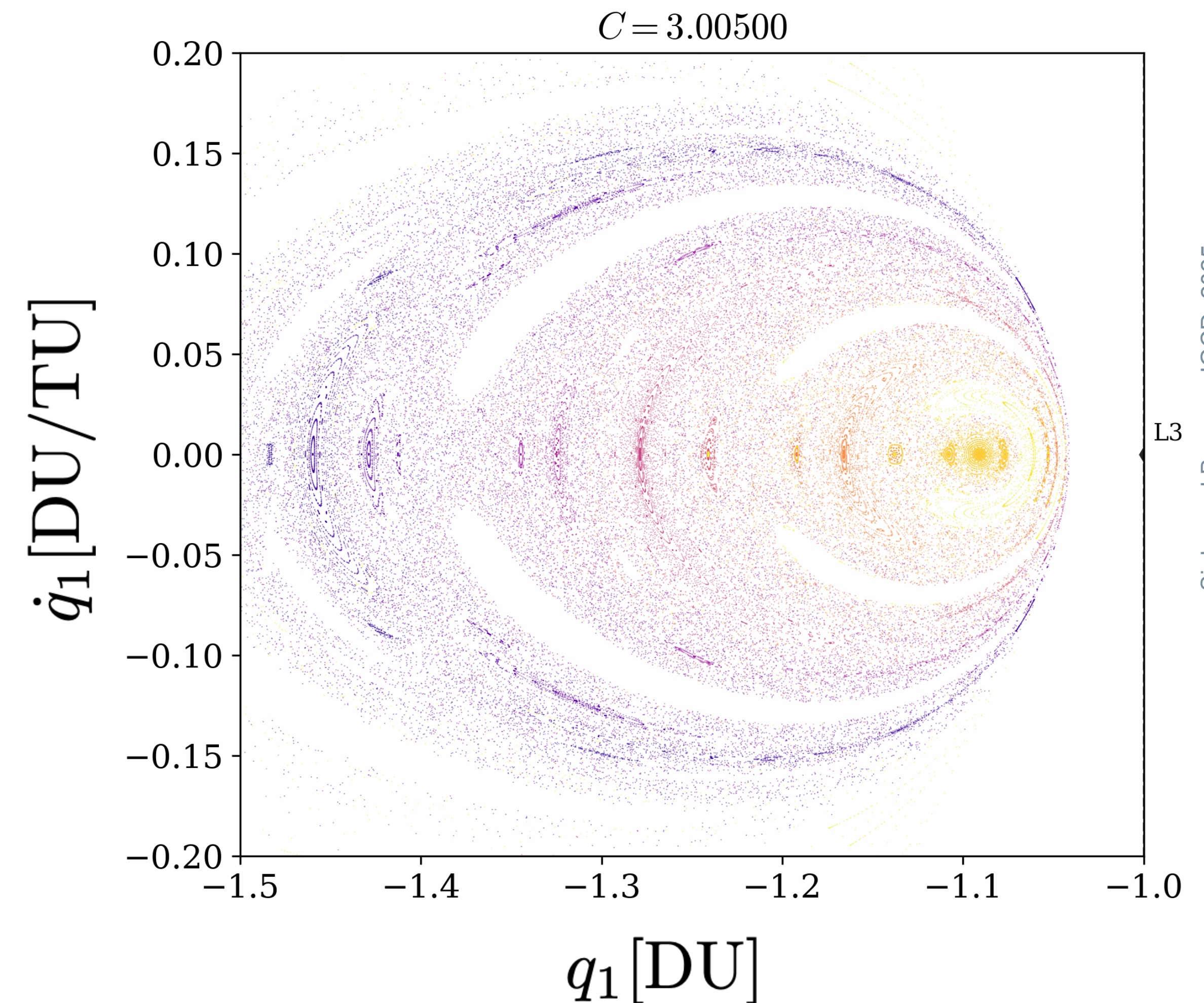
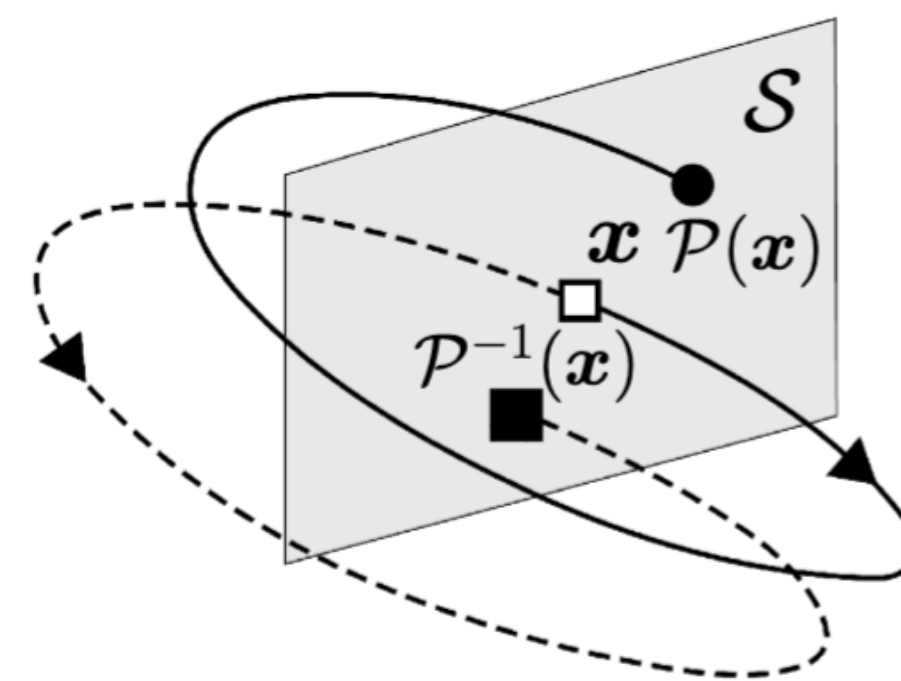
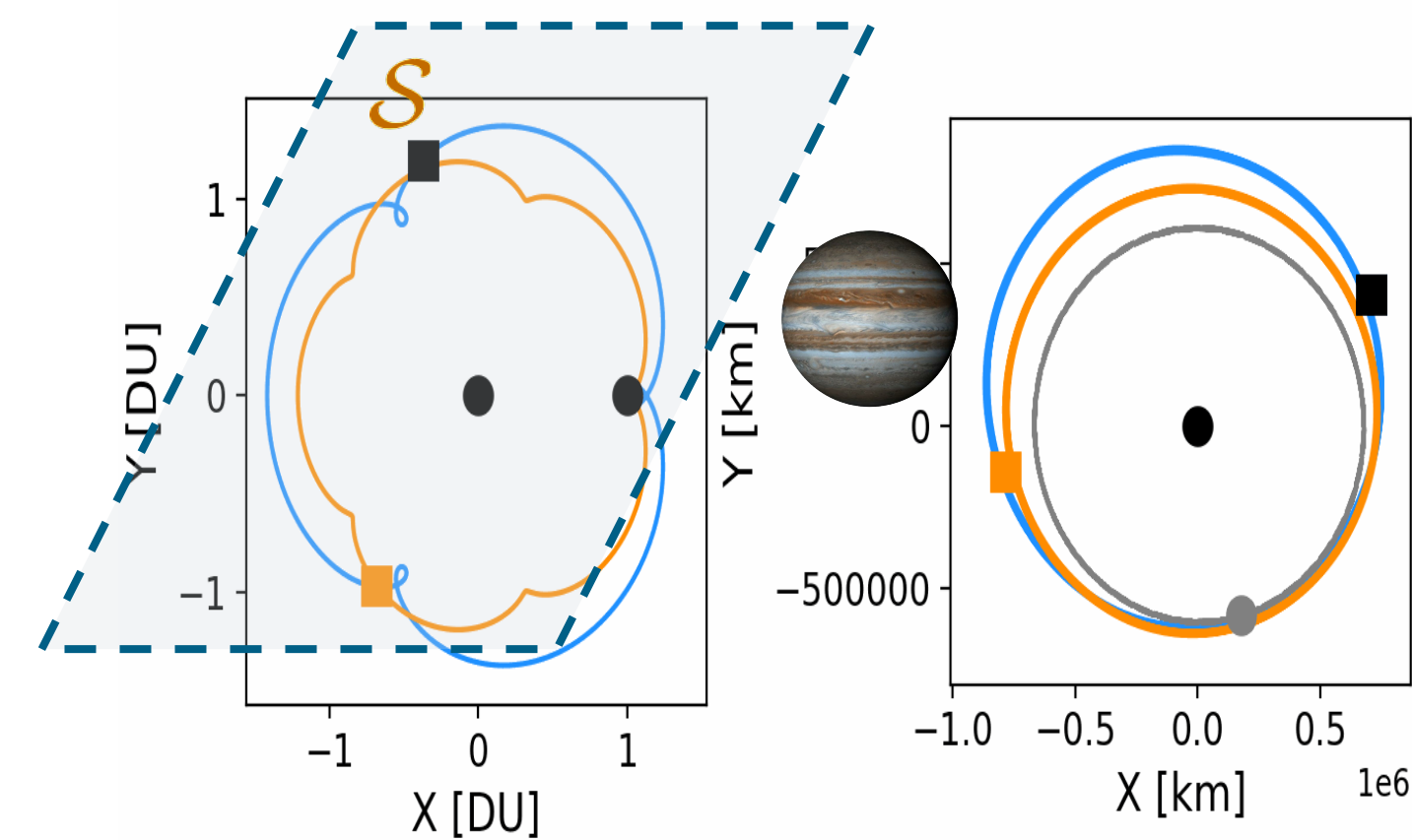


Sinha and Beeson. JGCD. 2025

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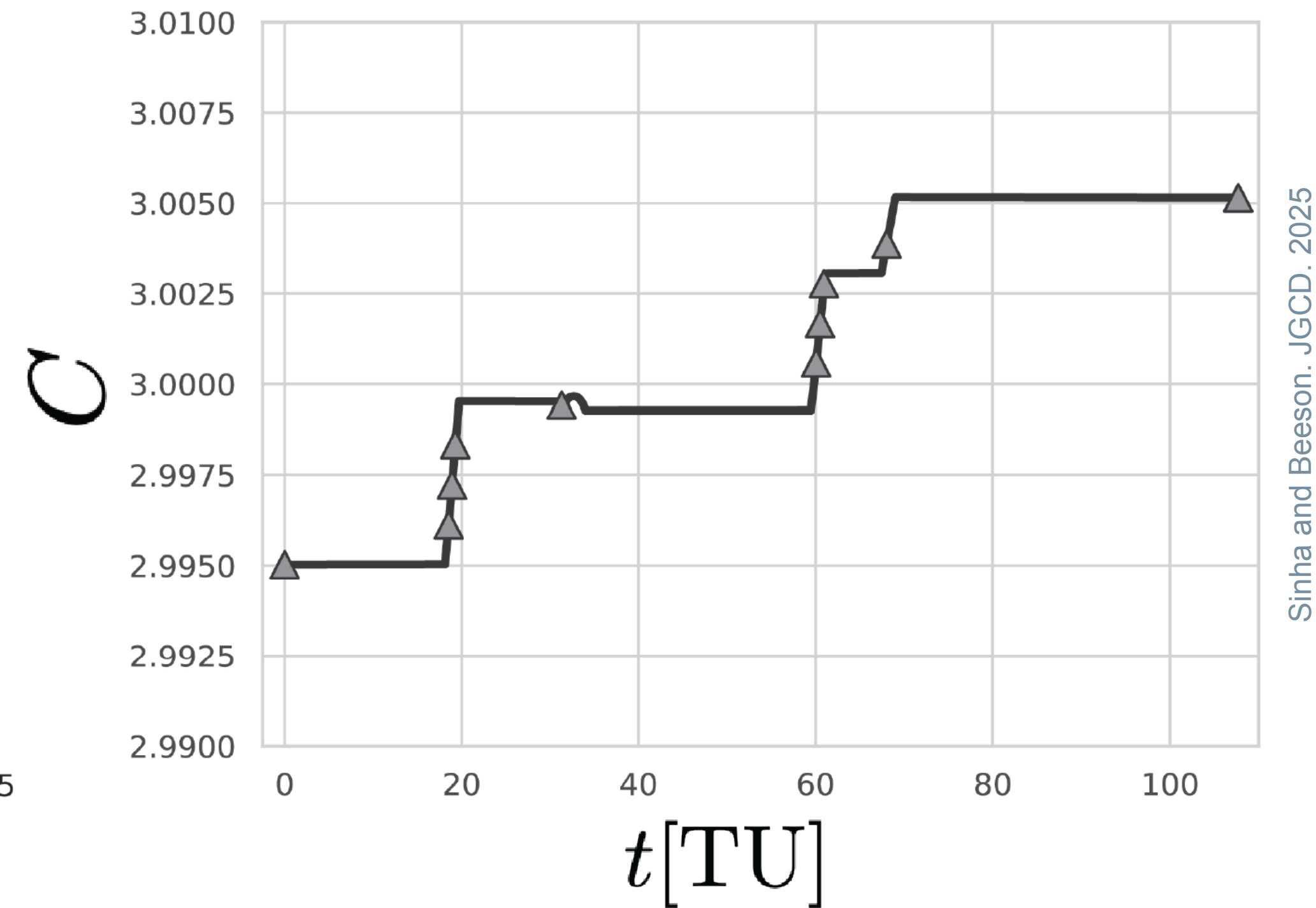
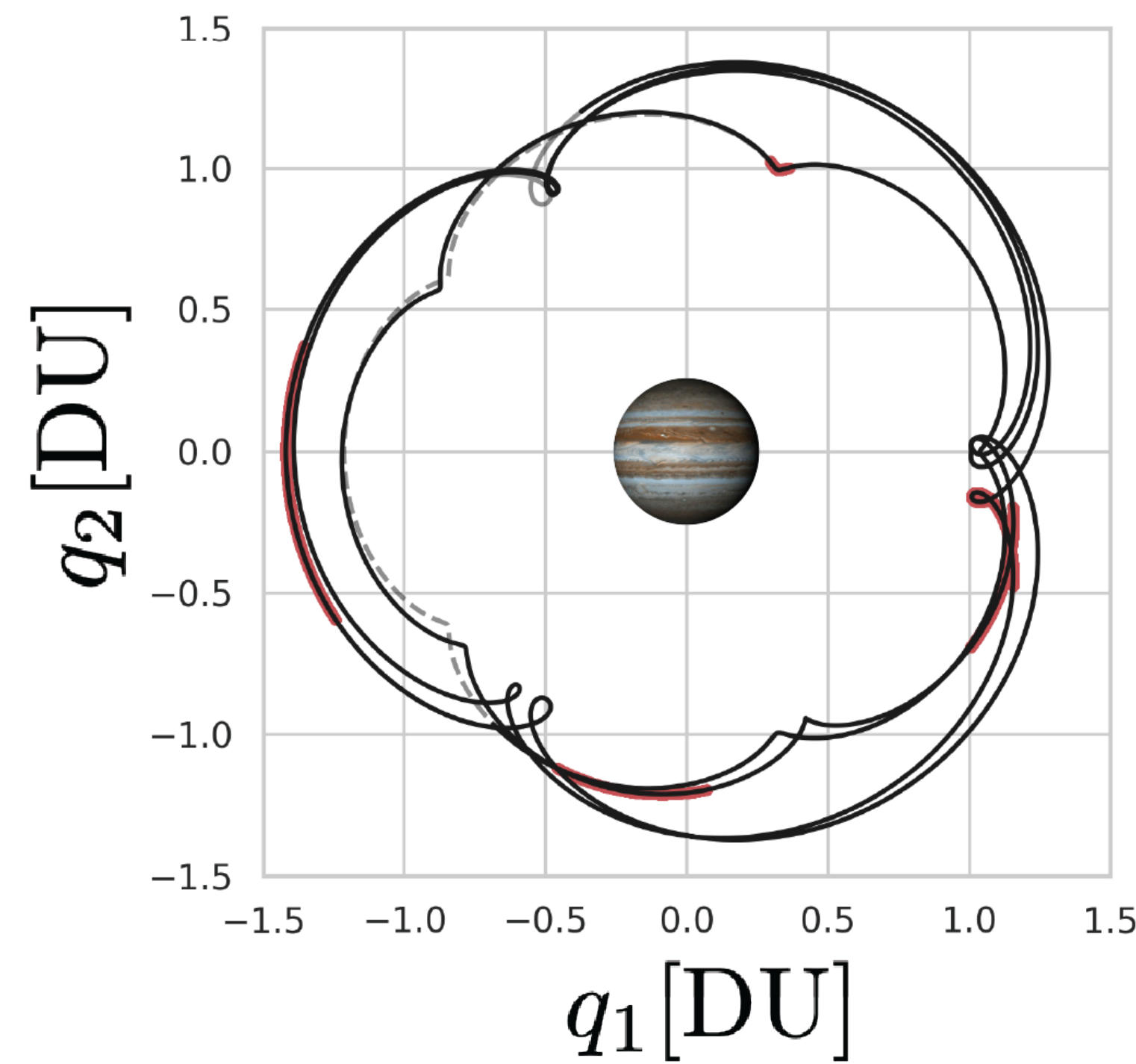
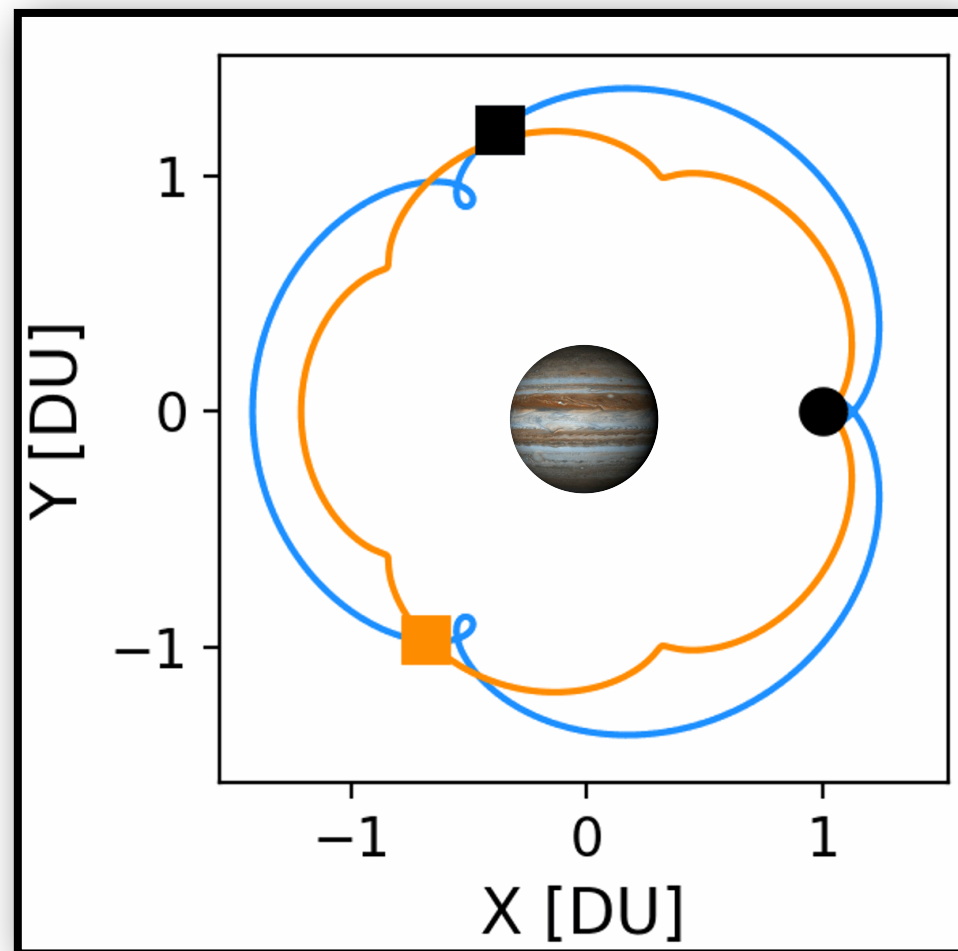
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Sinha and Beeson. JGCD. 2025

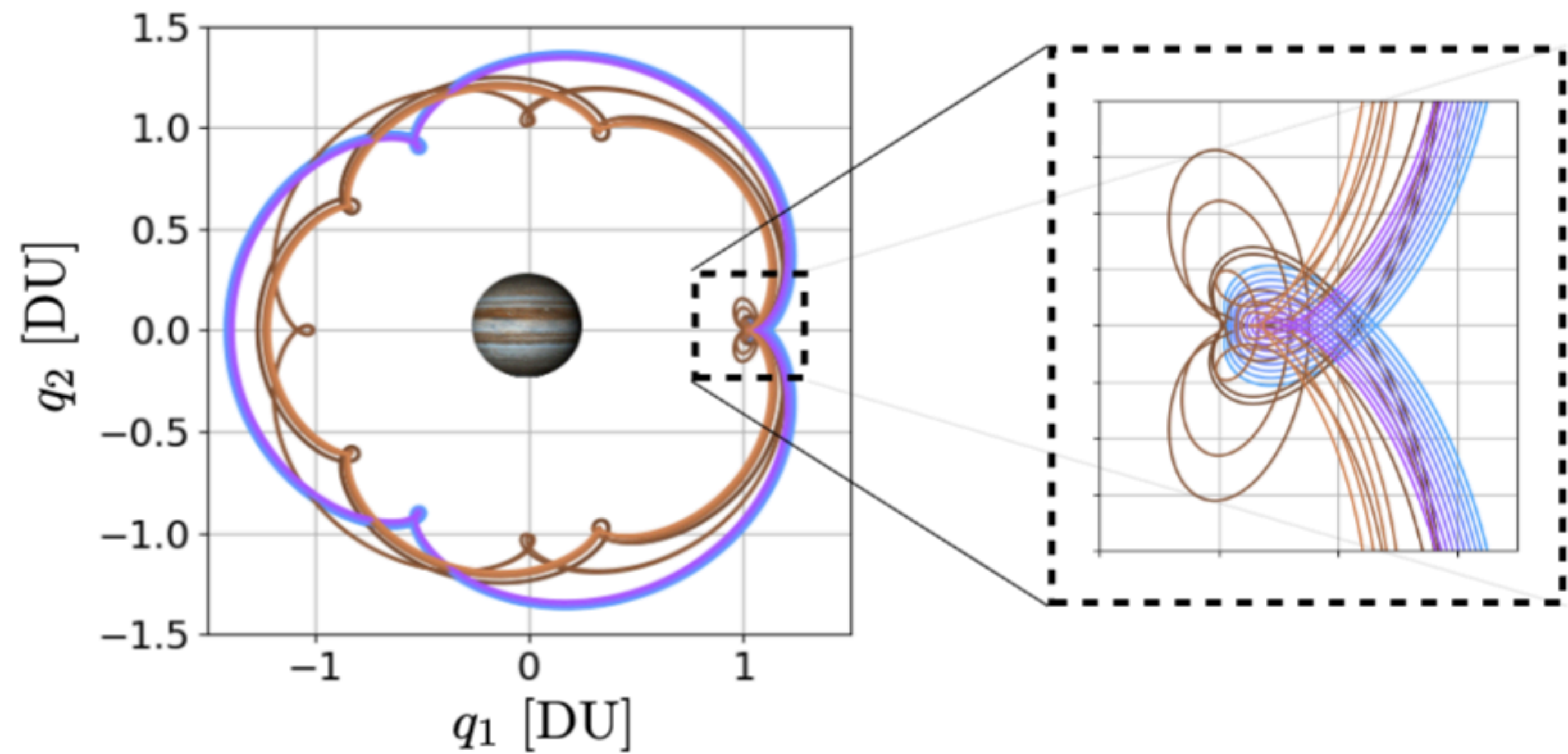
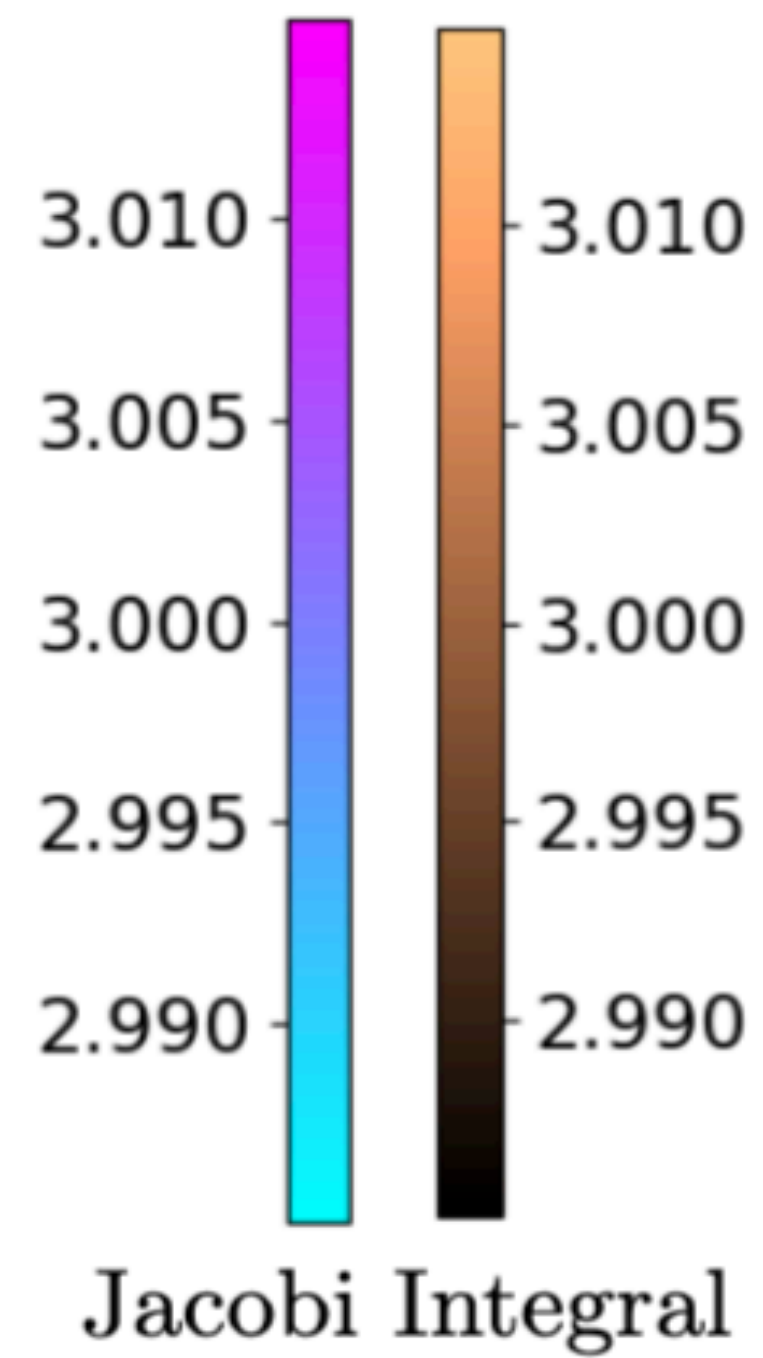
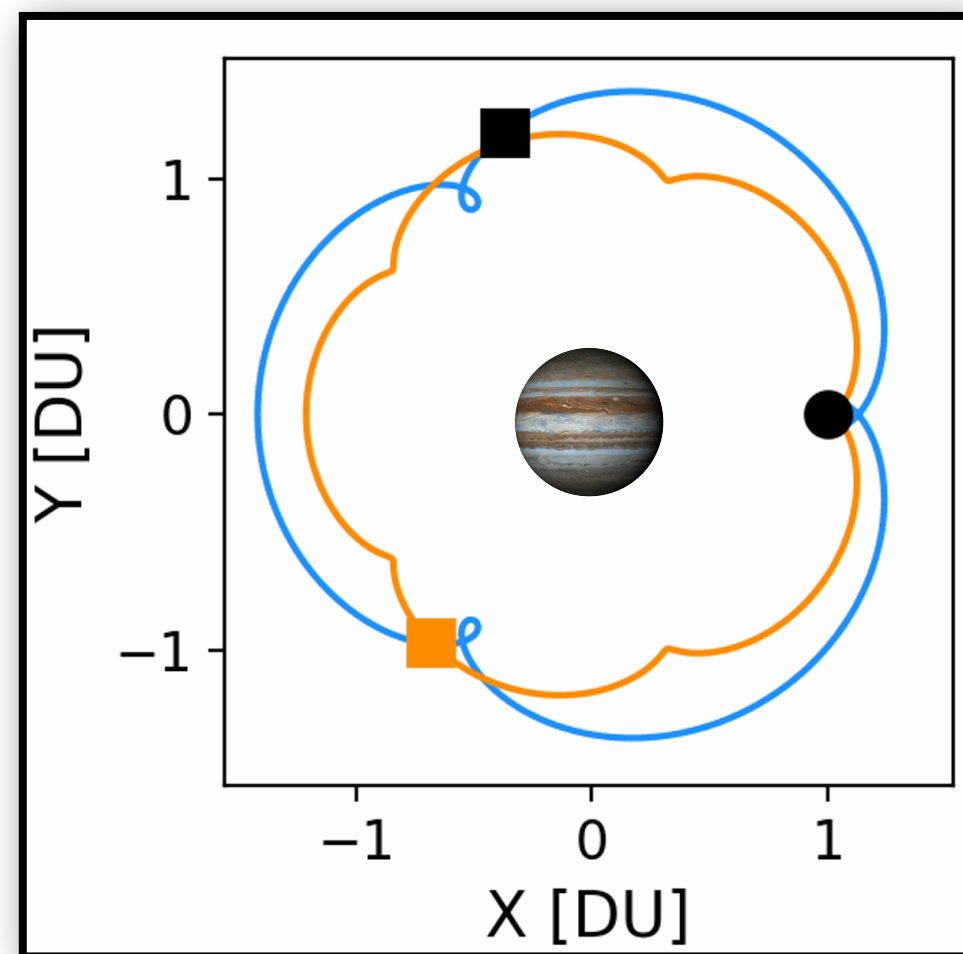
Sinha and Beeson. JGCD. 2025

The Jacobi constant changes continuously over the low-thrust solution



Sinha and Beeson, JGCD, 2025

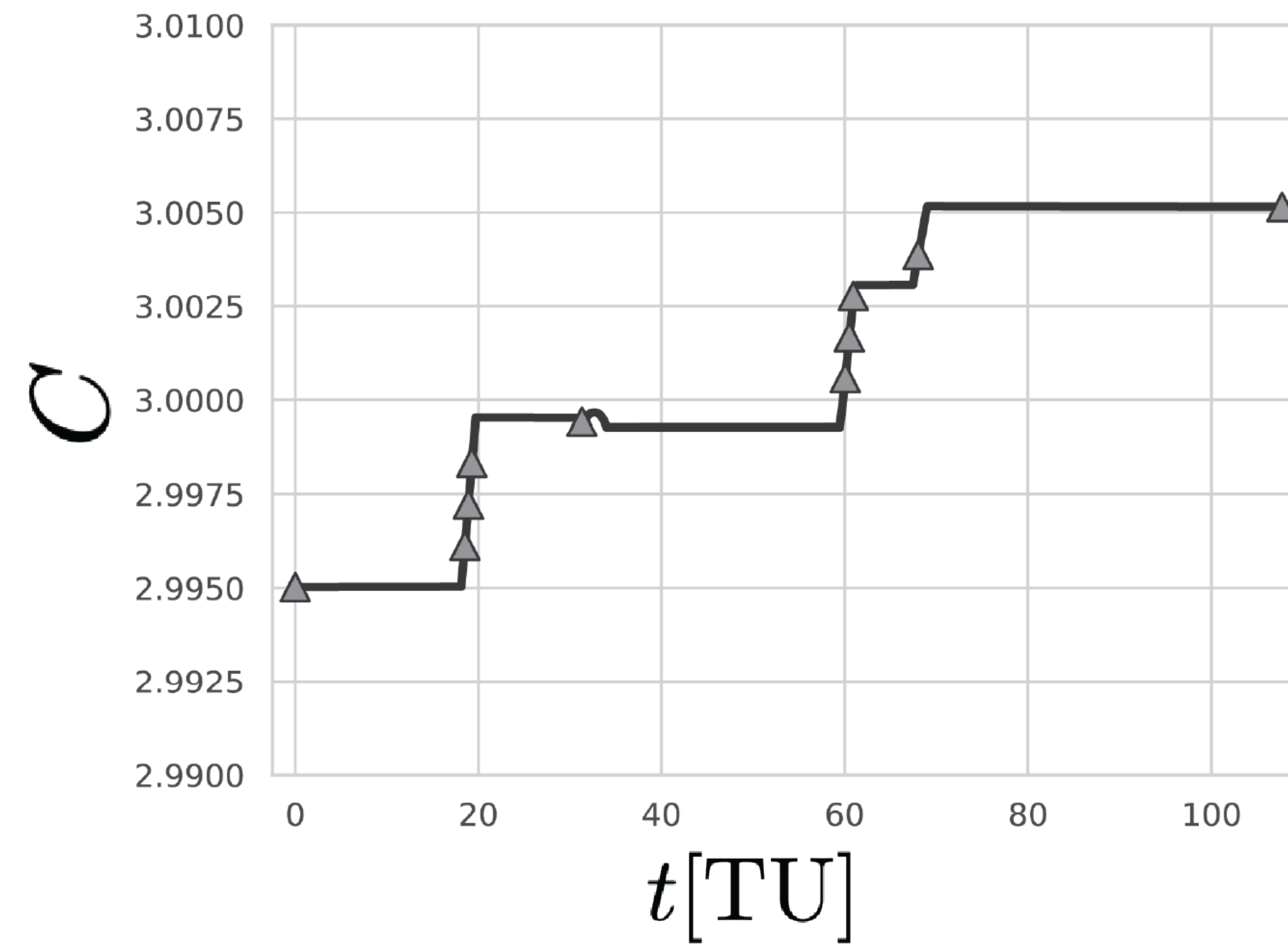
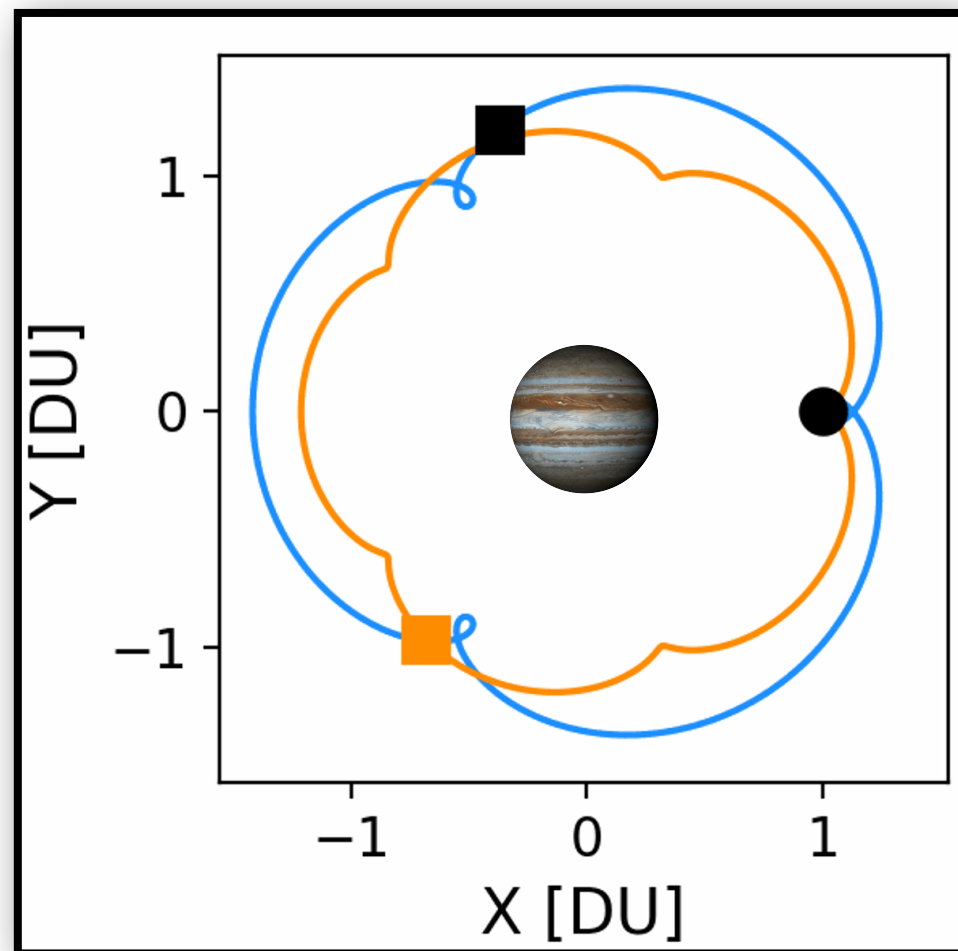
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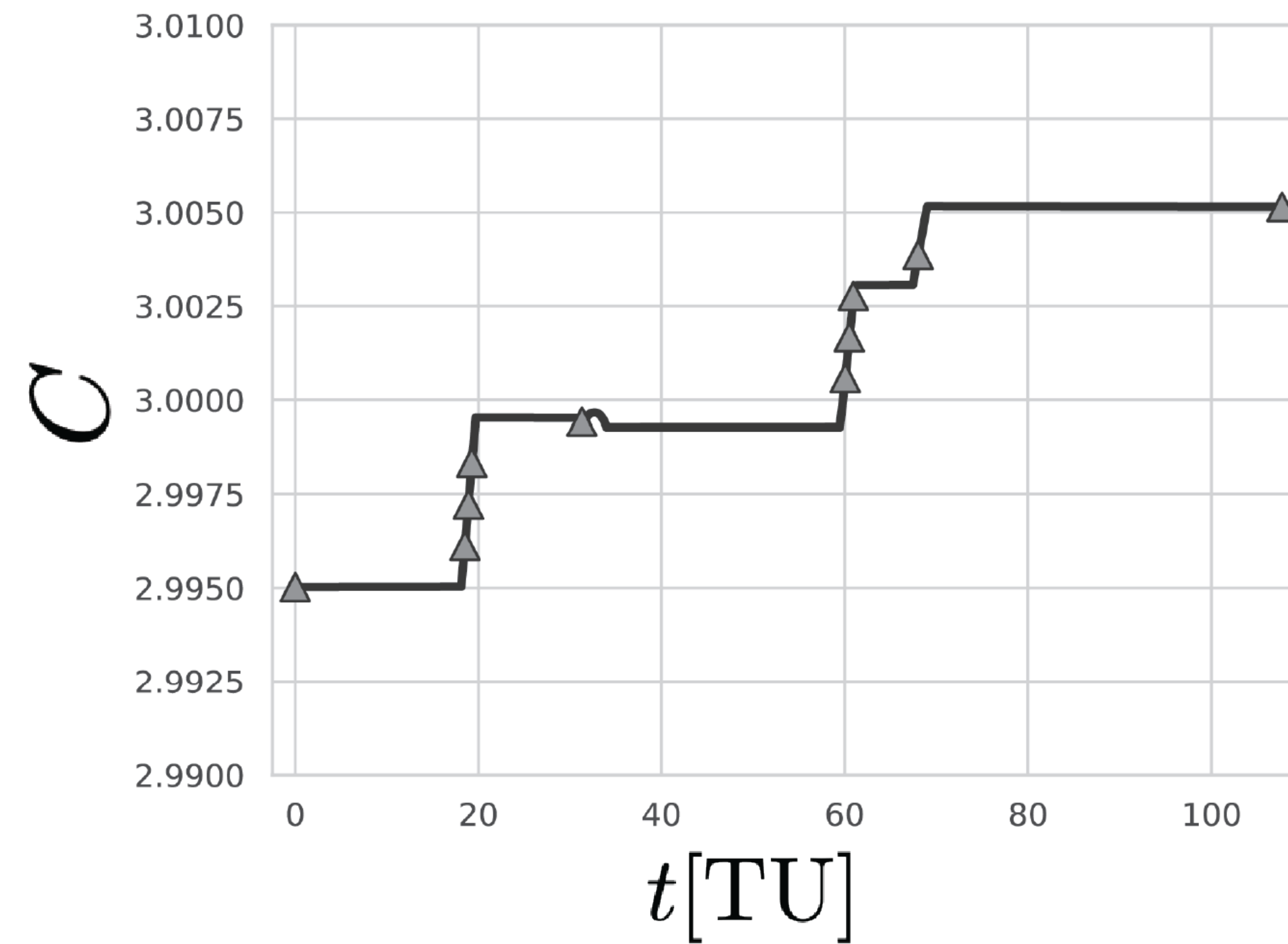
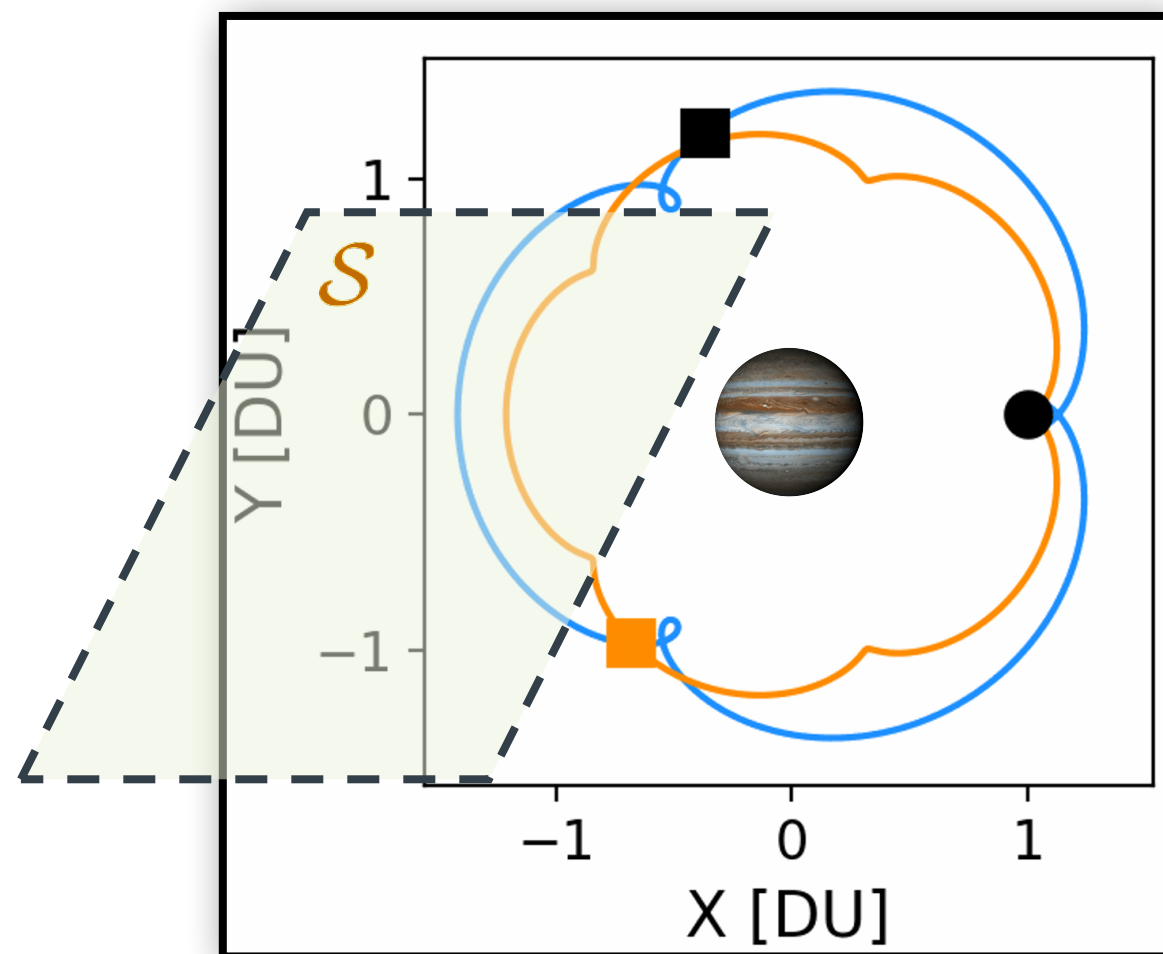
Families of 3:4 and 5:6 resonant orbits in JE CR3BP

Sinha and Beeson, JGCD, 2025

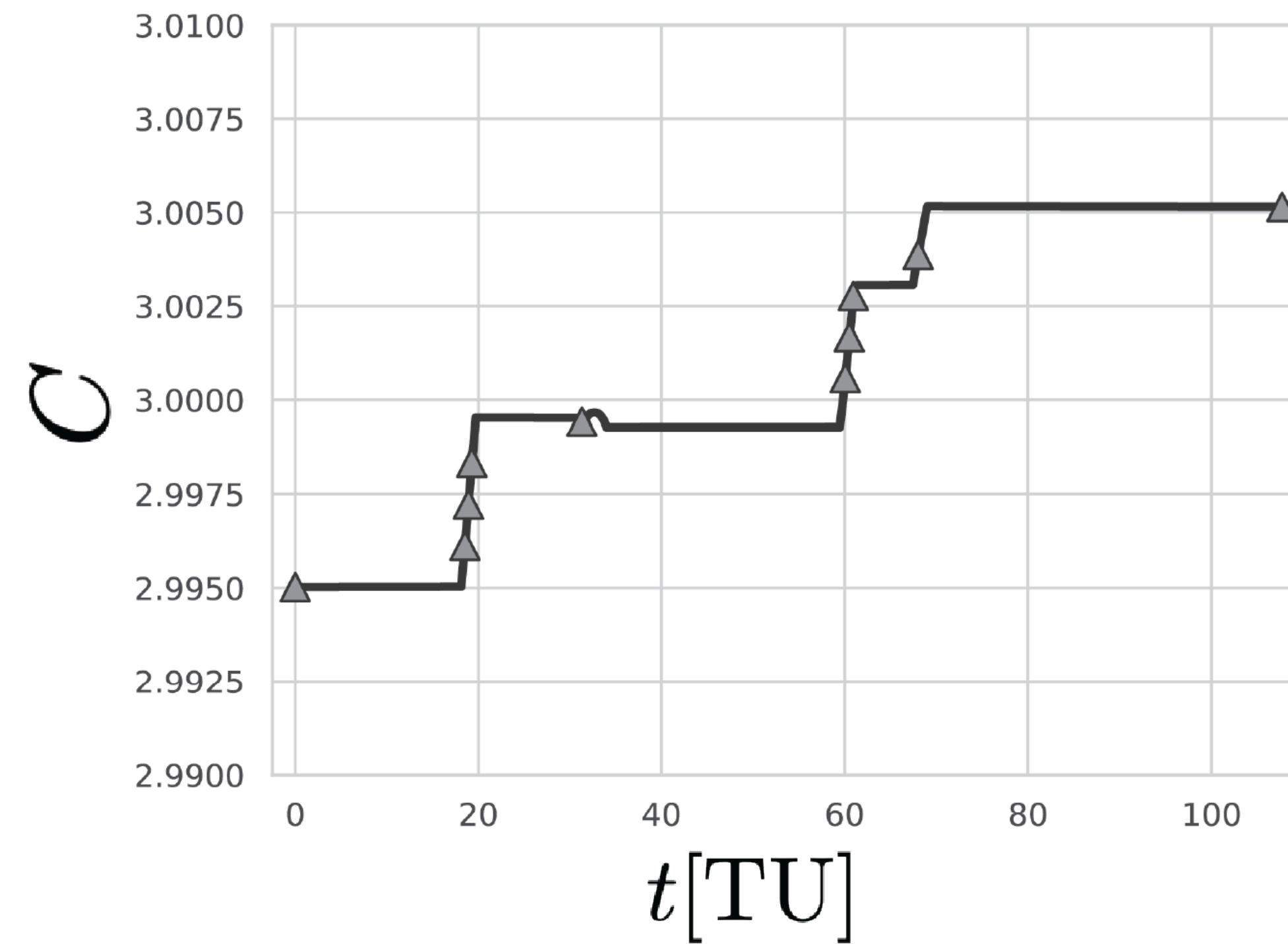
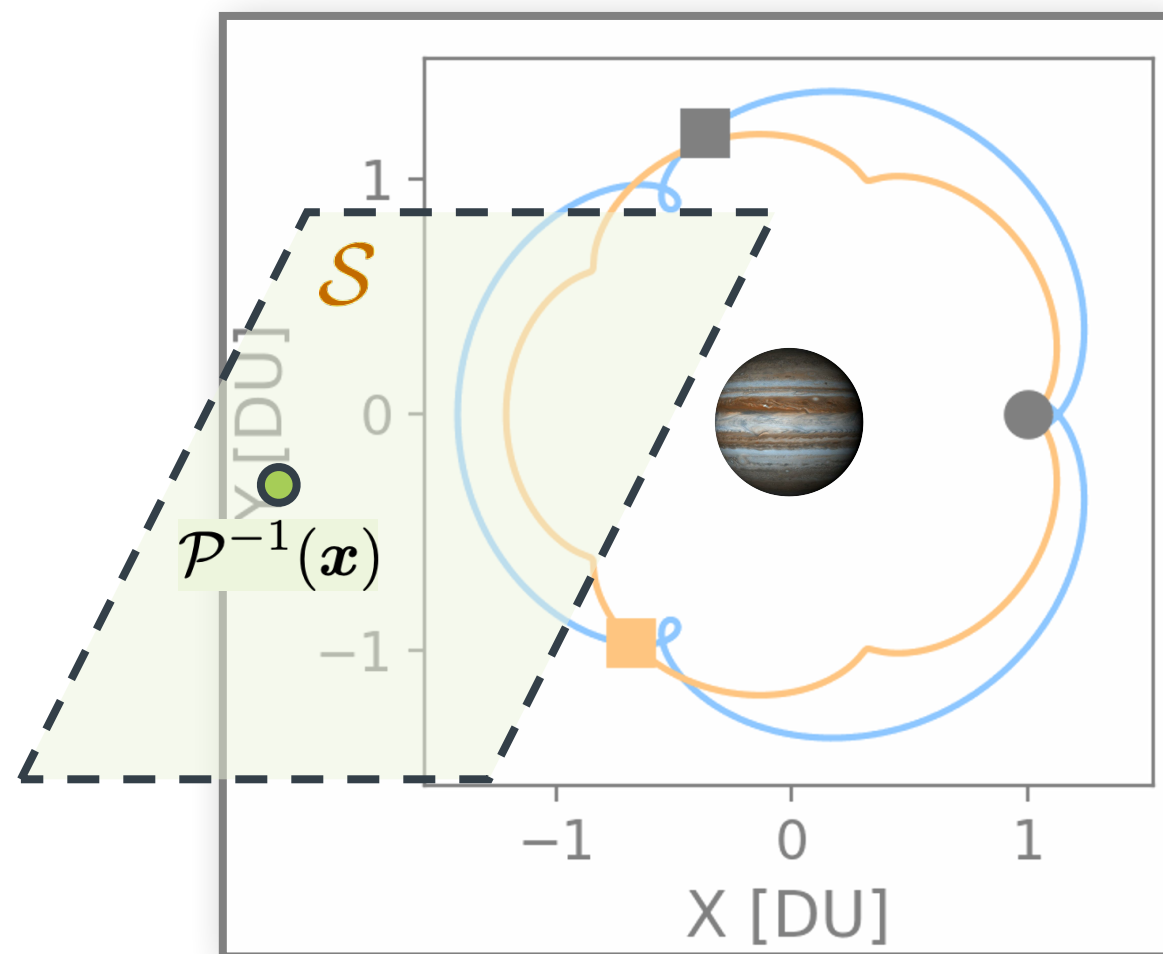
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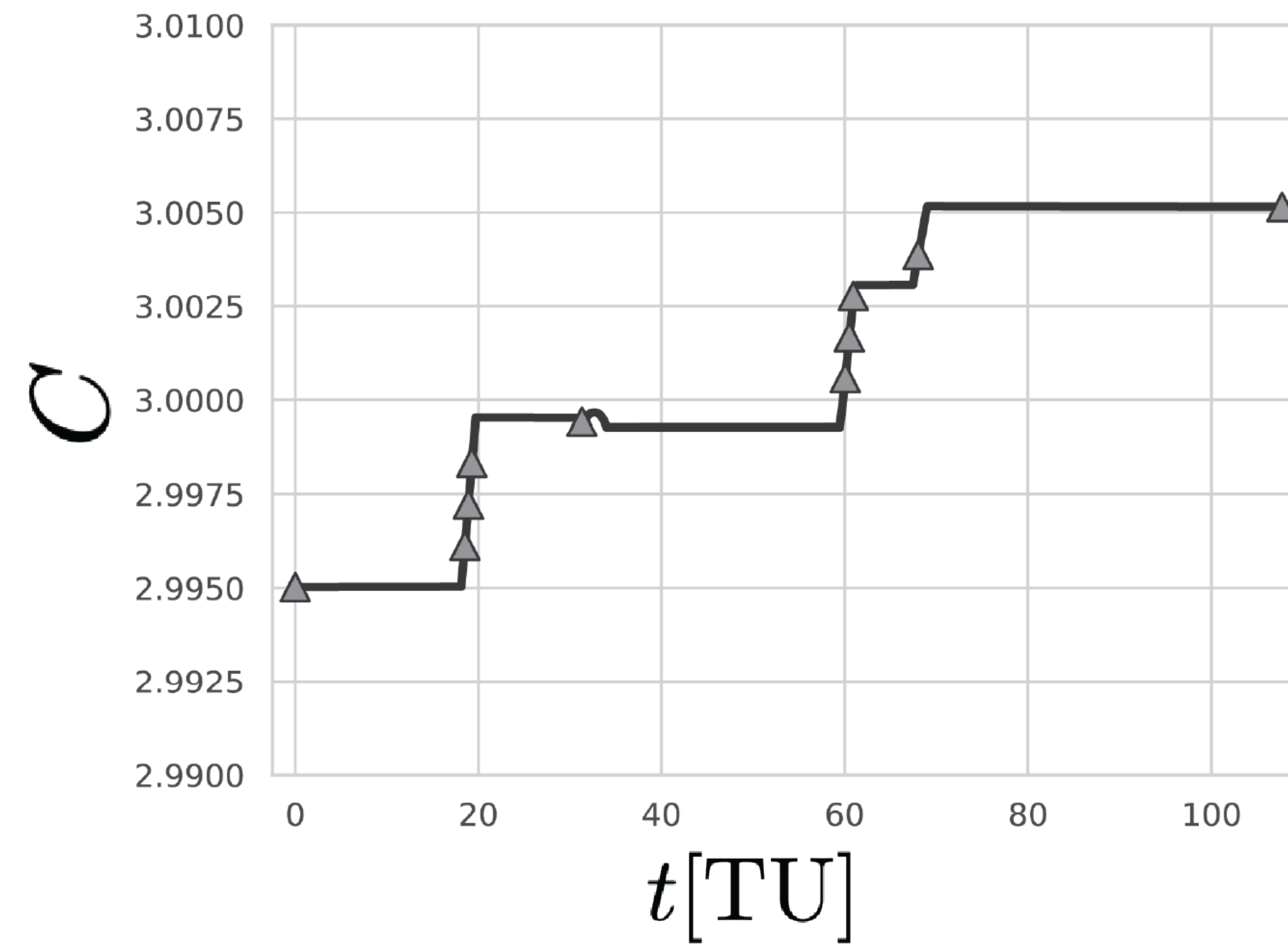
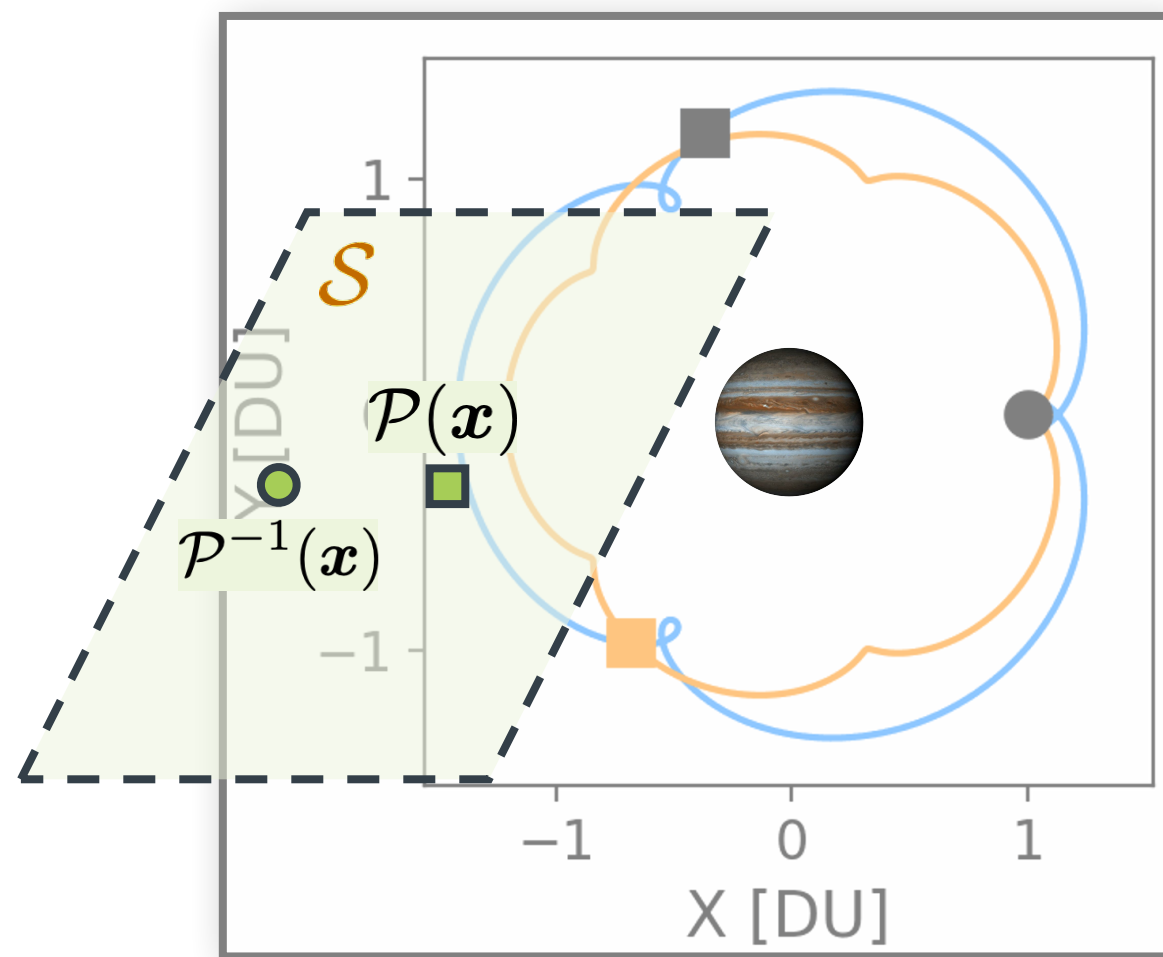
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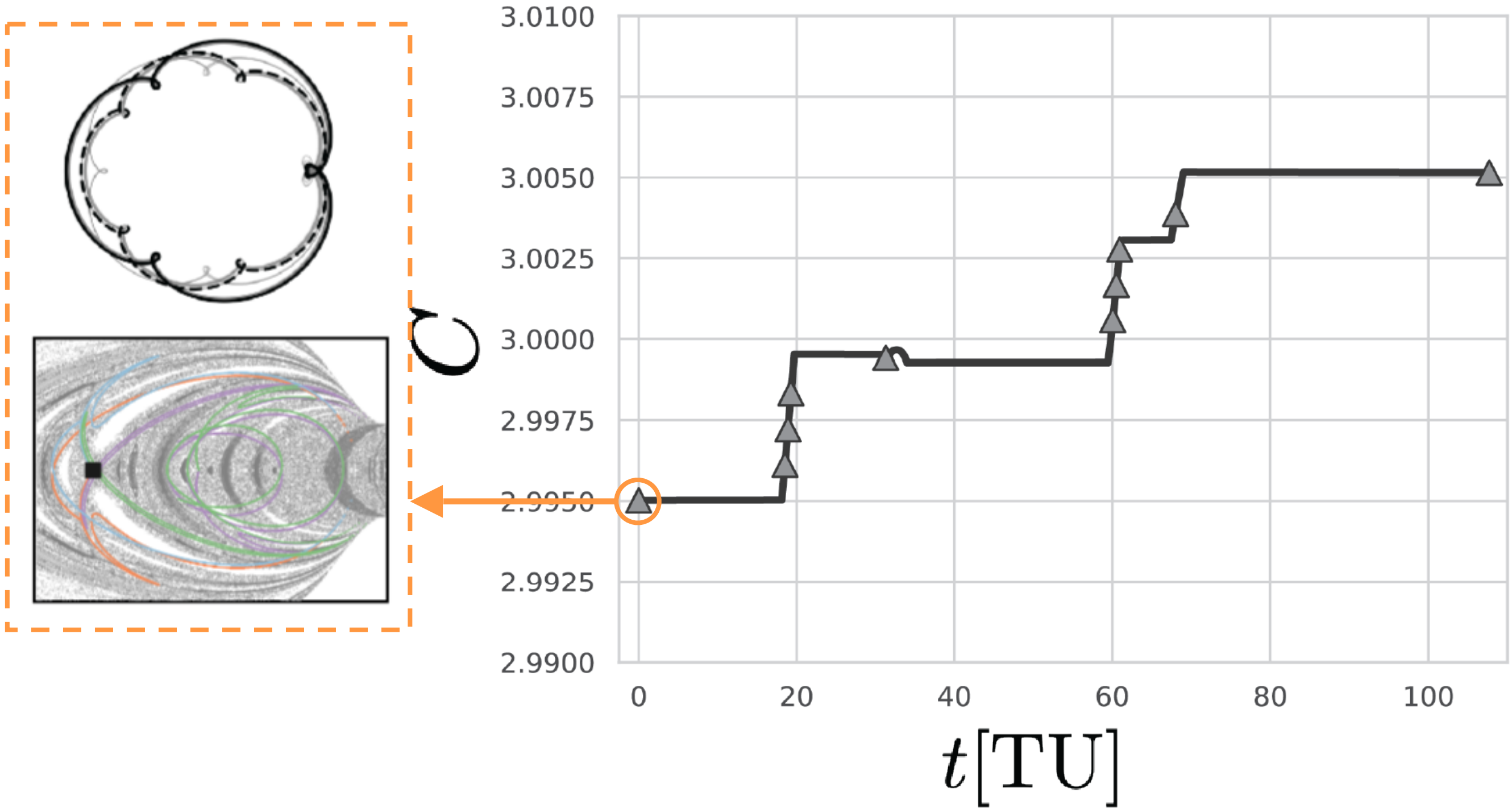
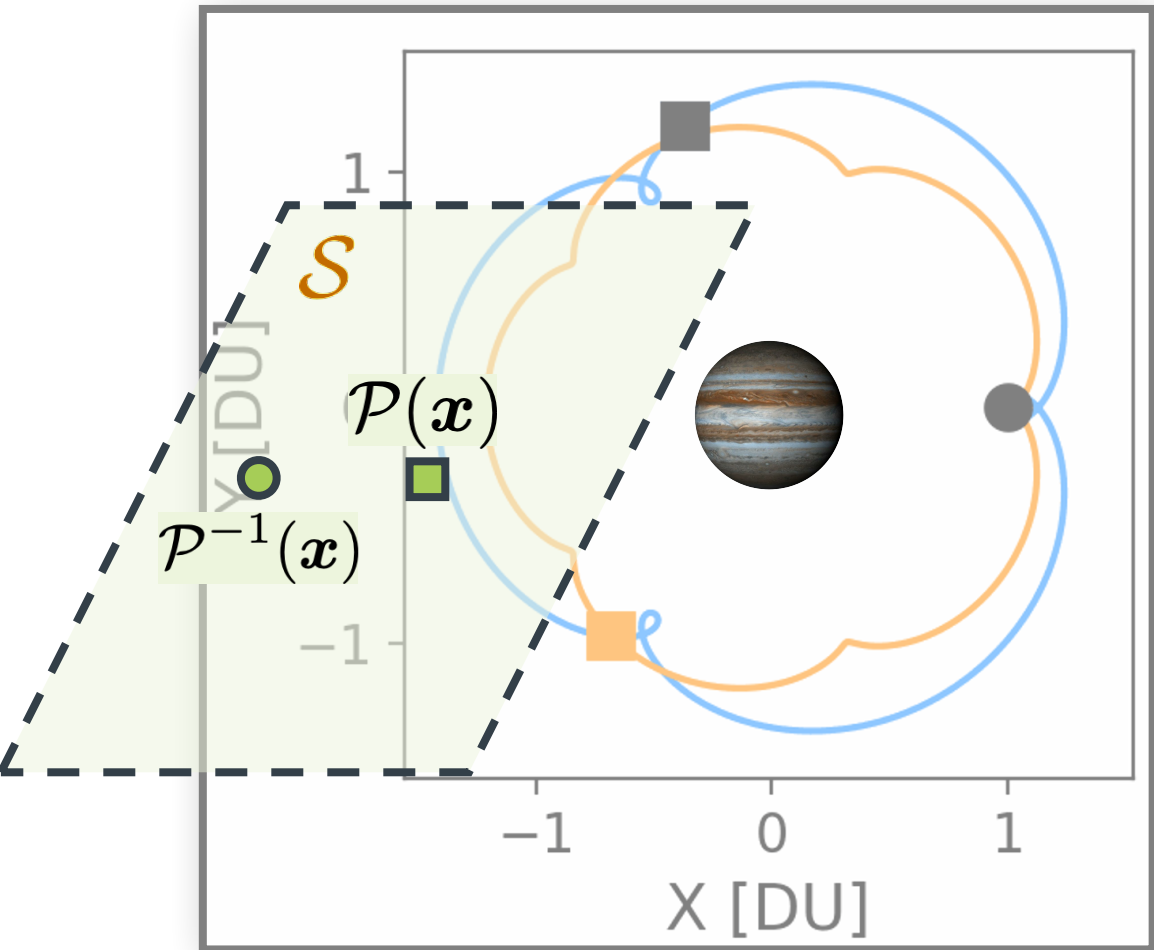


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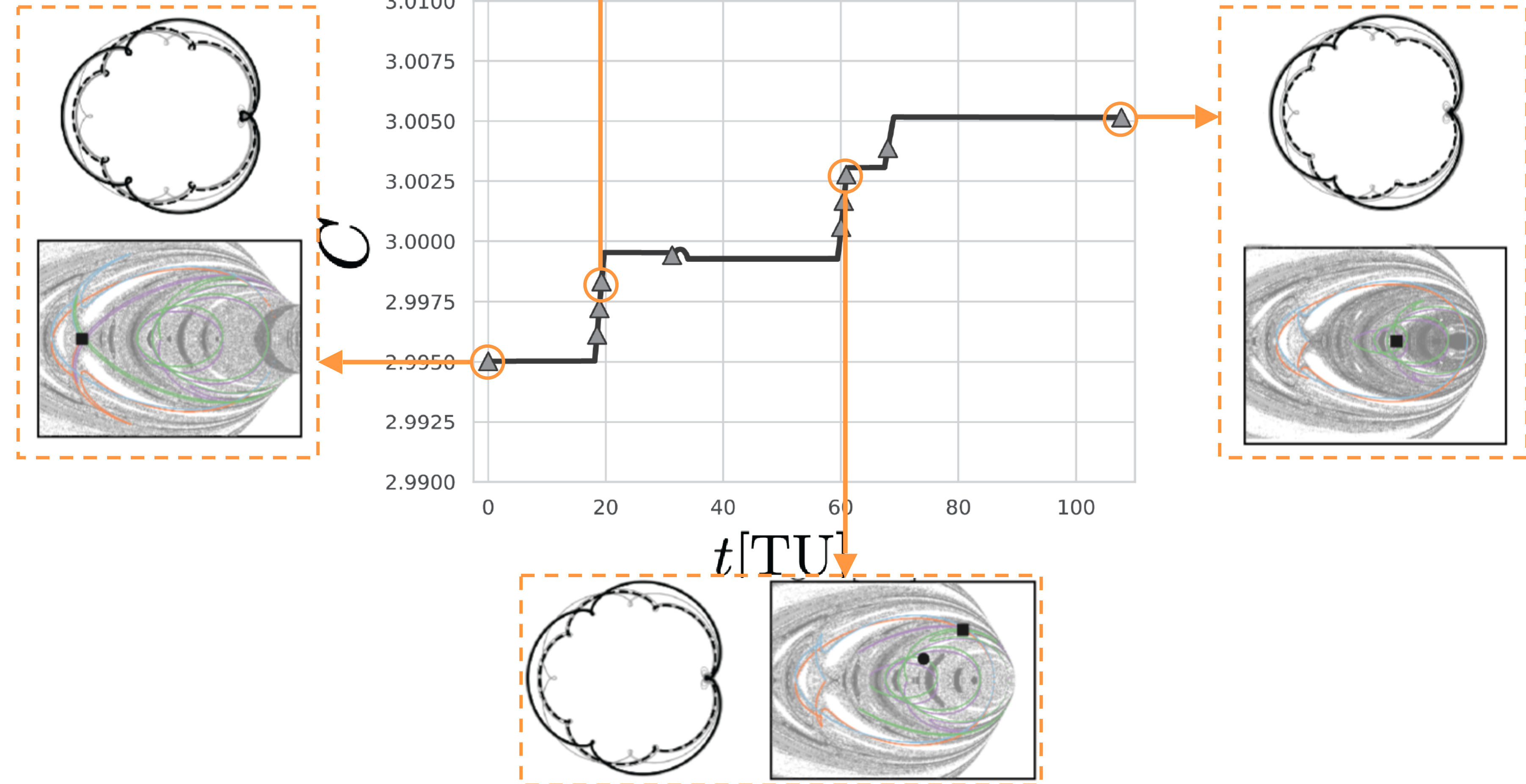
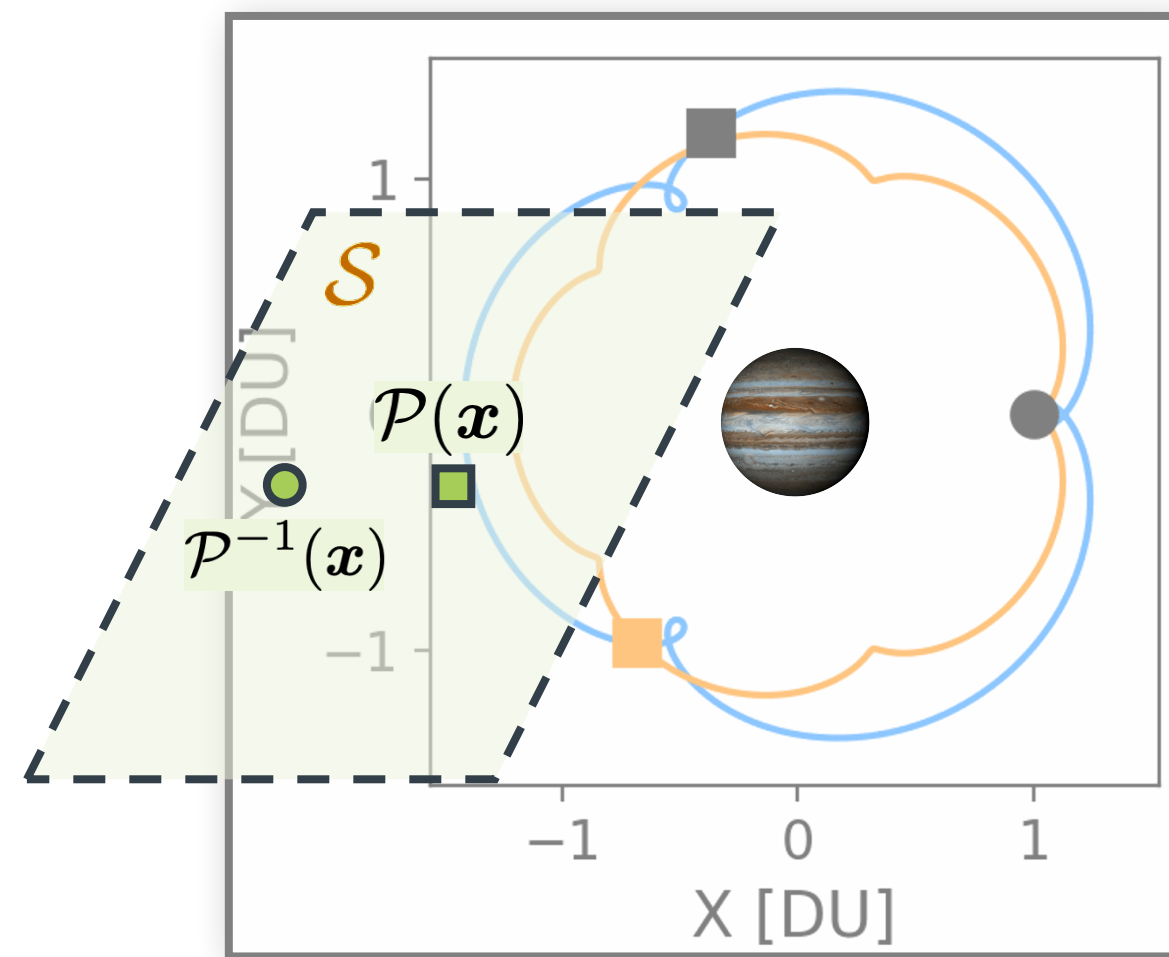
The Jacobi constant changes continuously over the low-thrust solution

$$\begin{matrix} W_U^{3:4} & W_U^{5:6} \\ W_S^{5:6} & W_S^{3:4} \end{matrix}$$



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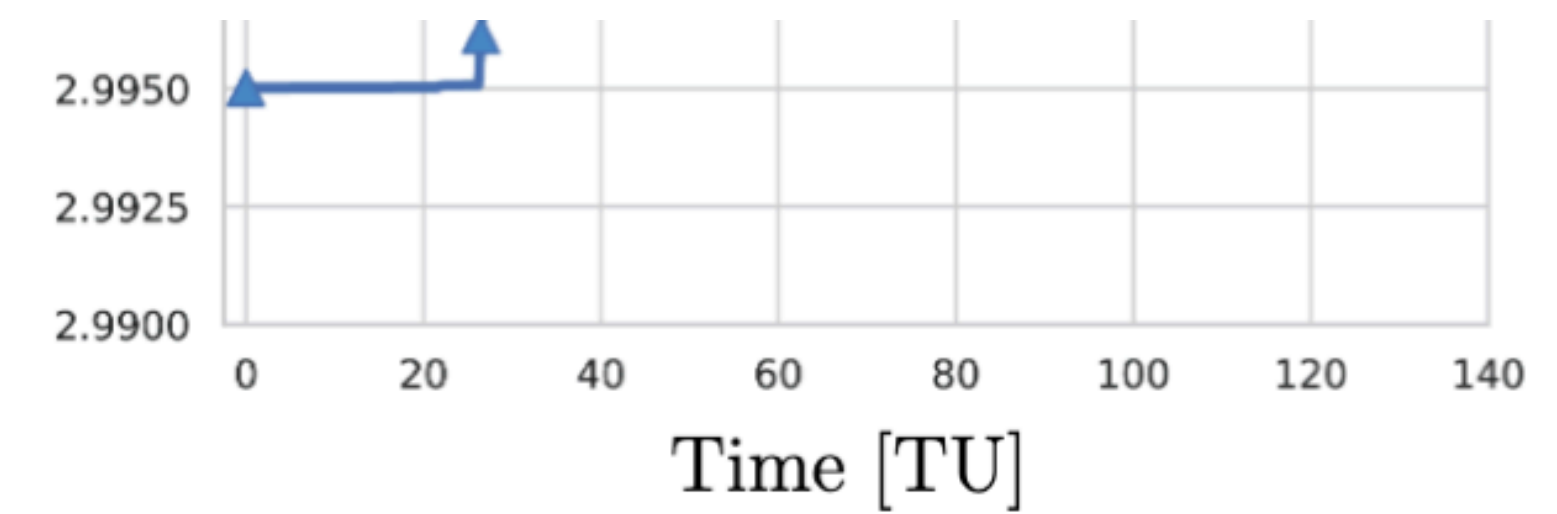
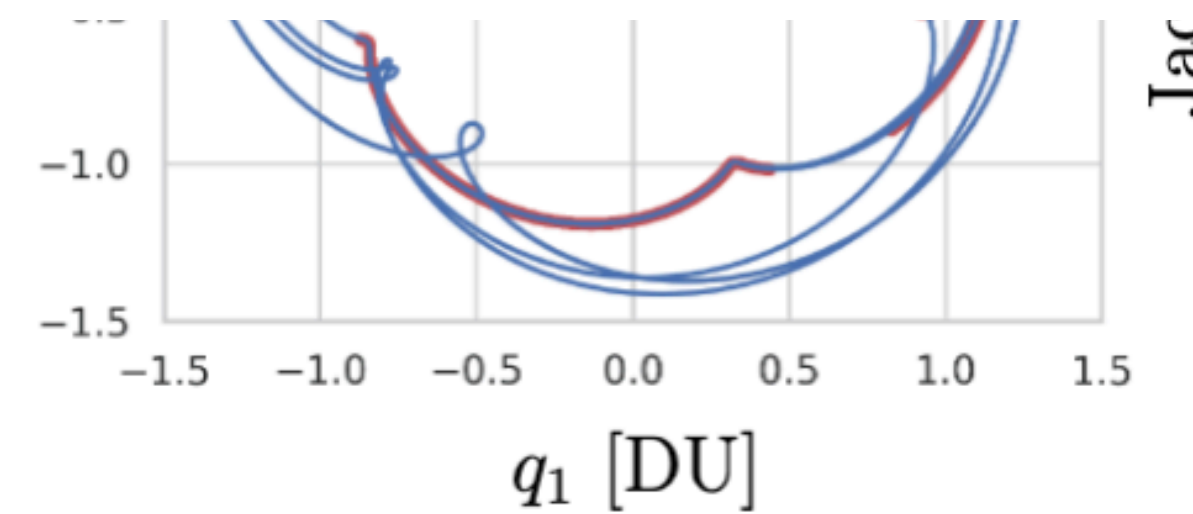
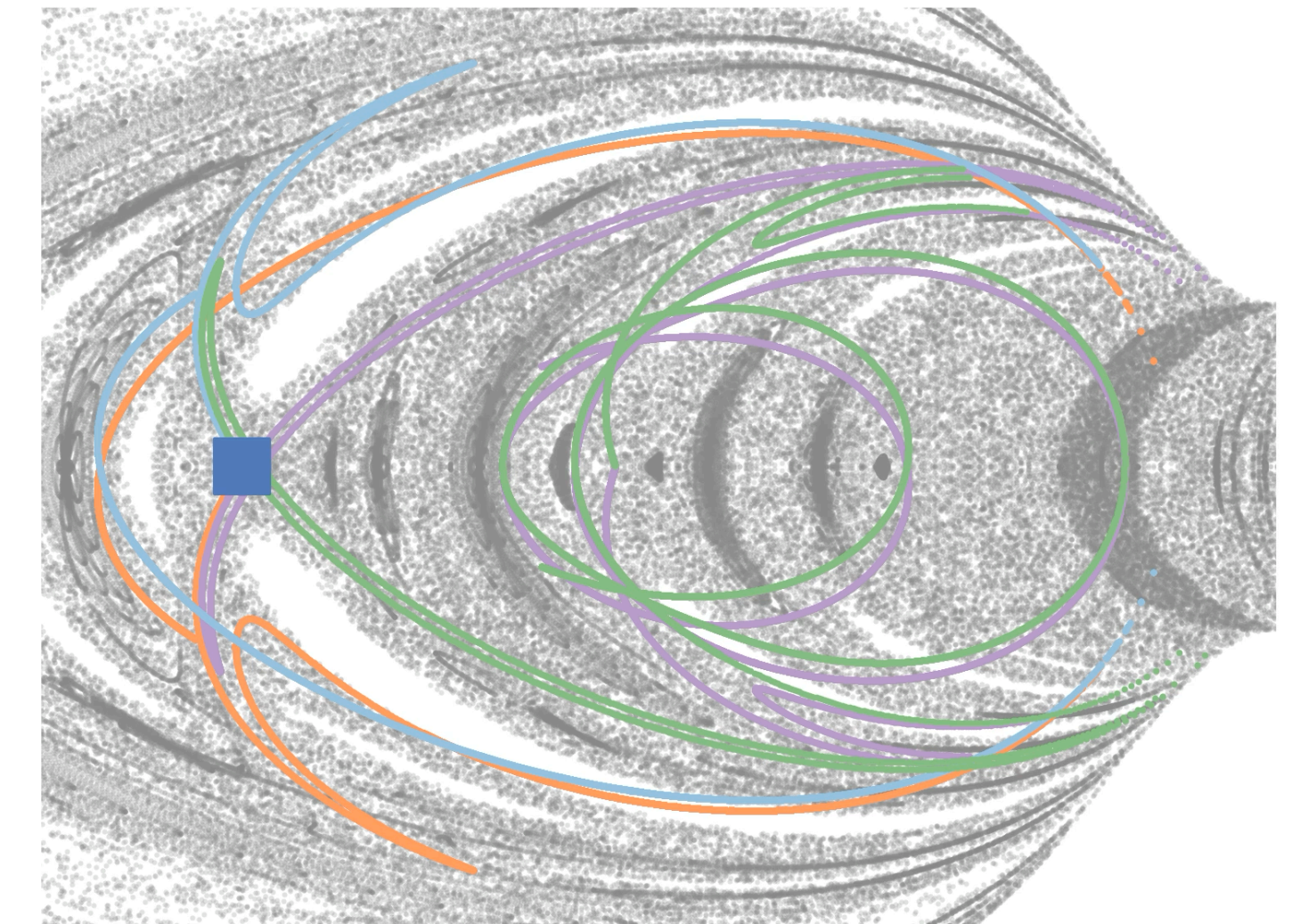
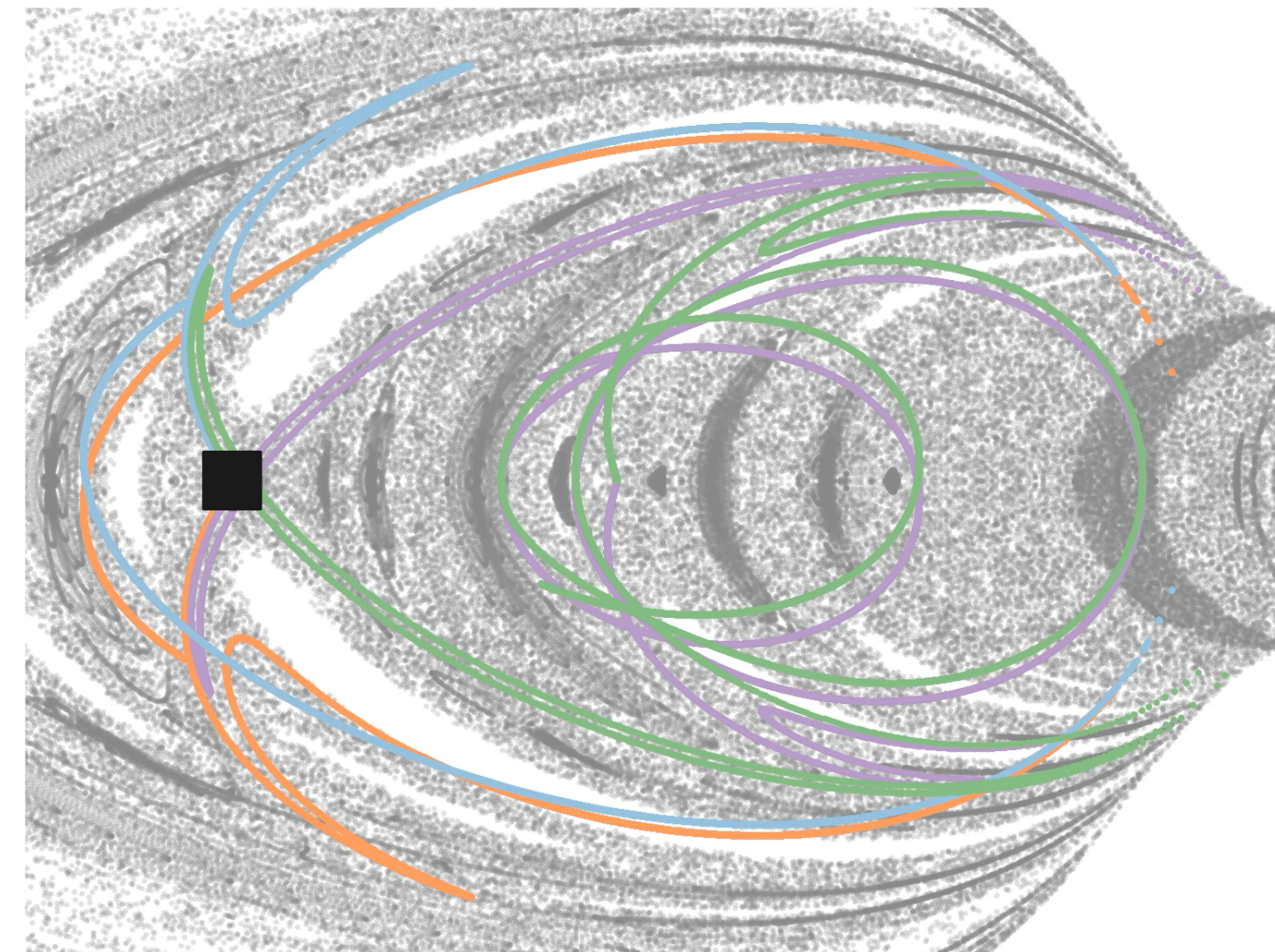
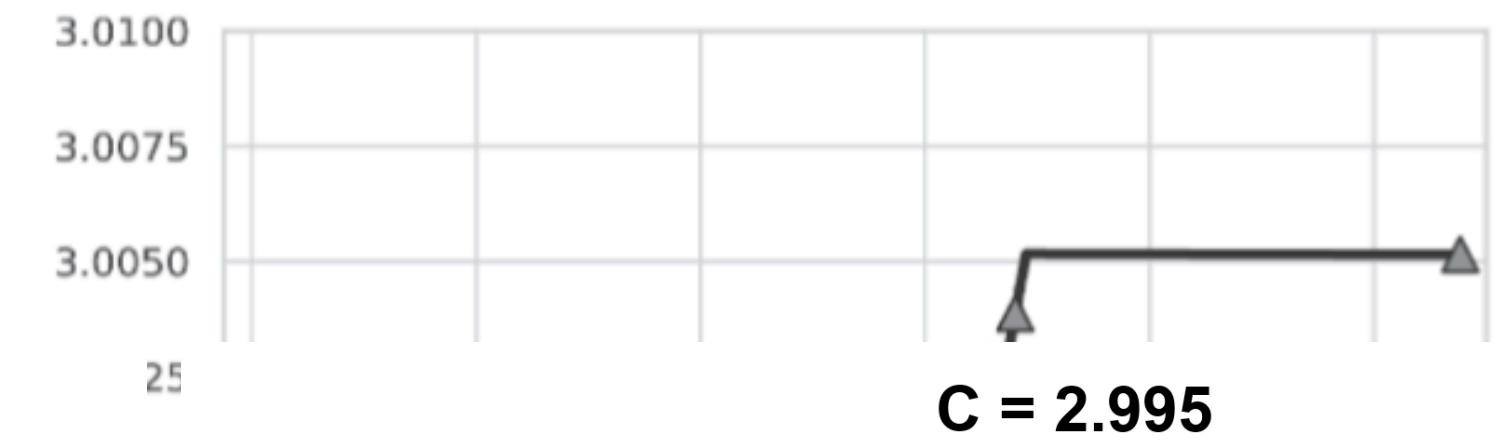
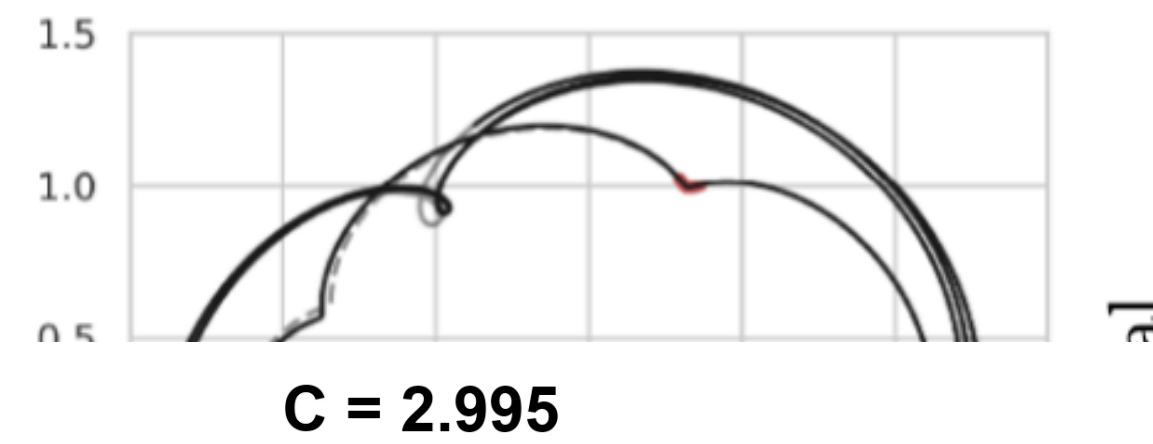
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Sinha and Beeson, JGCD, 2025

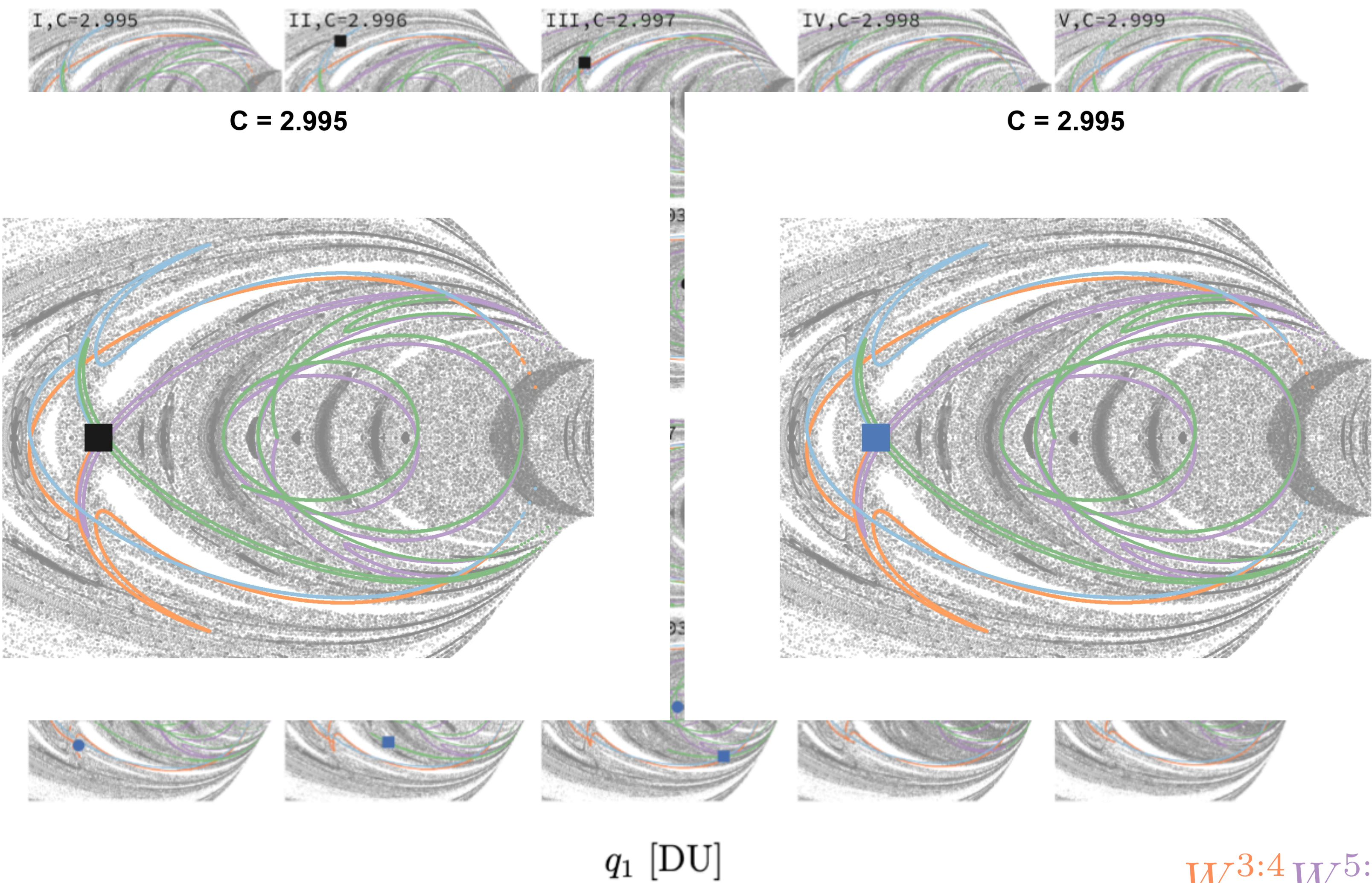
The study evaluates each strategy using two distance metrics:

- Orthogonal distance d_T
- Arc length d_A



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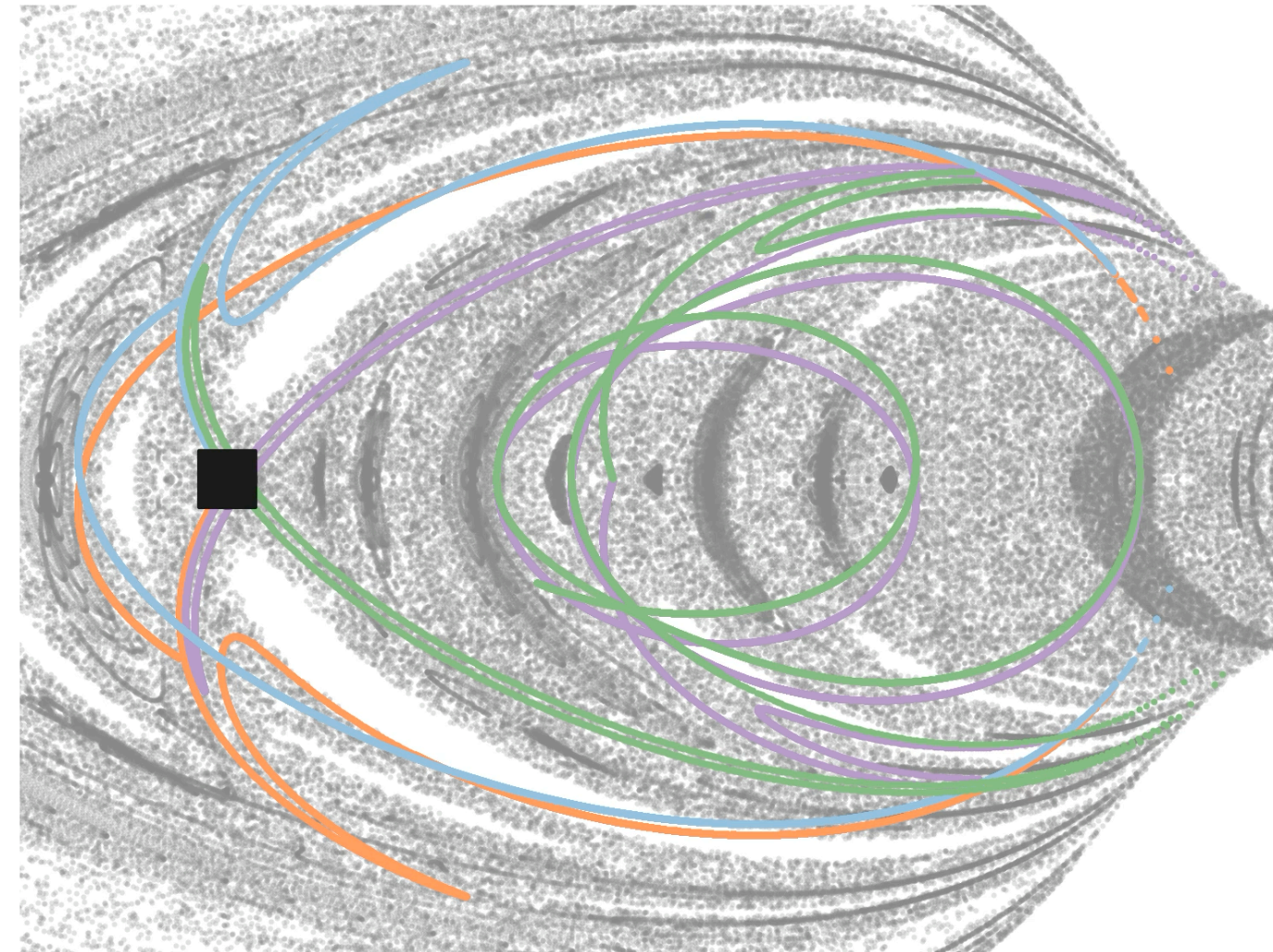


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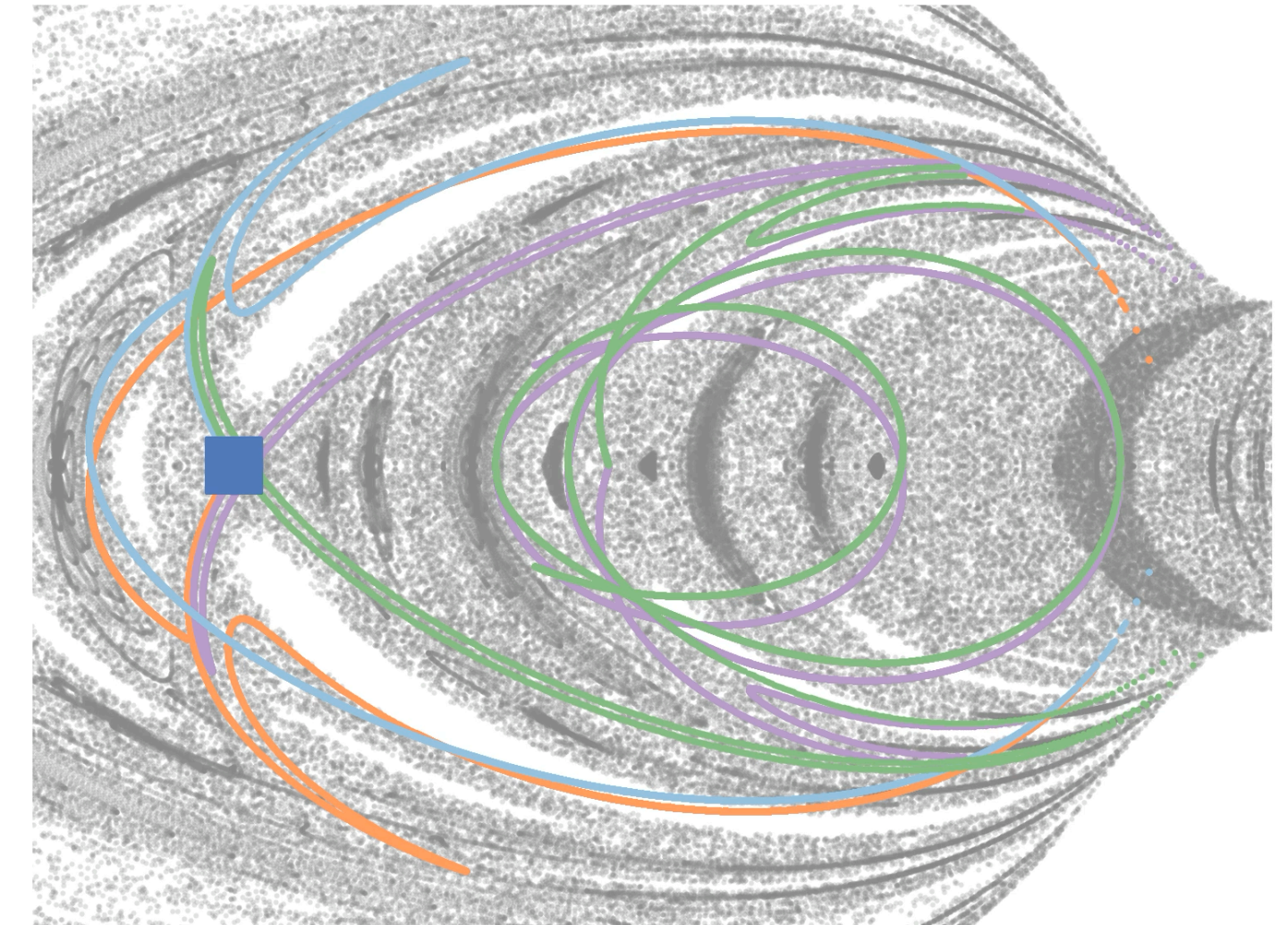
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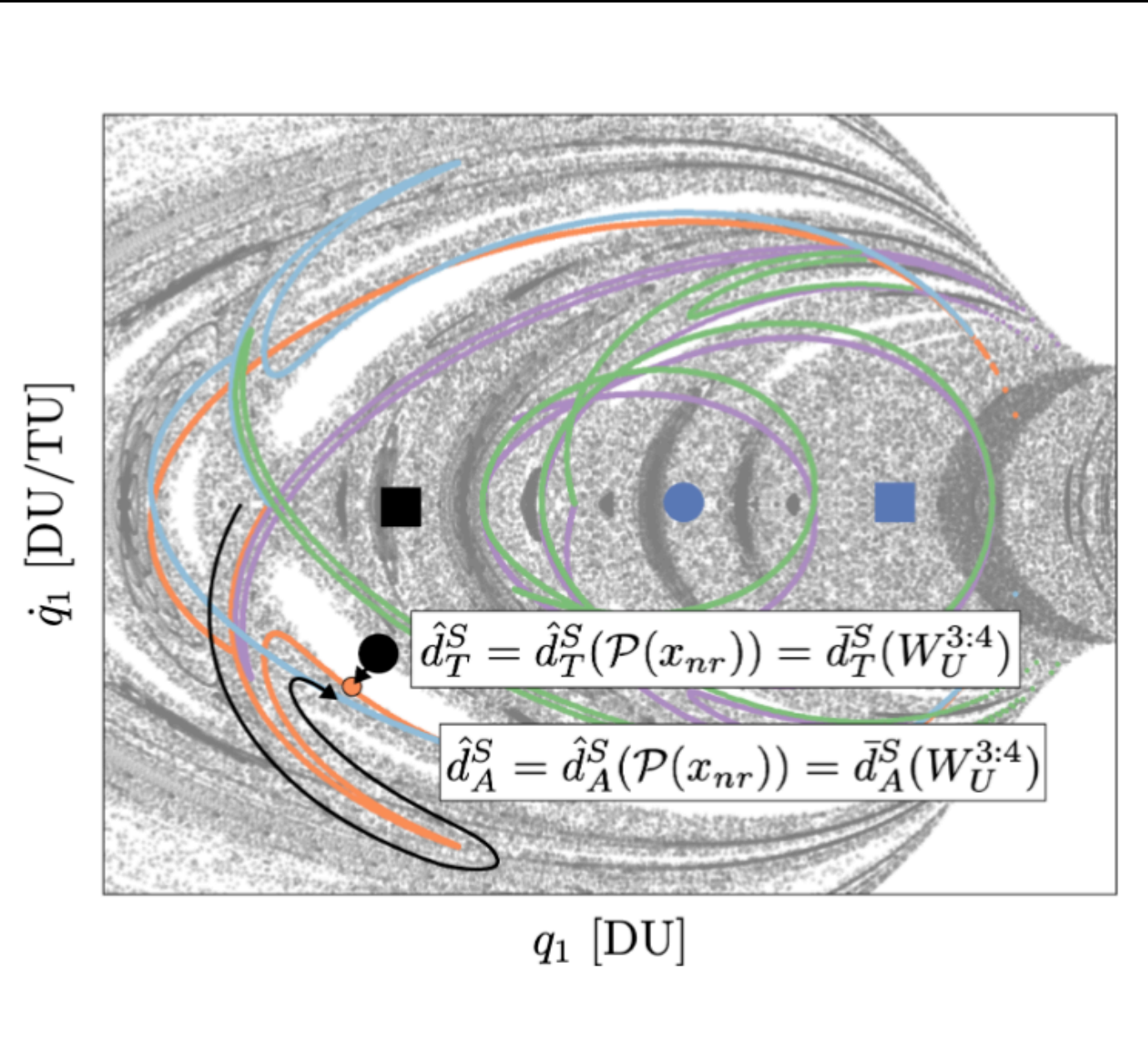


Sinha and Beeson. JGCD. 2025

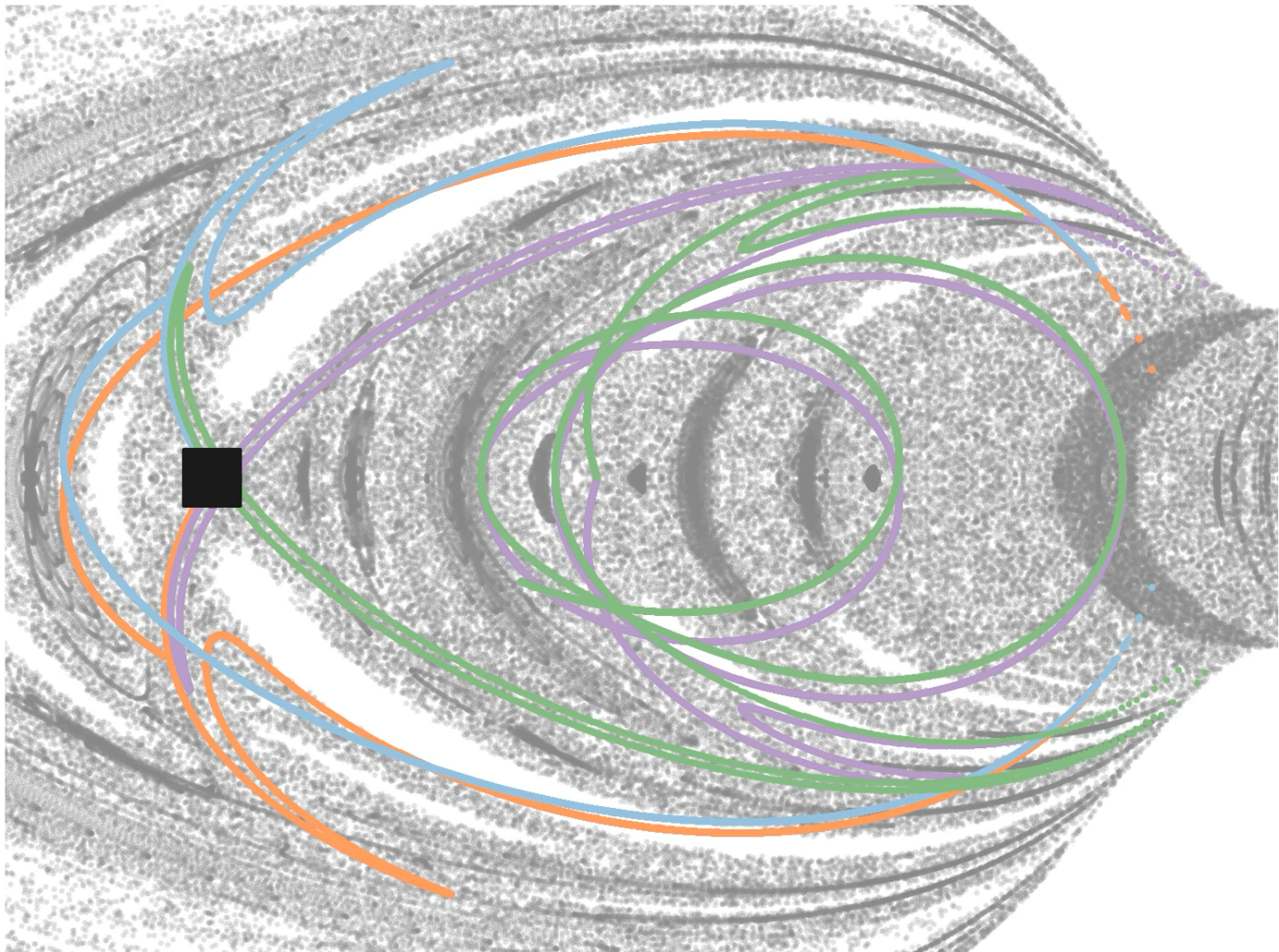
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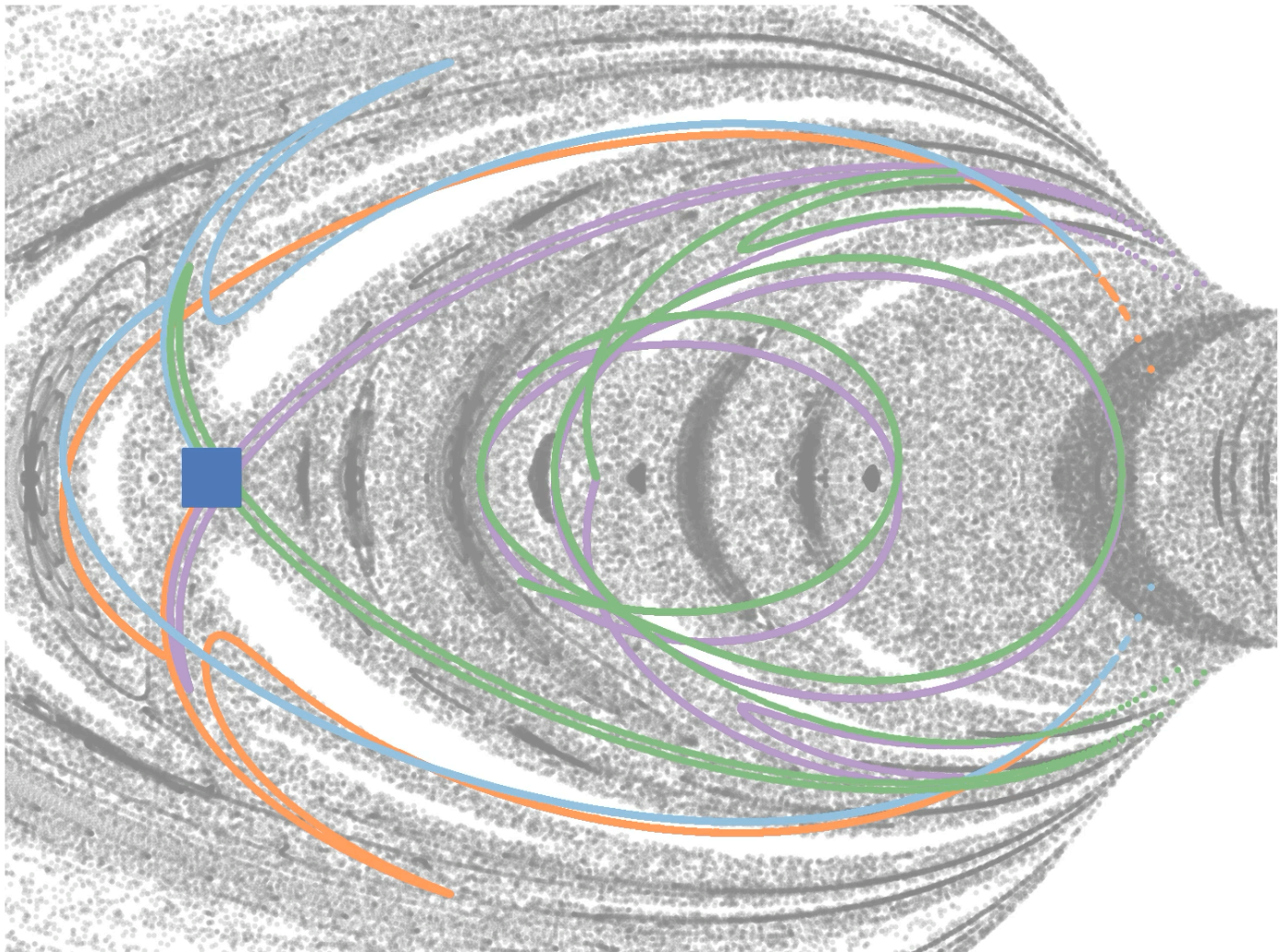
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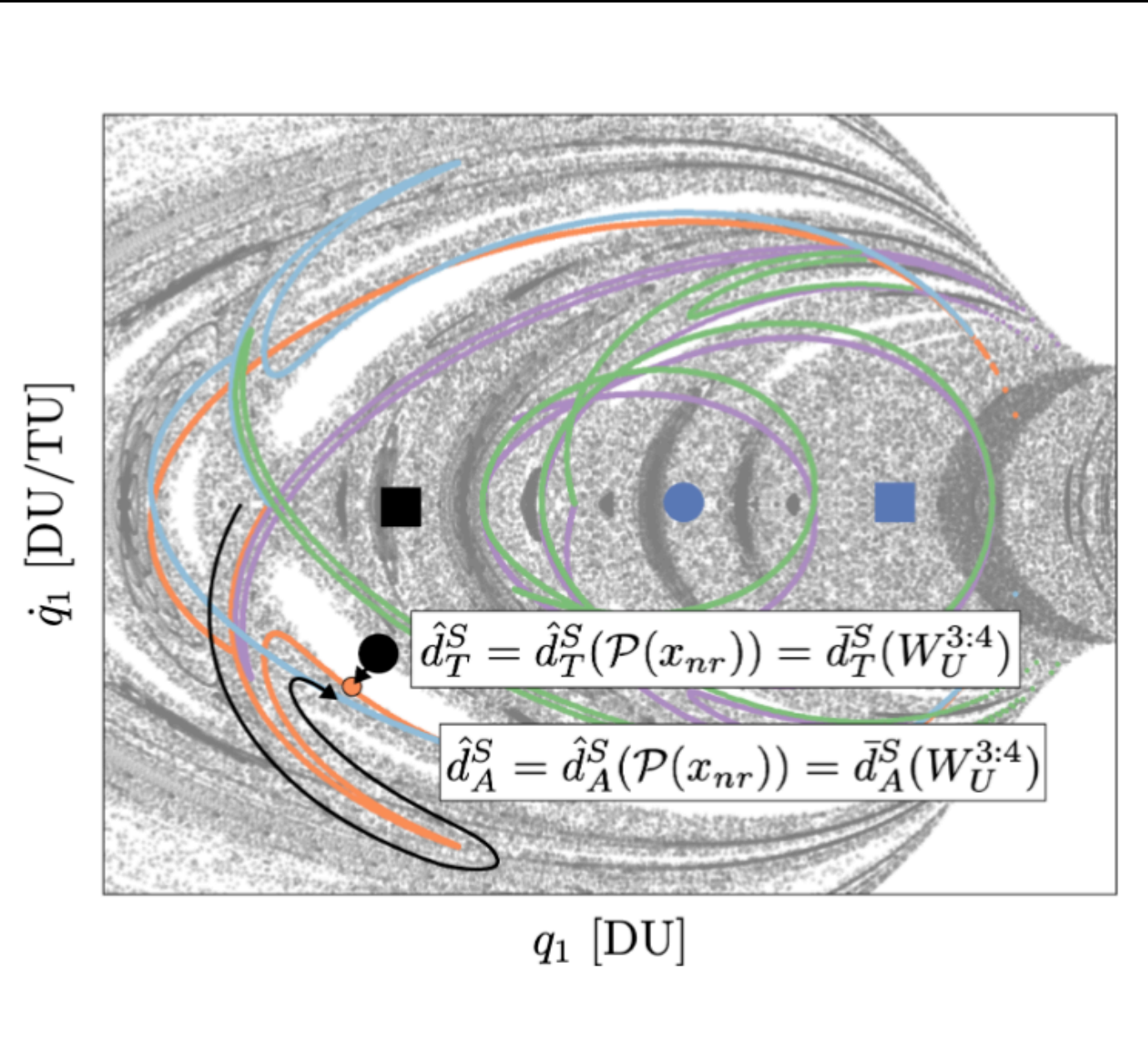


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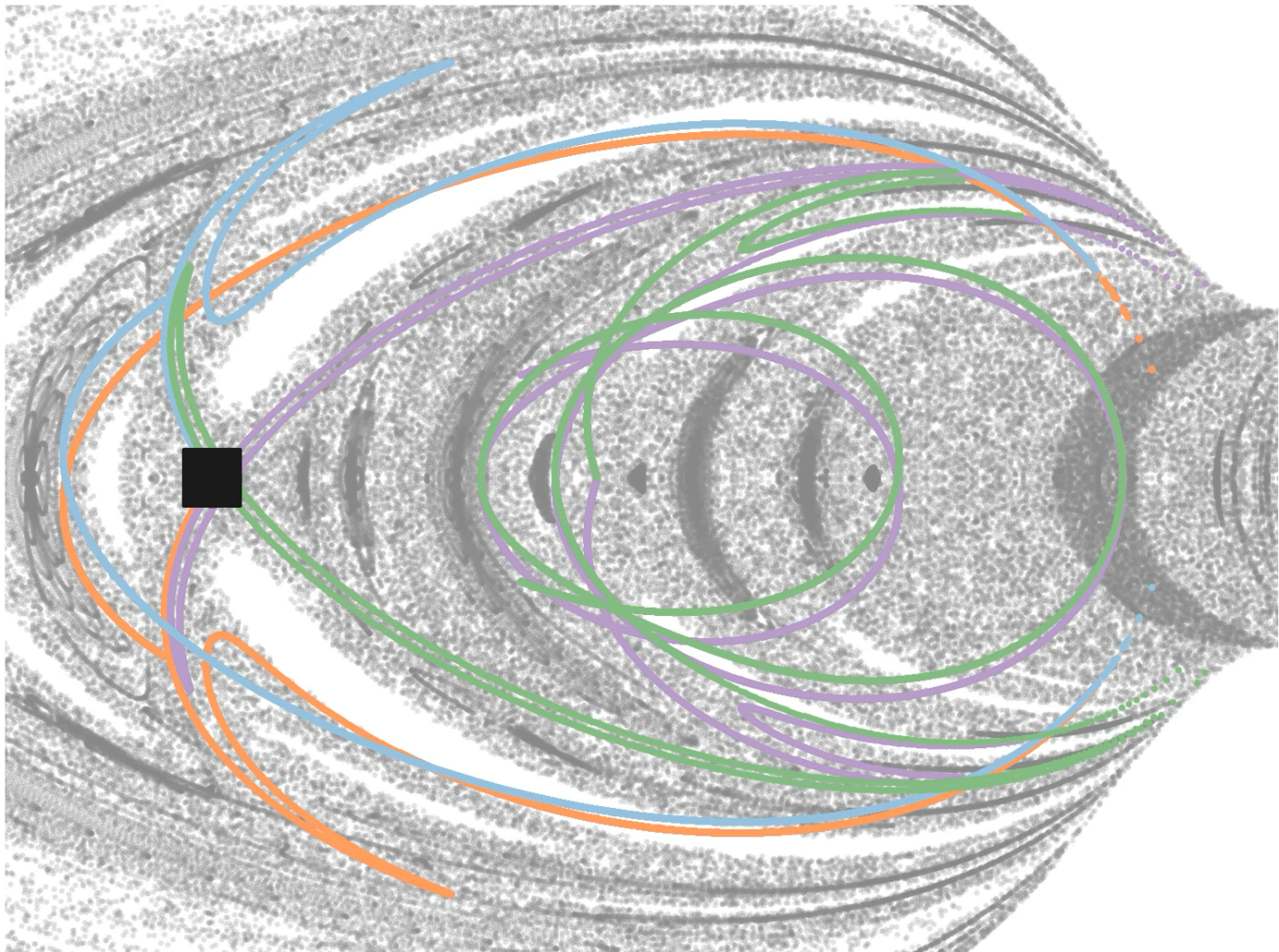
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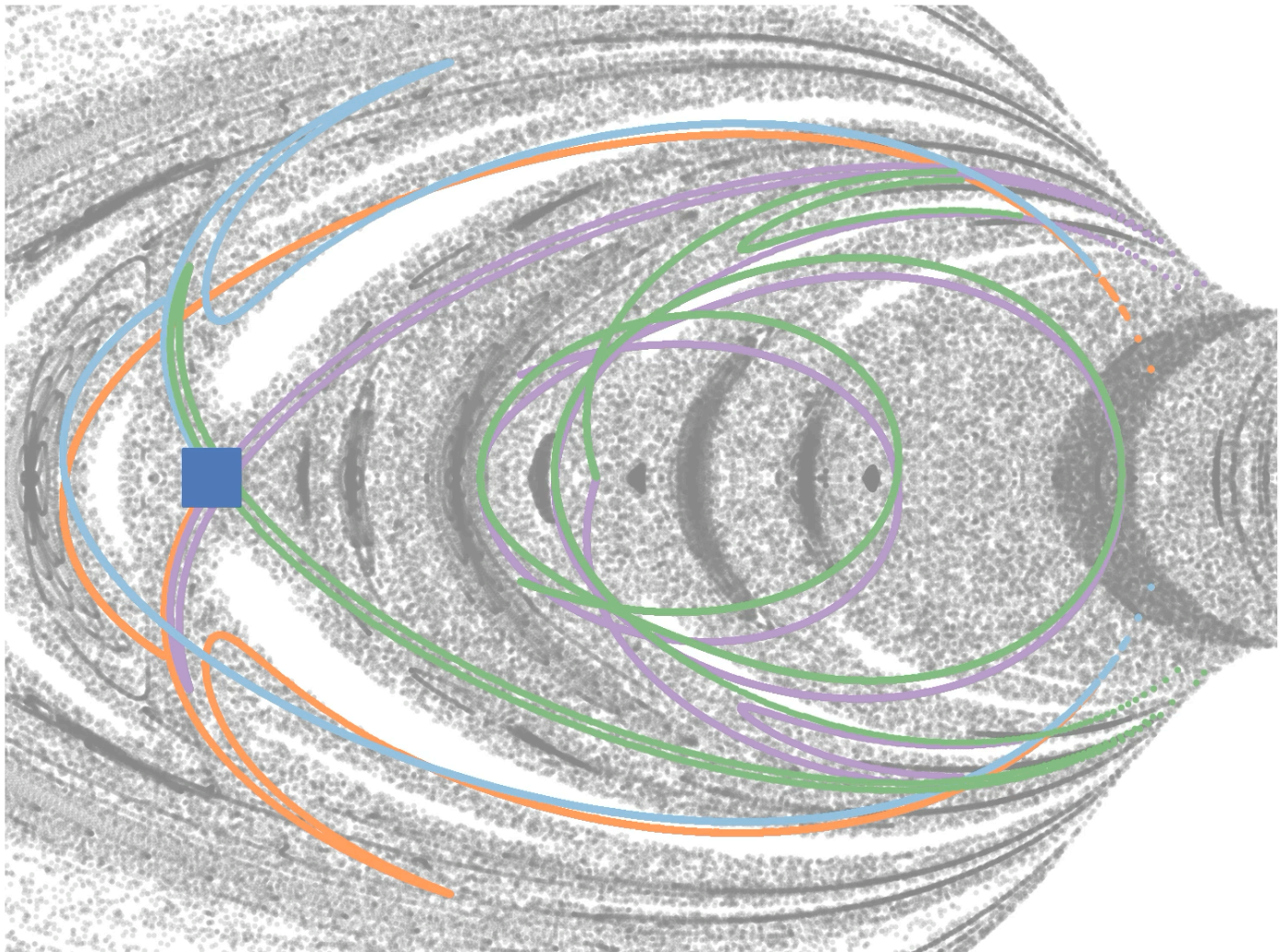
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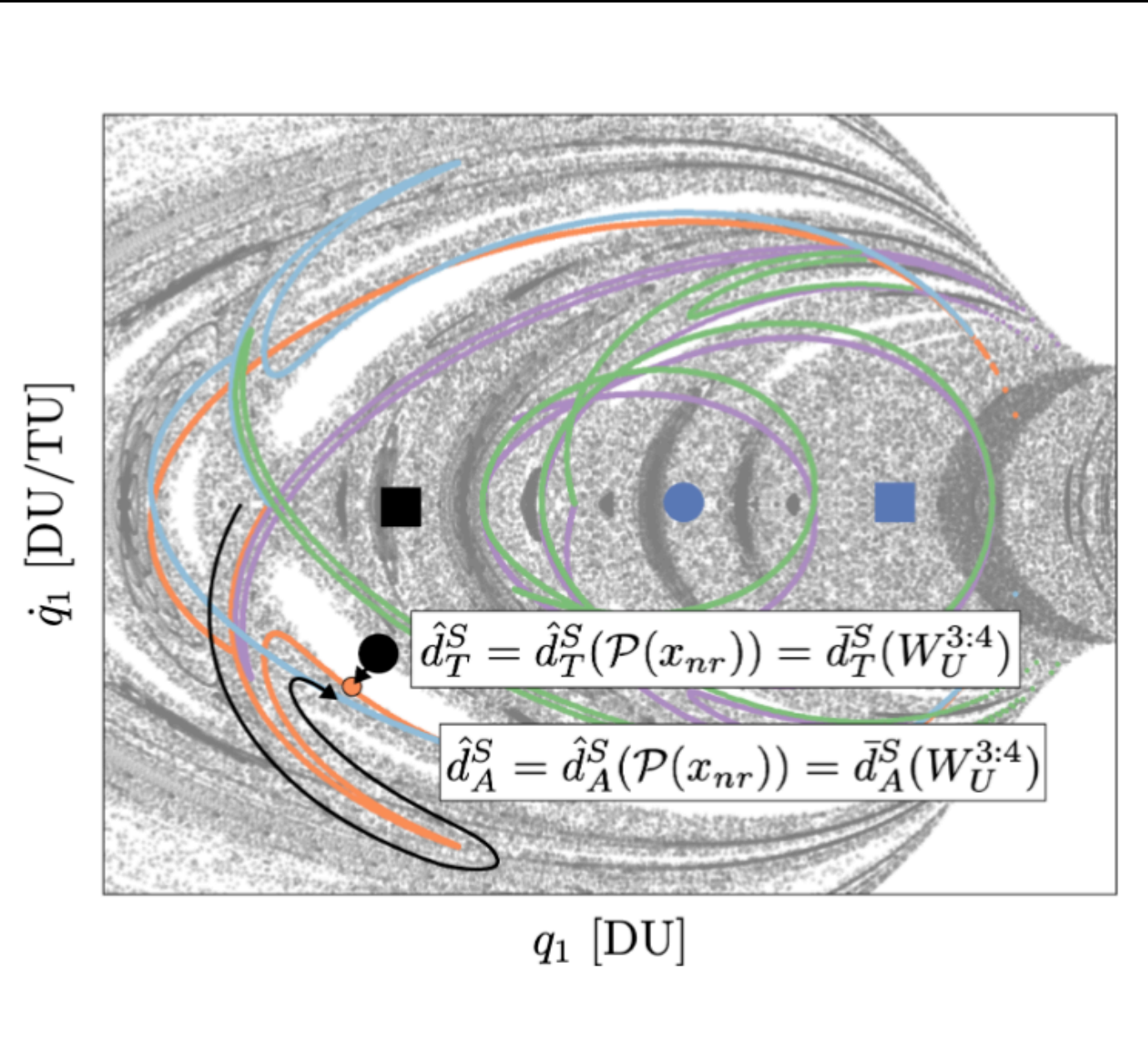


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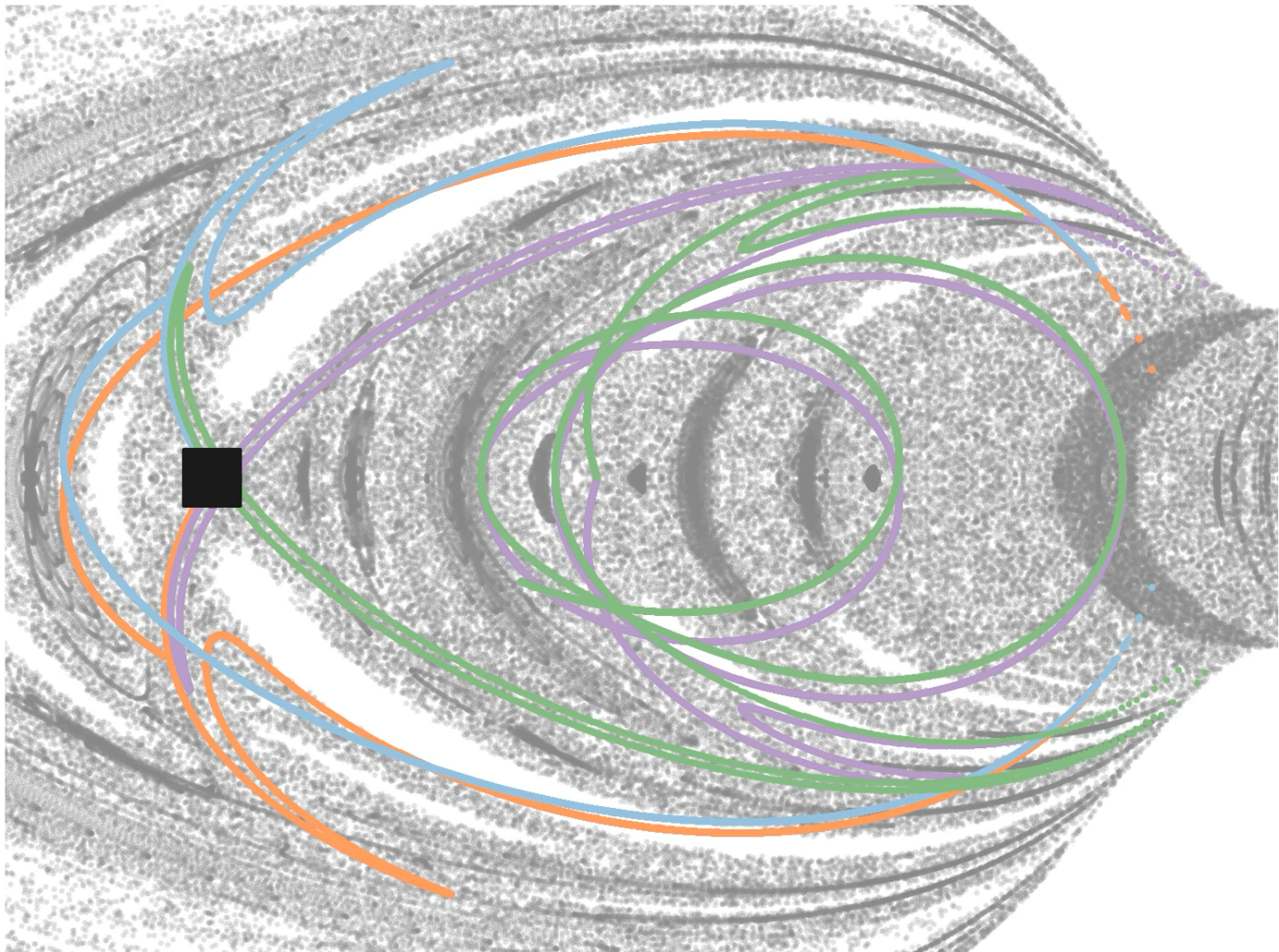
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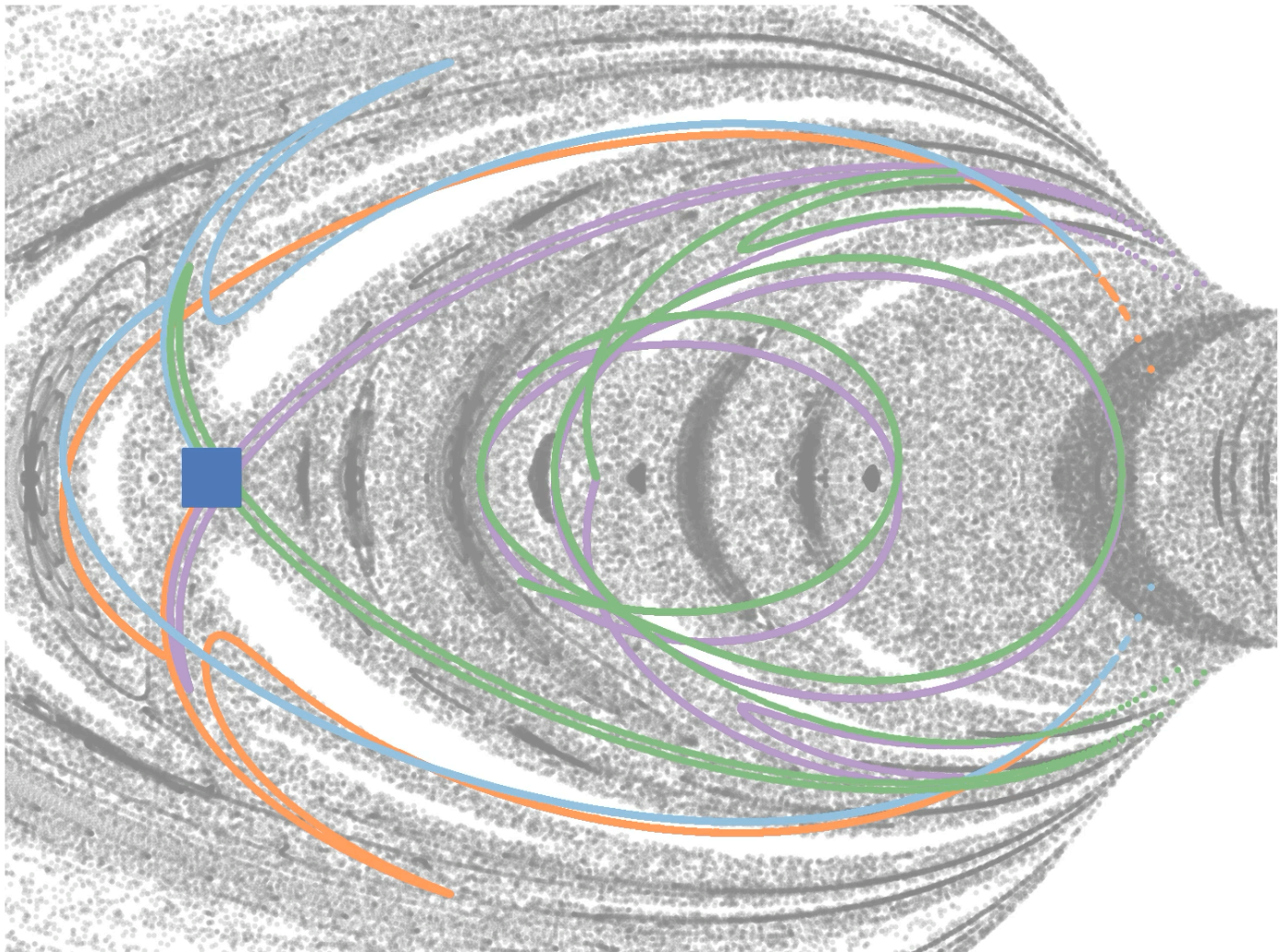
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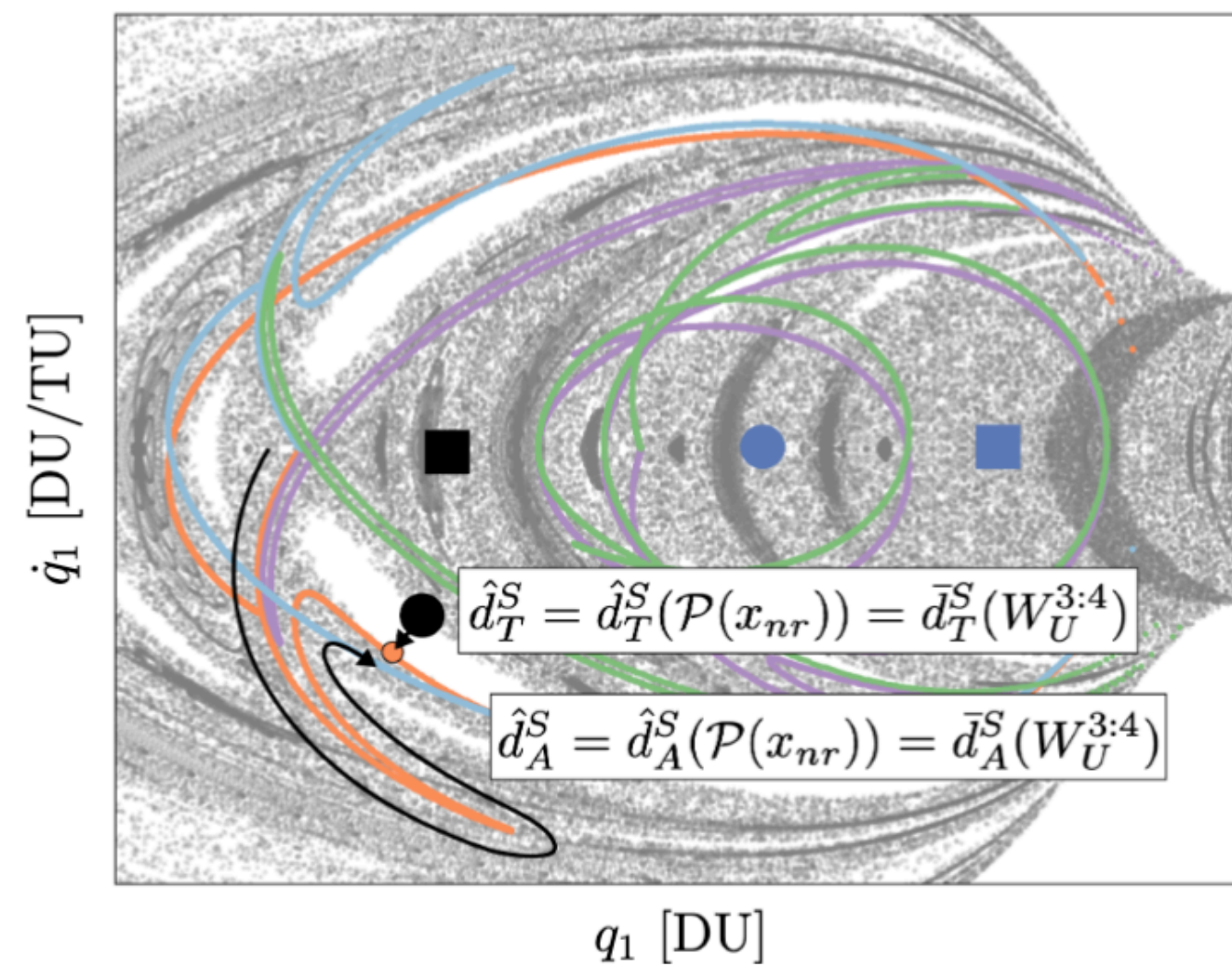


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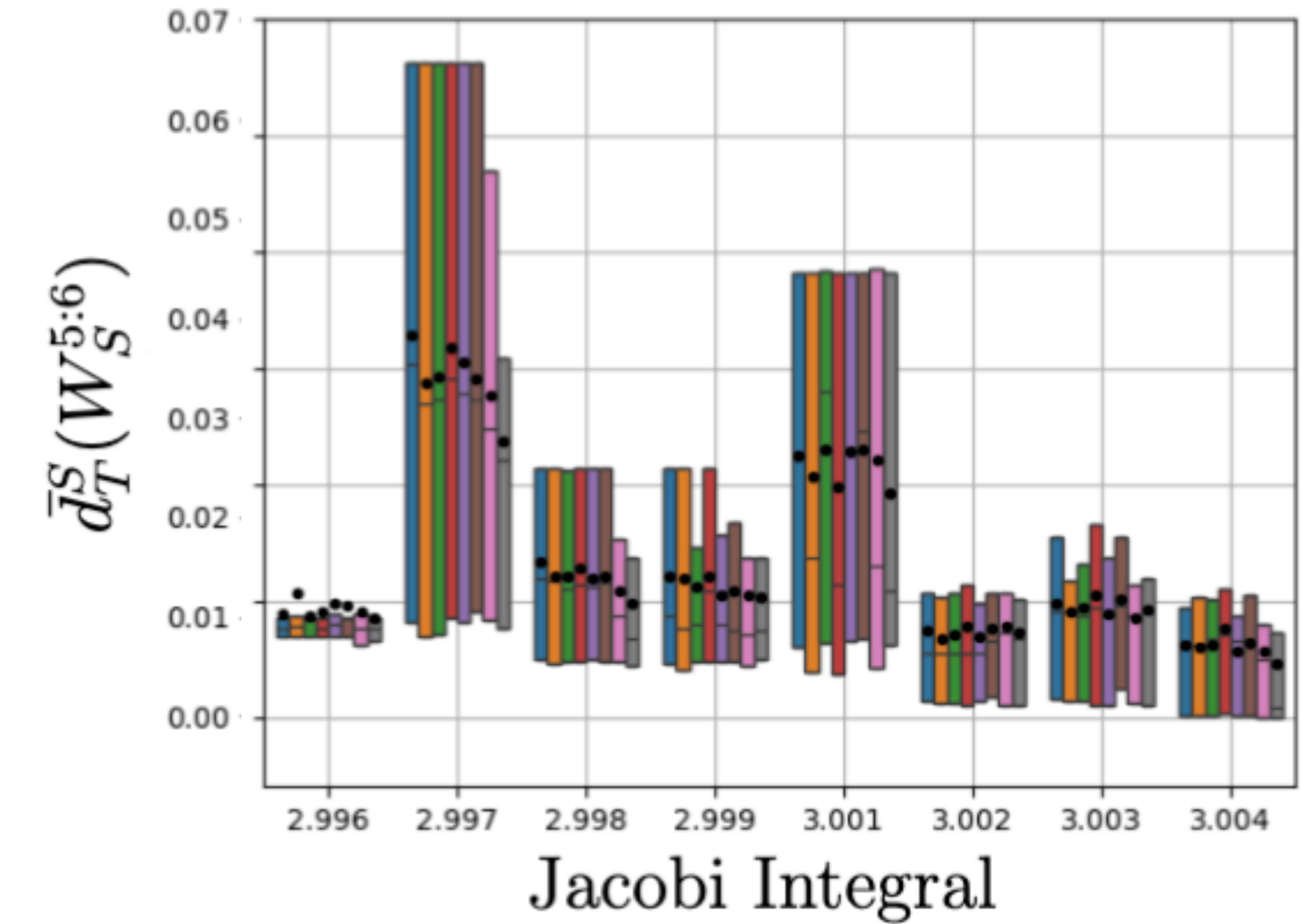
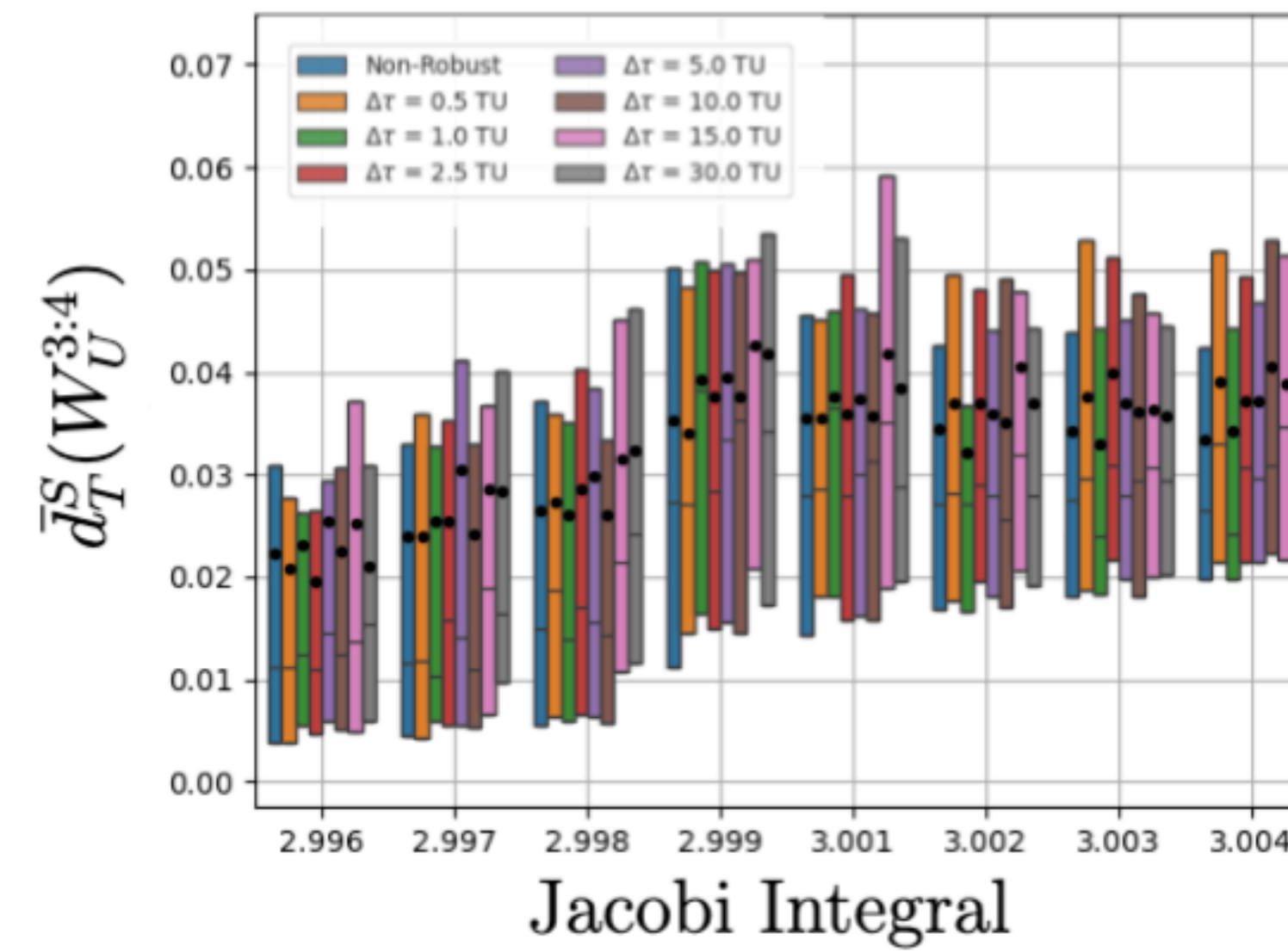
The robust low-thrust solutions leverage the invariant manifolds closely, just like the robust solutions.



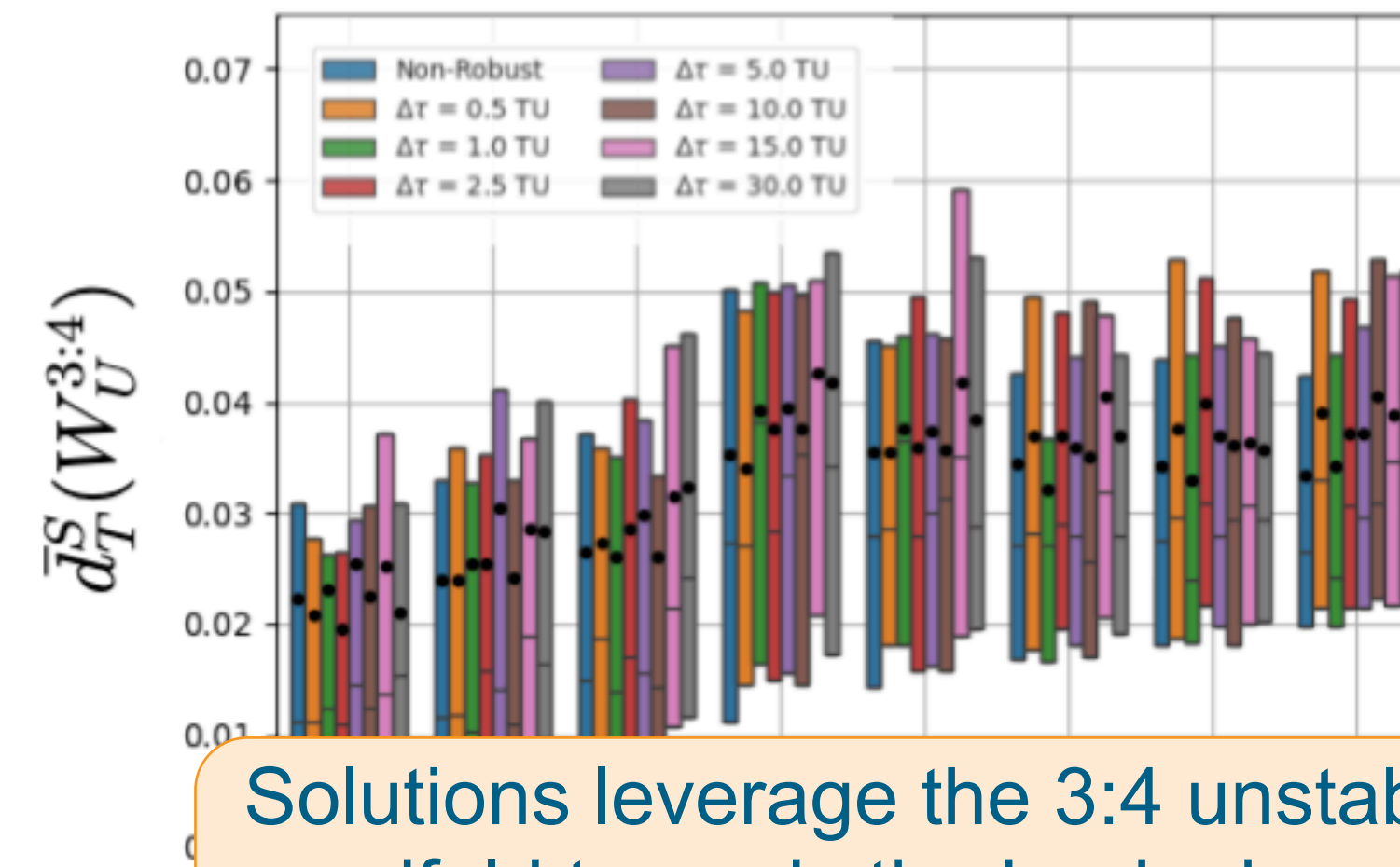
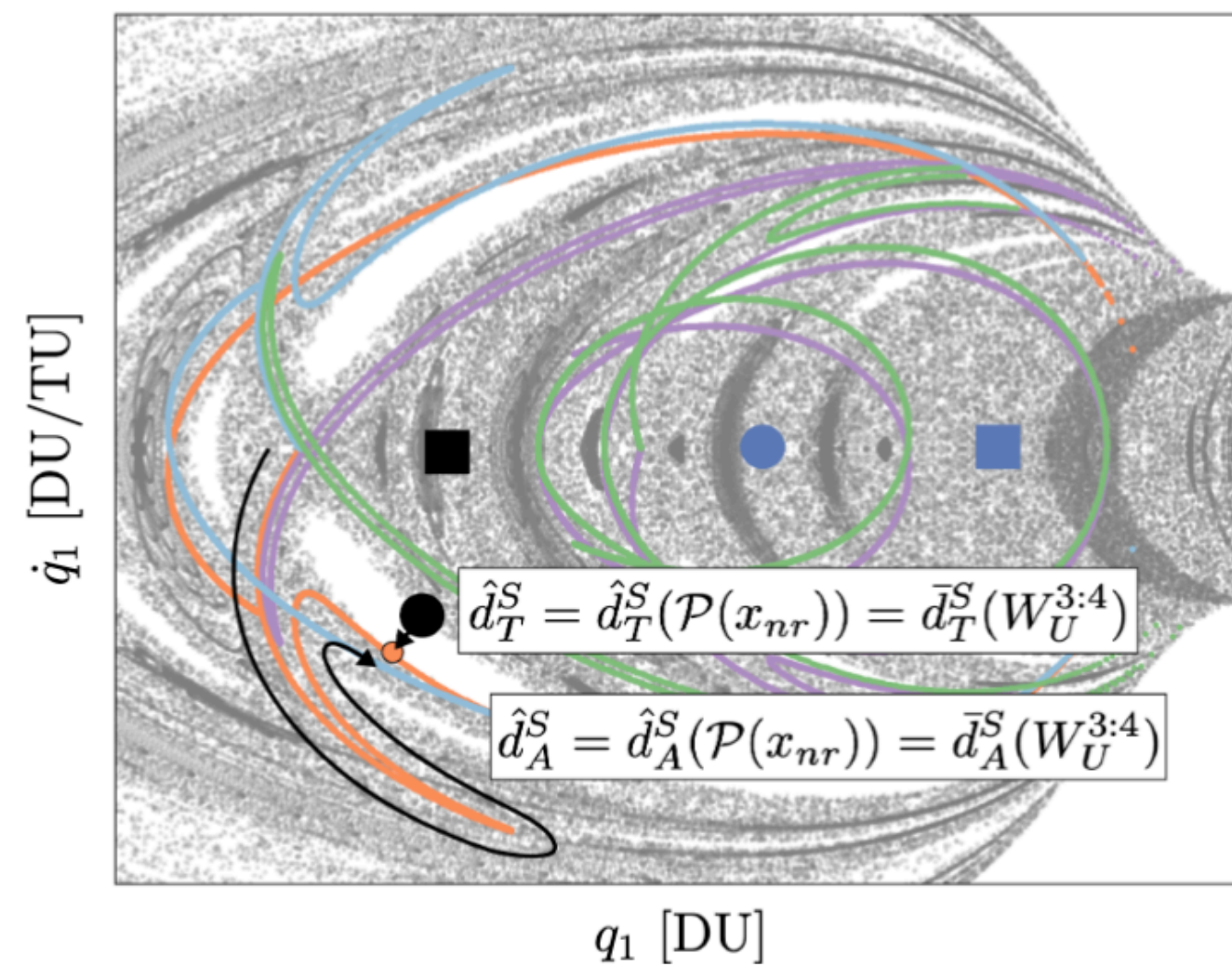
For more details:



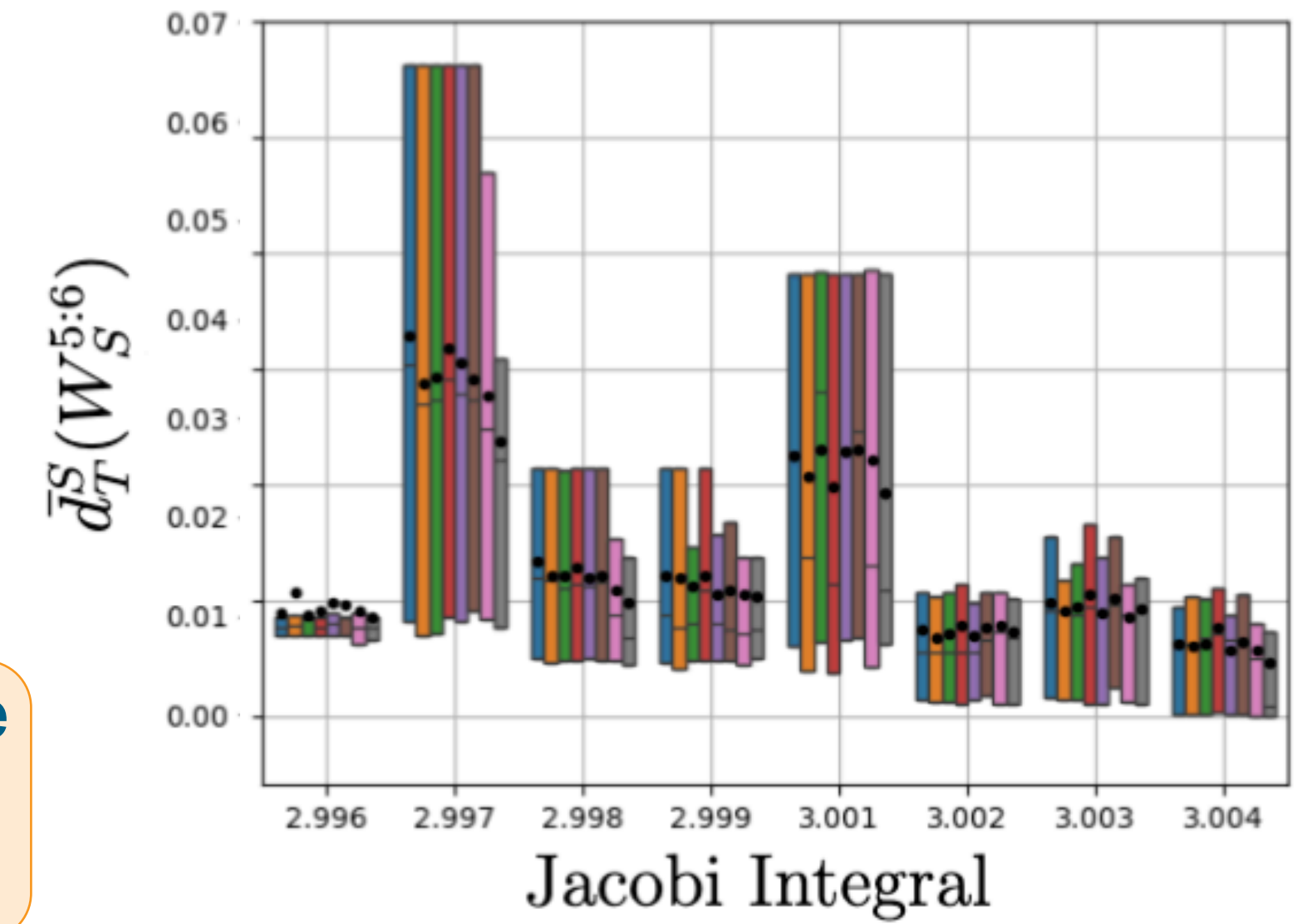
Sinha and Beeson (JGCD, 2025)



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Solutions leverage the 3:4 unstable manifold towards the beginning of the transfer

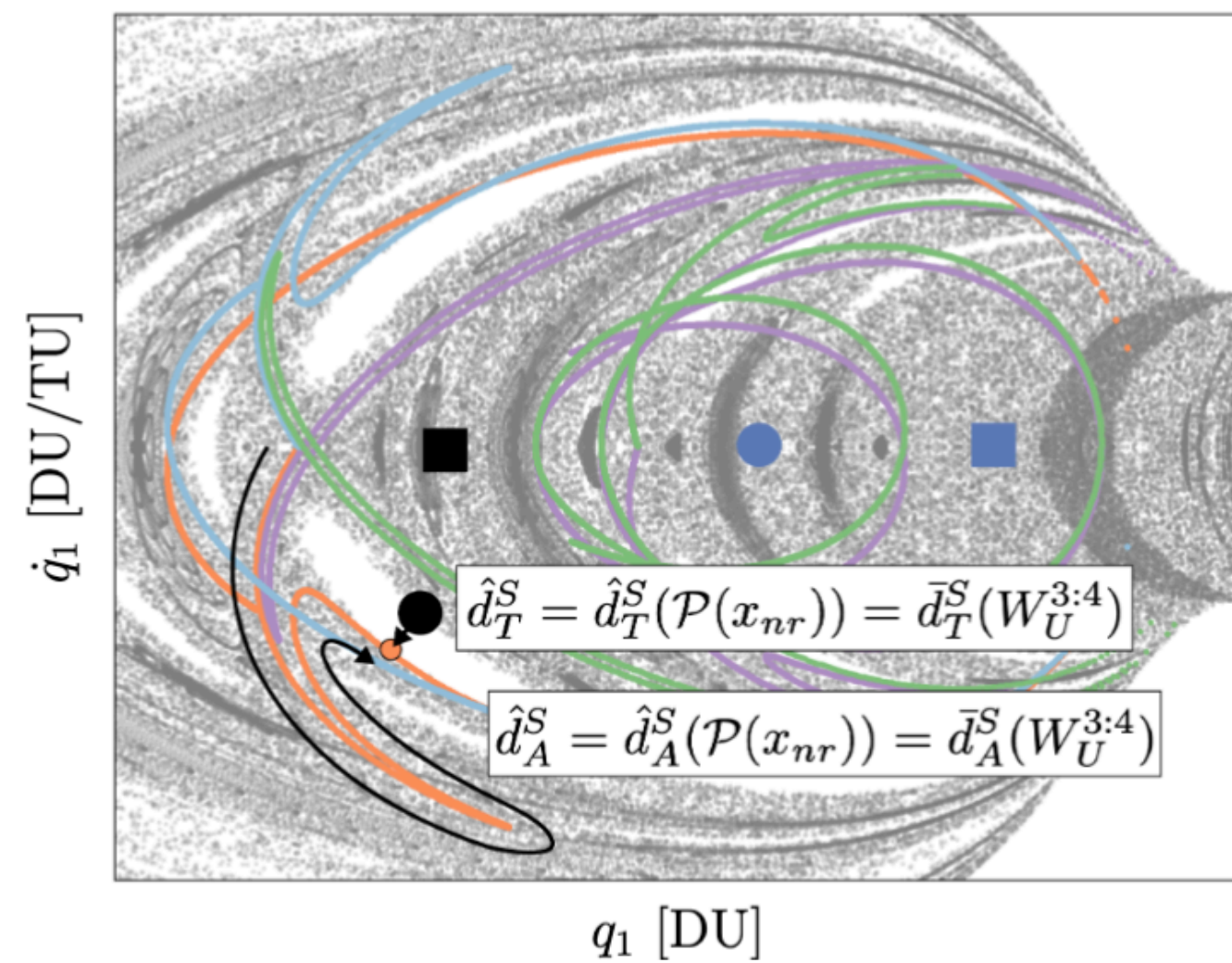


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Sinha and Beeson (JGCD, 2025)

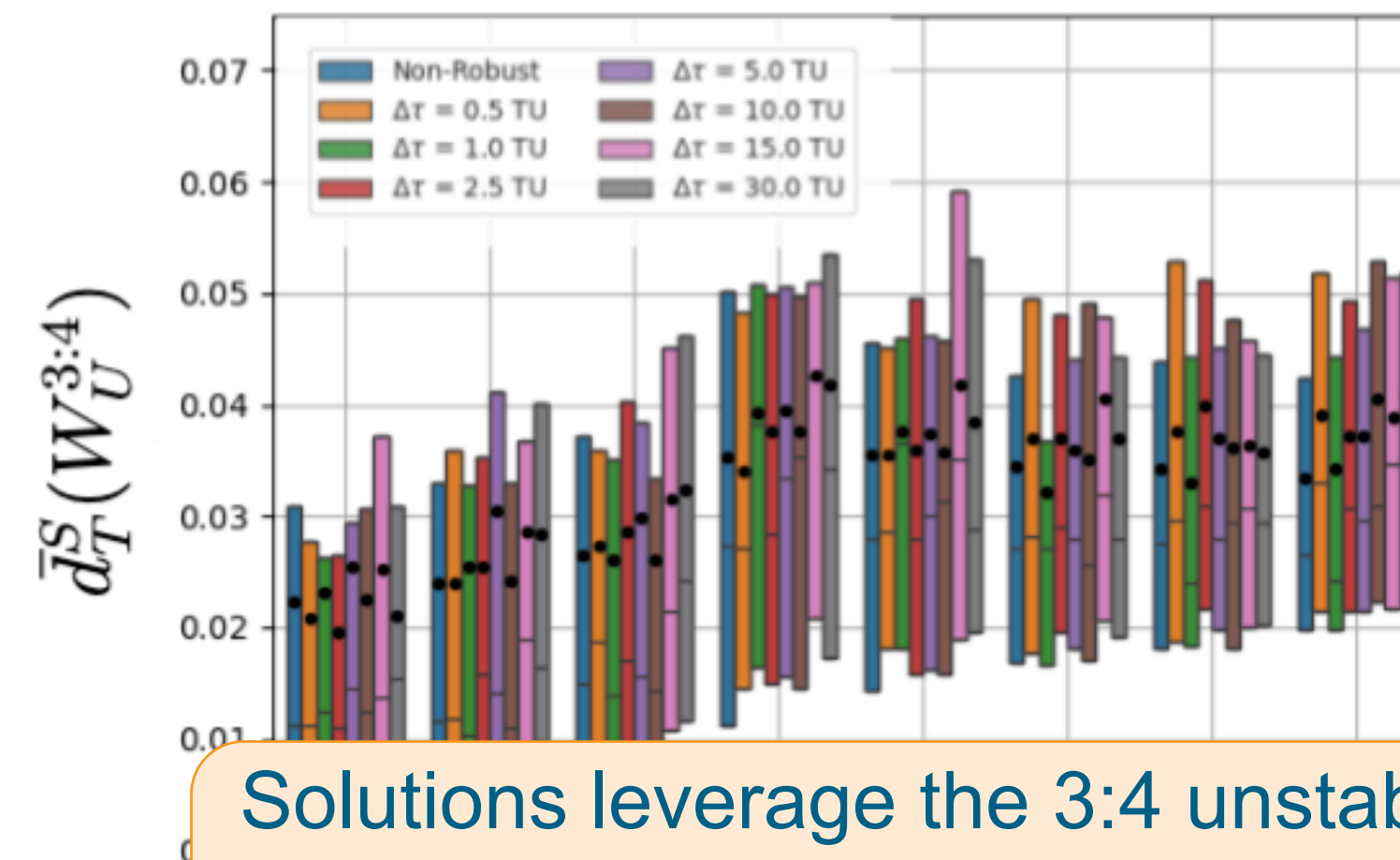
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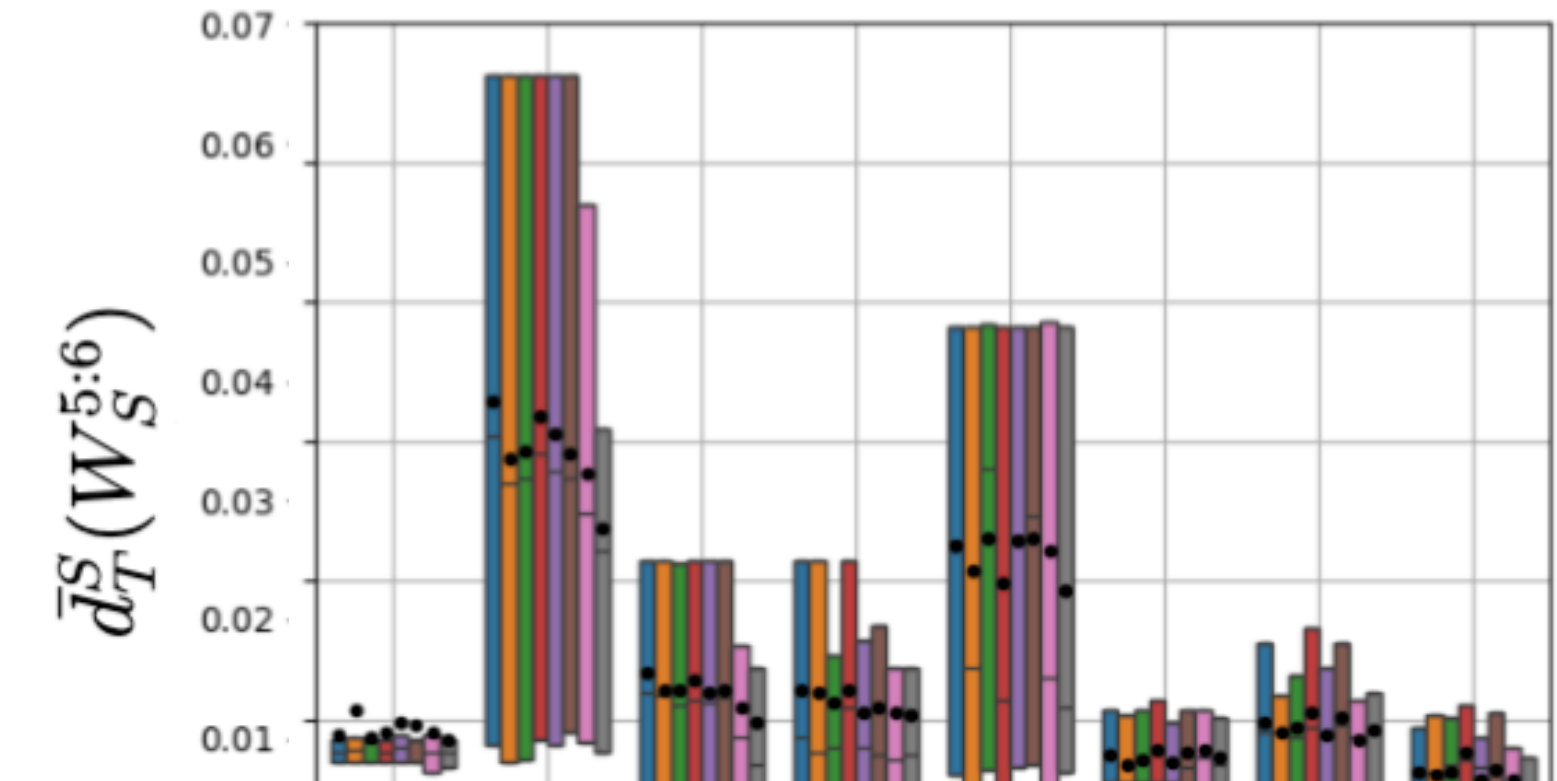
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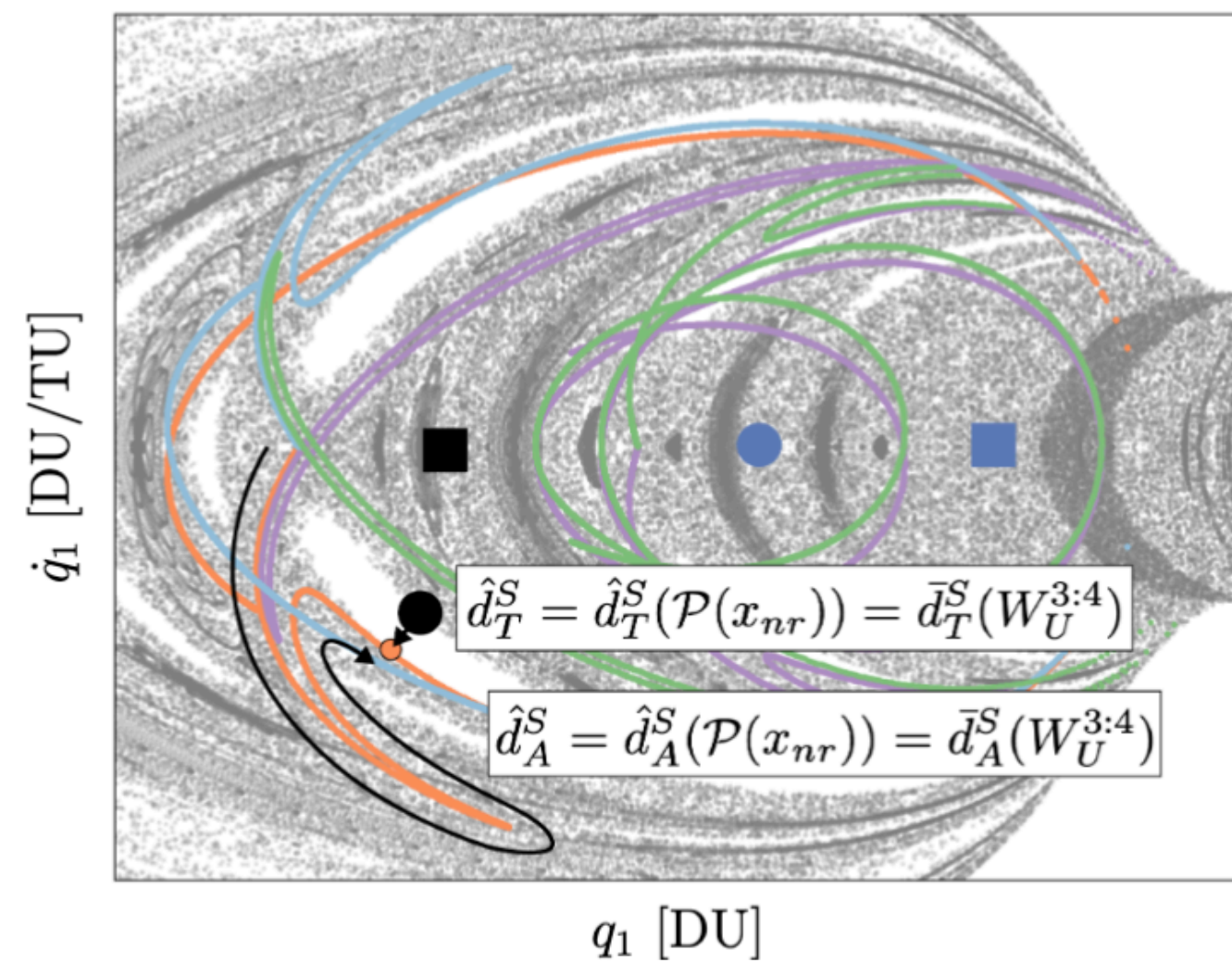


Solutions leverage the 3:4 unstable manifold towards the beginning of the transfer



Solutions leverage the 5:6 stable manifold towards the end of the transfer

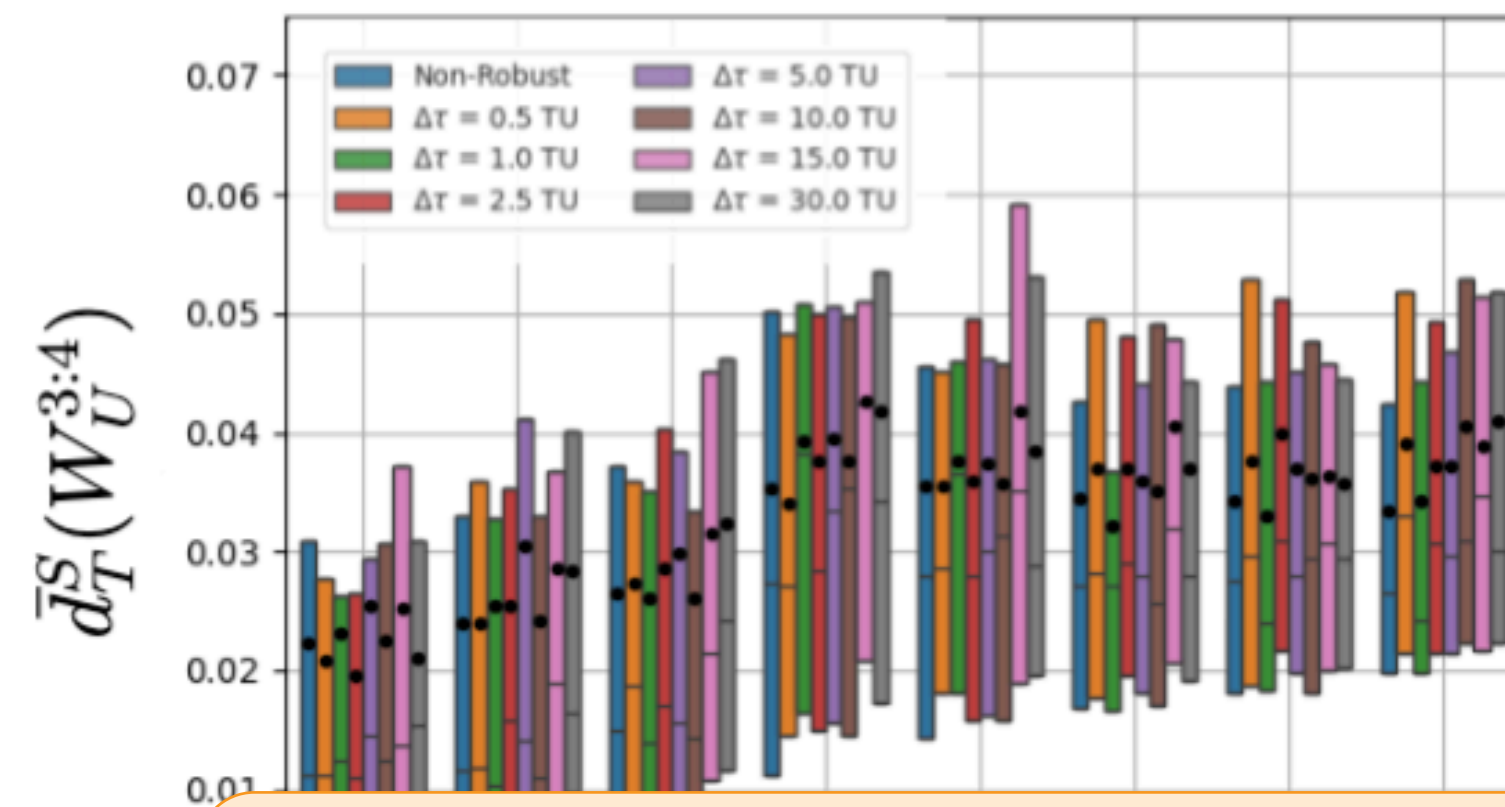
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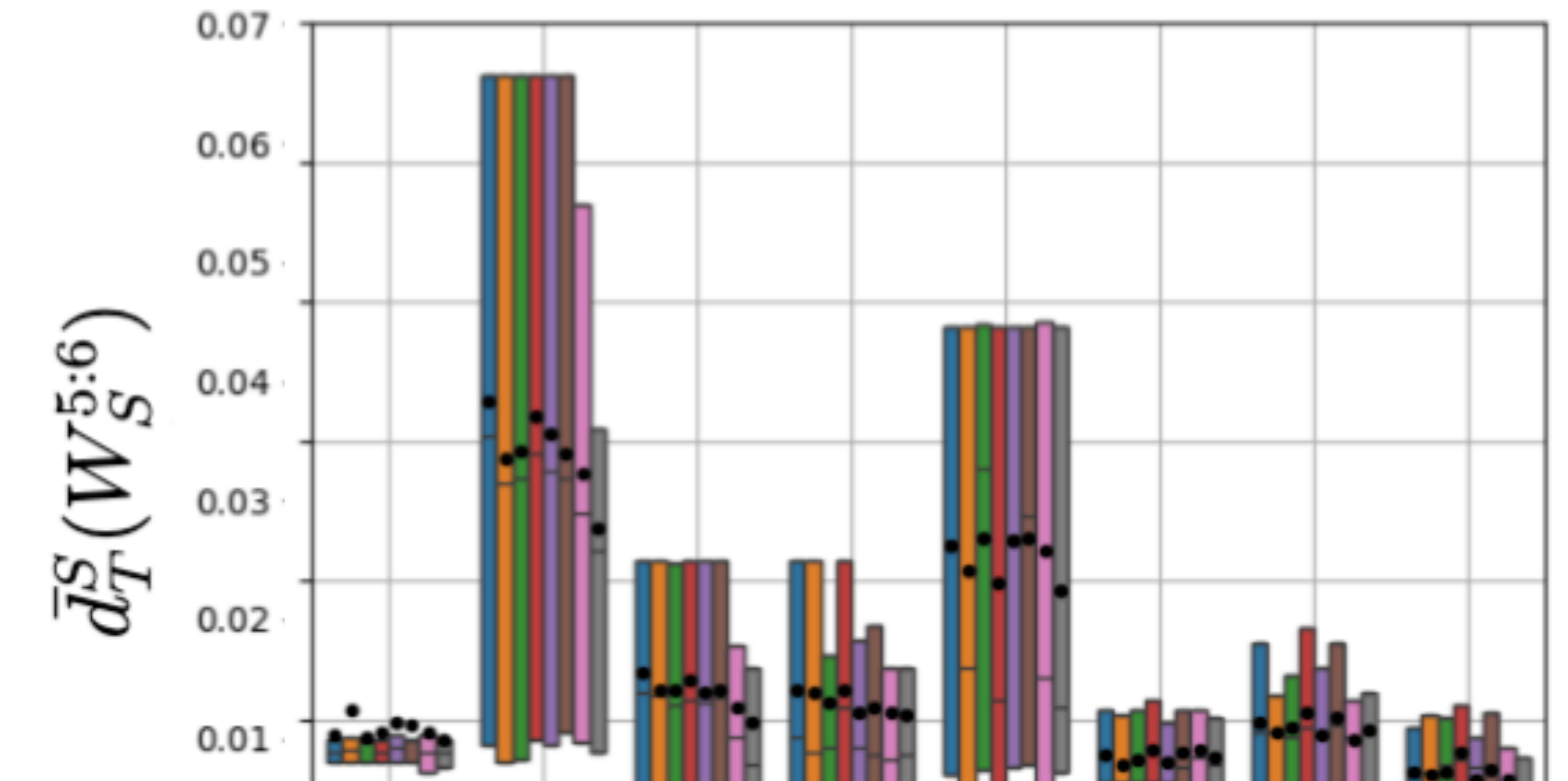
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Solutions leverage the 3:4 unstable manifold towards the beginning of the transfer

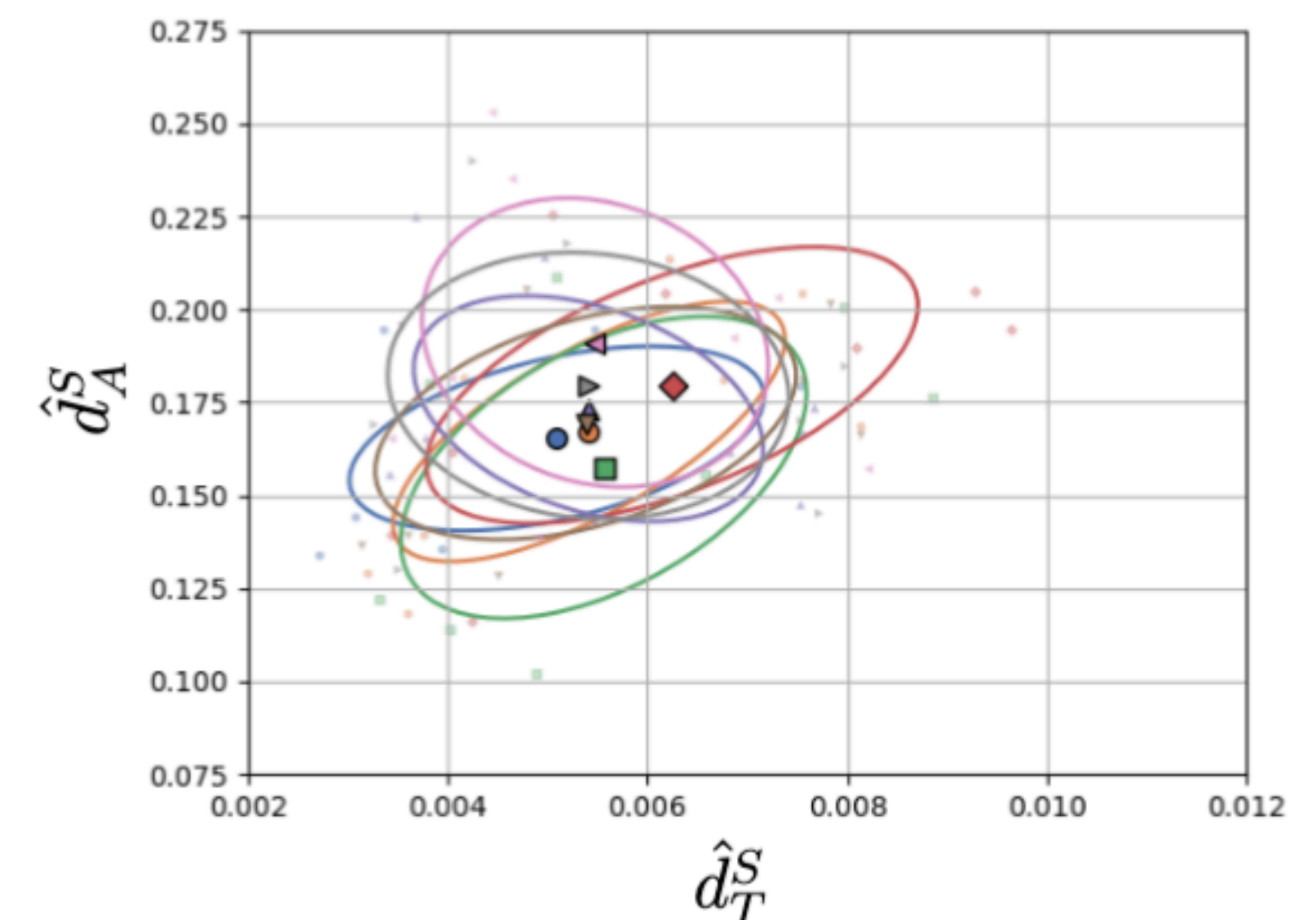
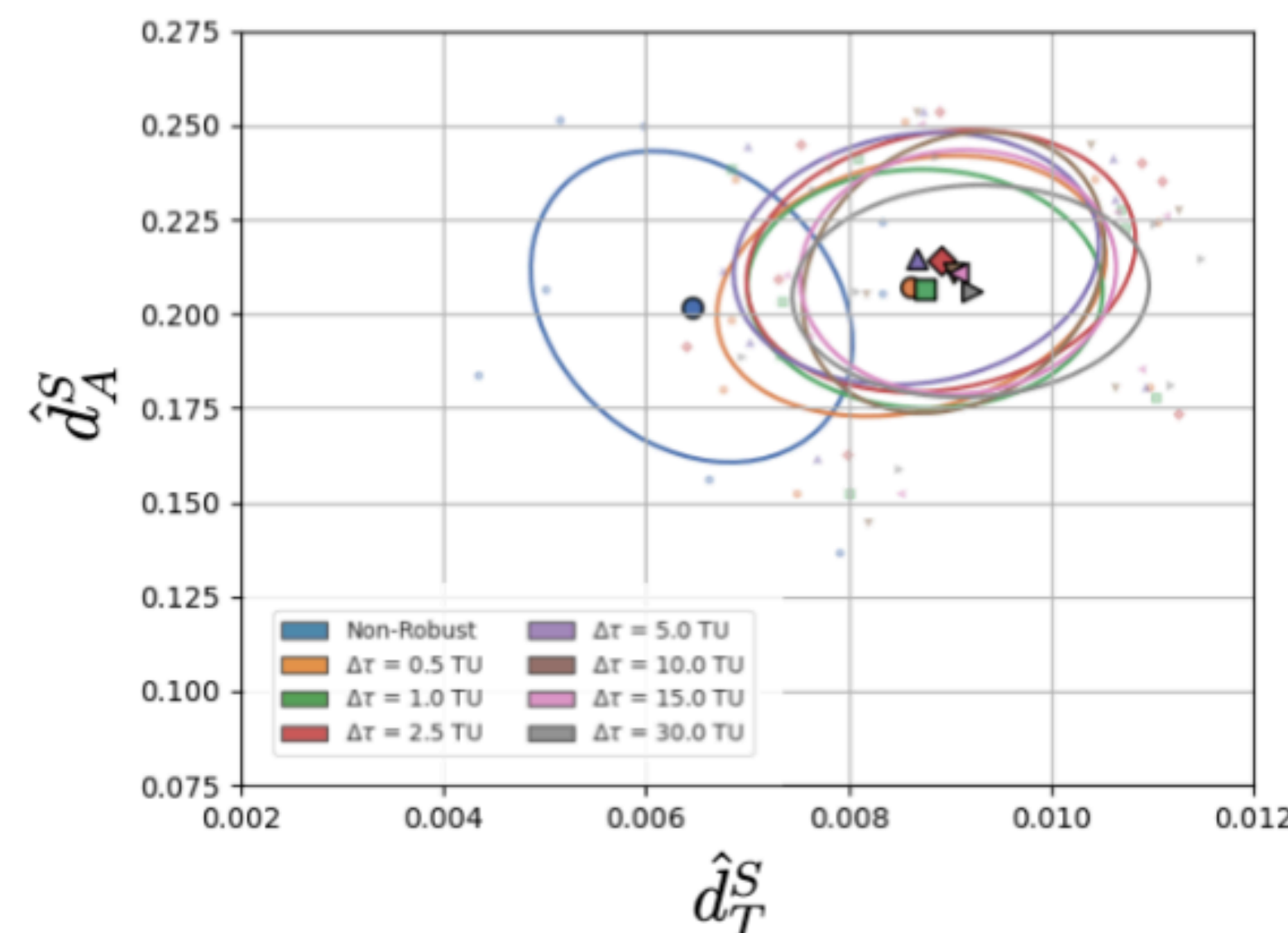


Solutions leverage the 5:6 stable manifold towards the end of the transfer

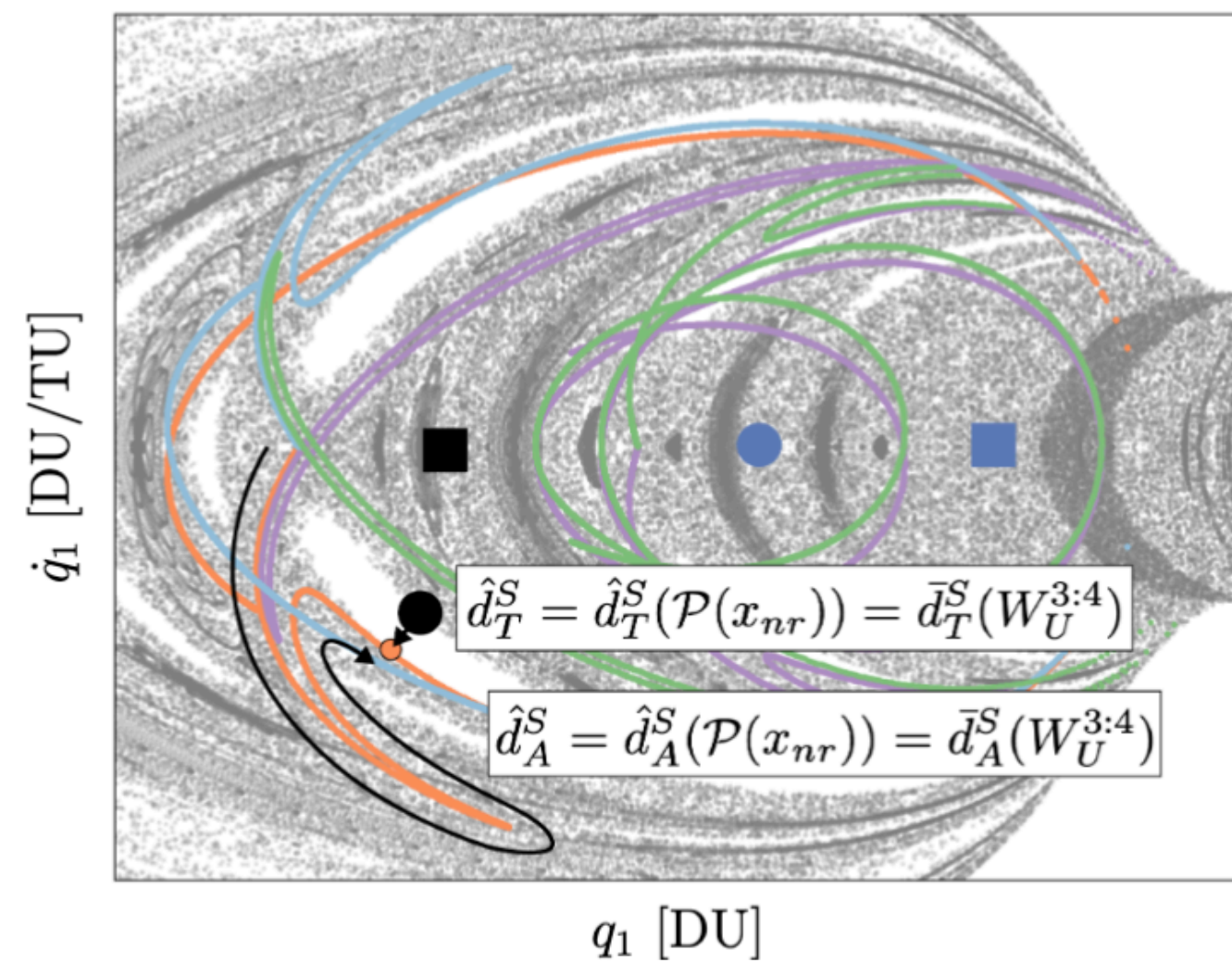
Aggregate statistical comparison across all energy levels

Feasible

Optimal



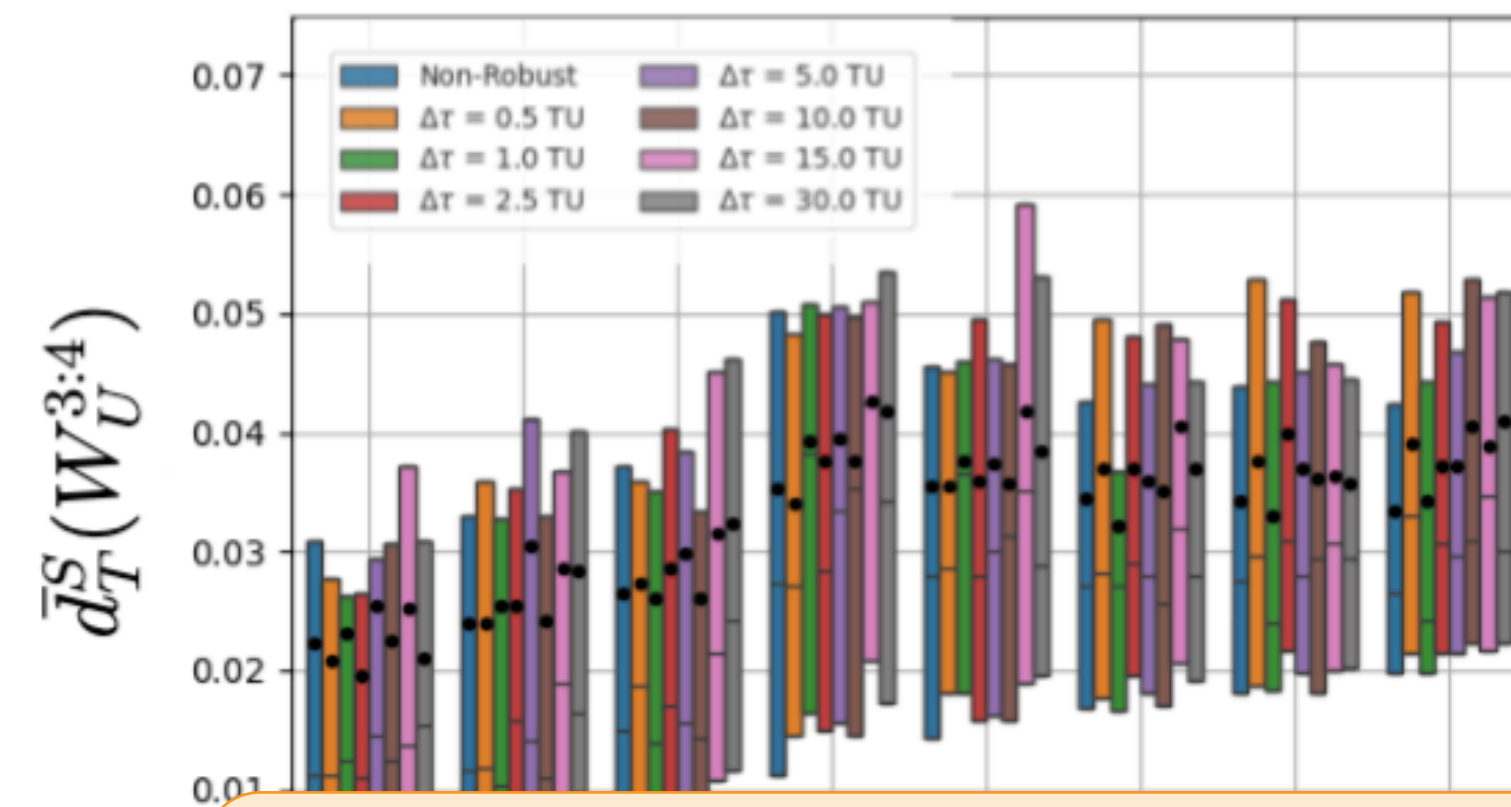
The robust low-thrust solutions leverage the invariant manifolds closely, just like the robust solutions.



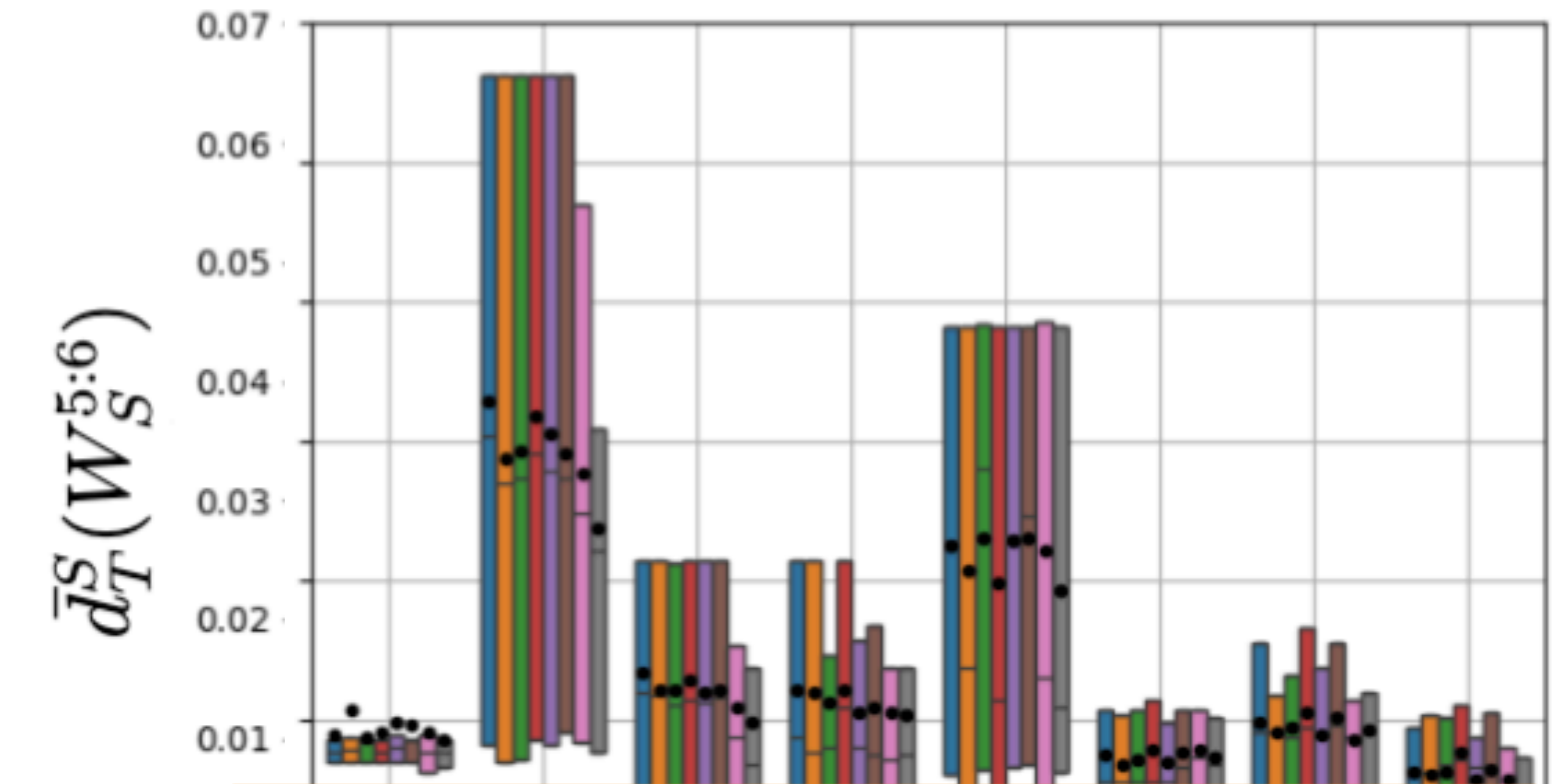
For more details:



Sinha and Beeson (JGCD, 2025)

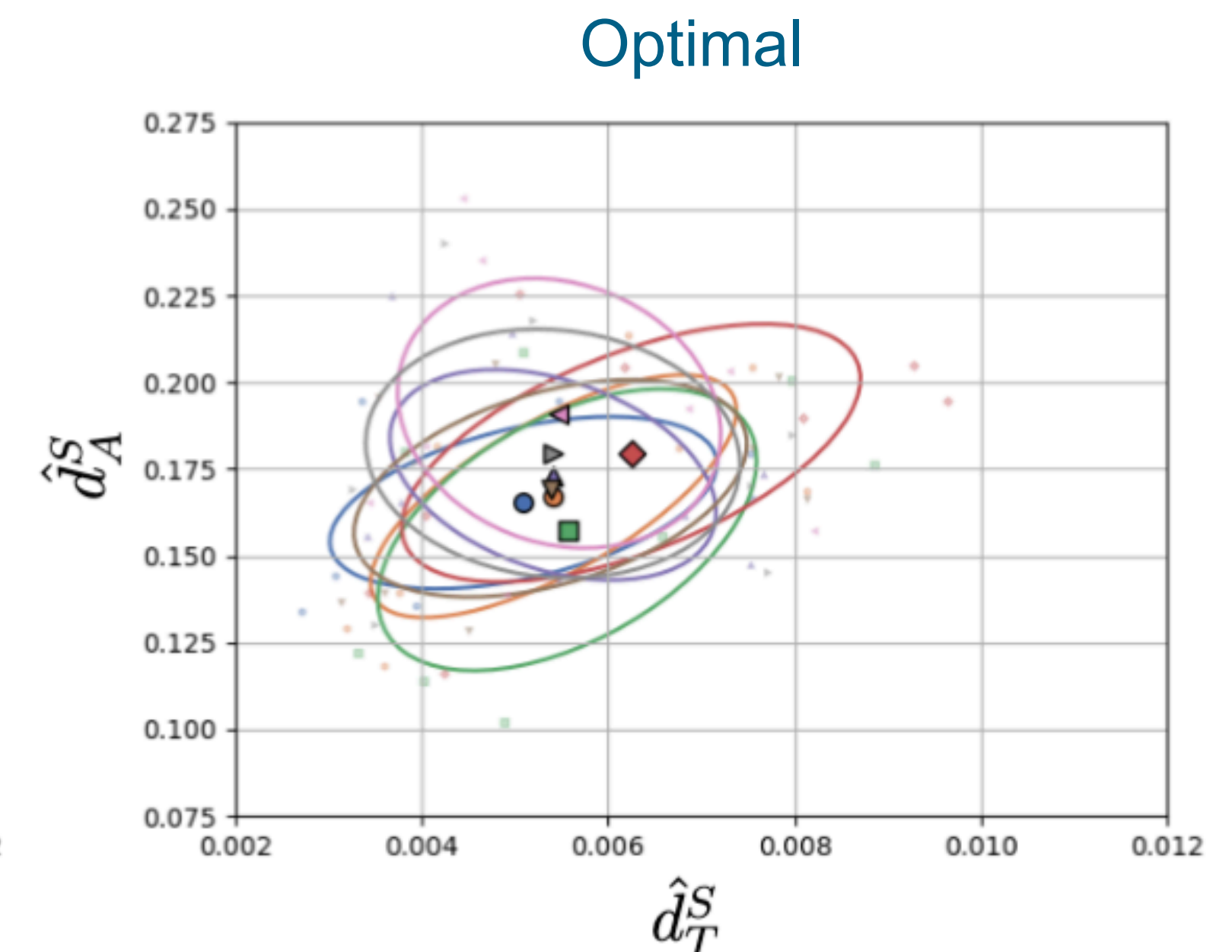
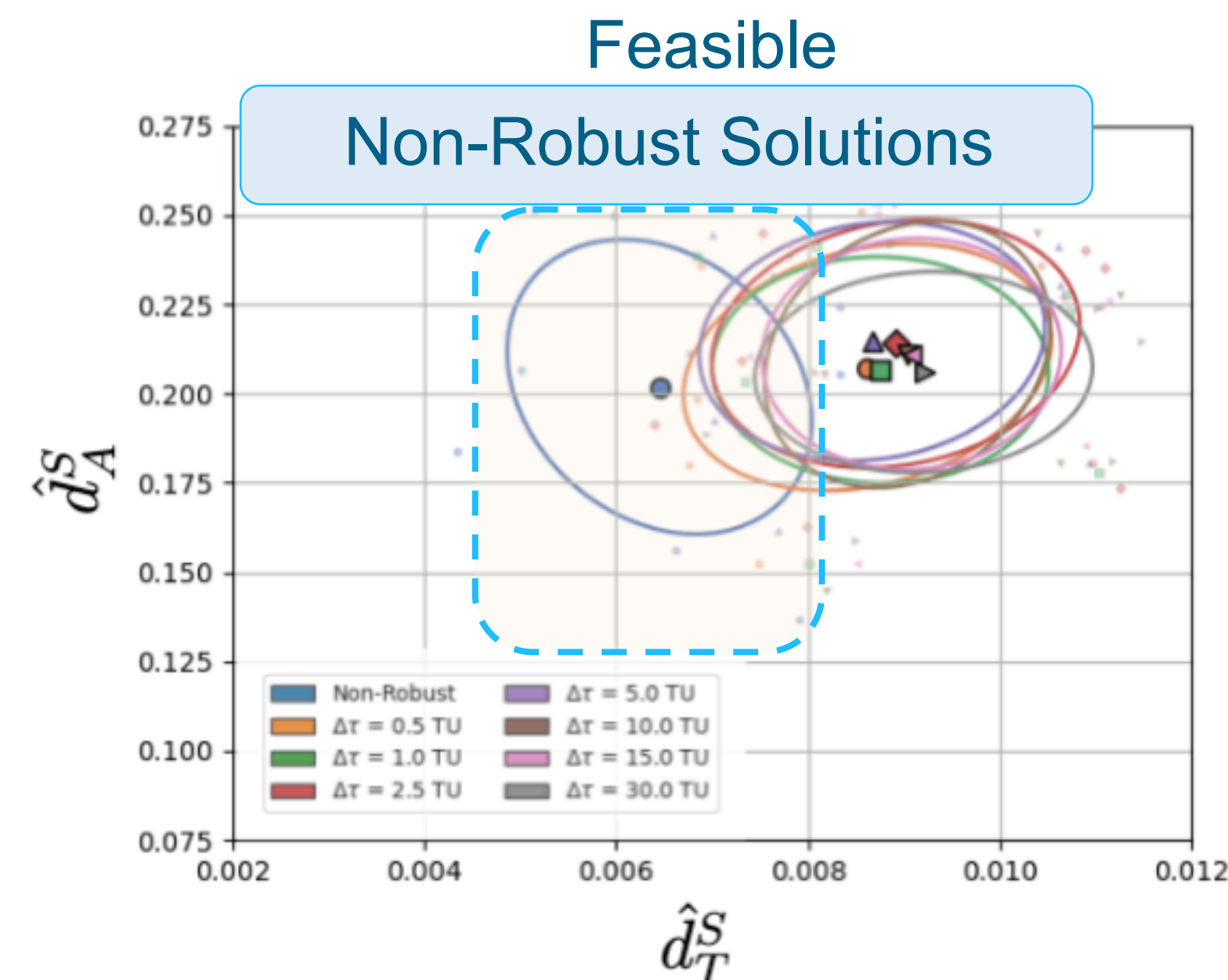


Solutions leverage the 3:4 unstable manifold towards the beginning of the transfer

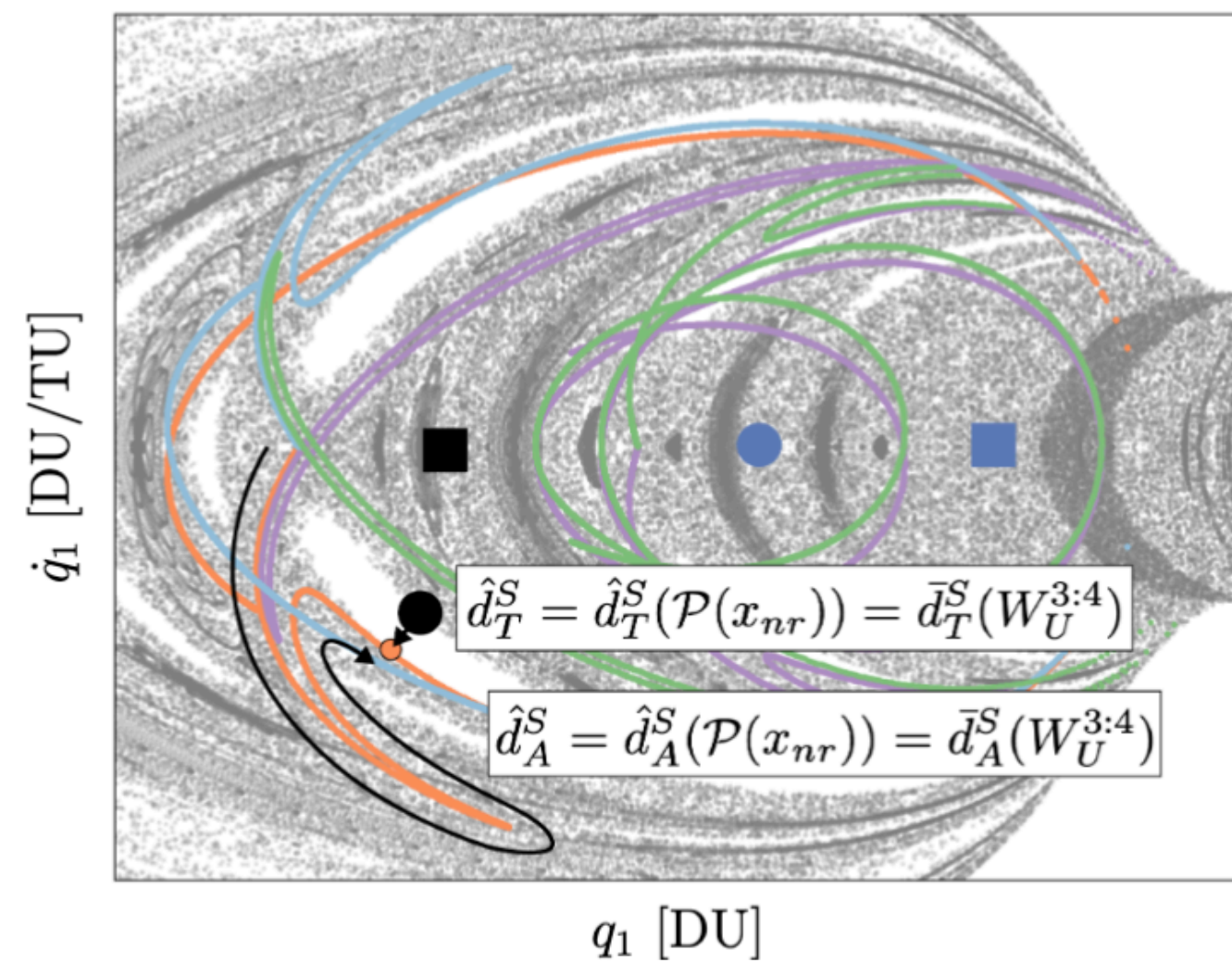


Solutions leverage the 5:6 stable manifold towards the end of the transfer

Aggregate statistical comparison across all energy levels



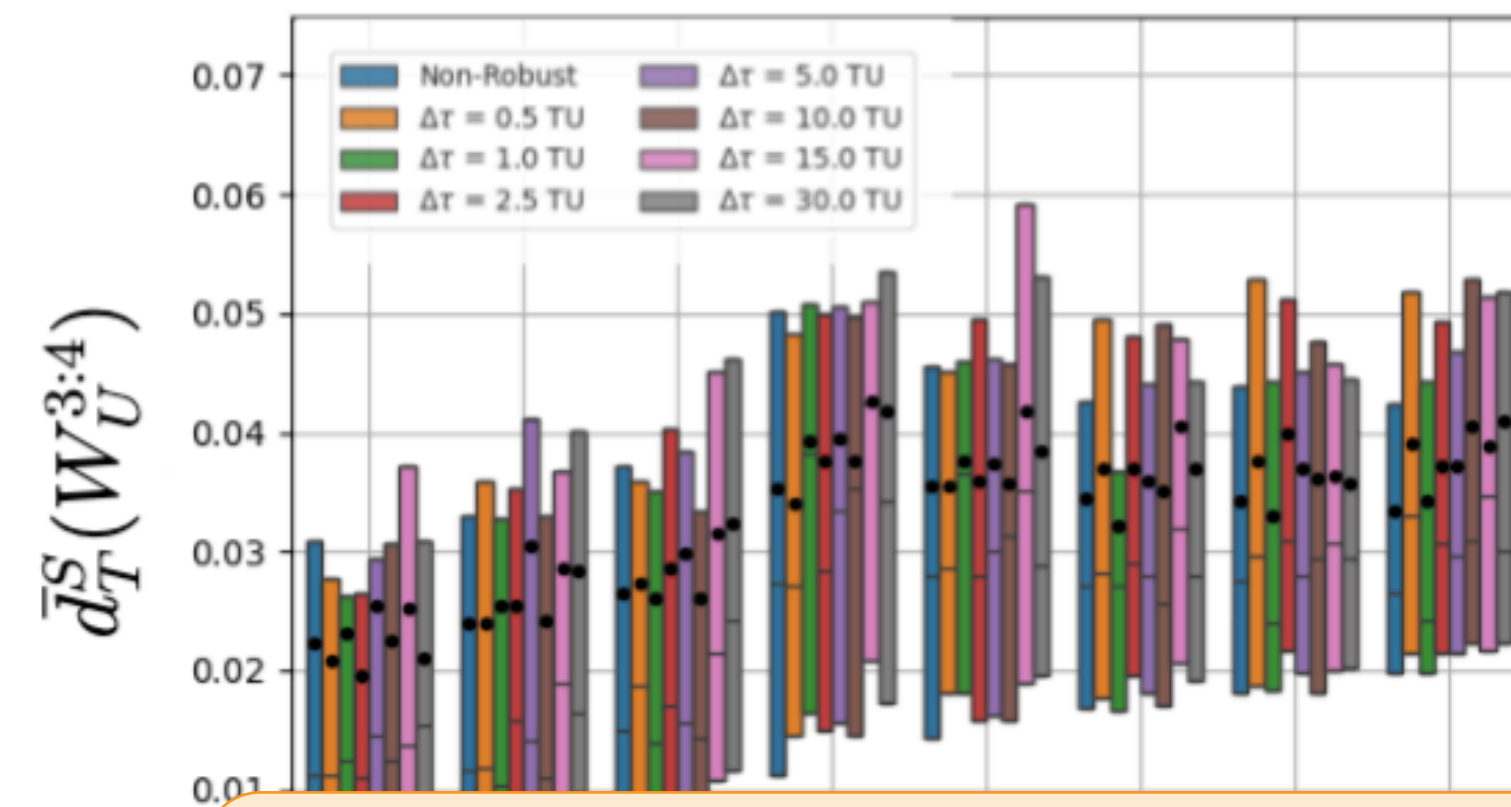
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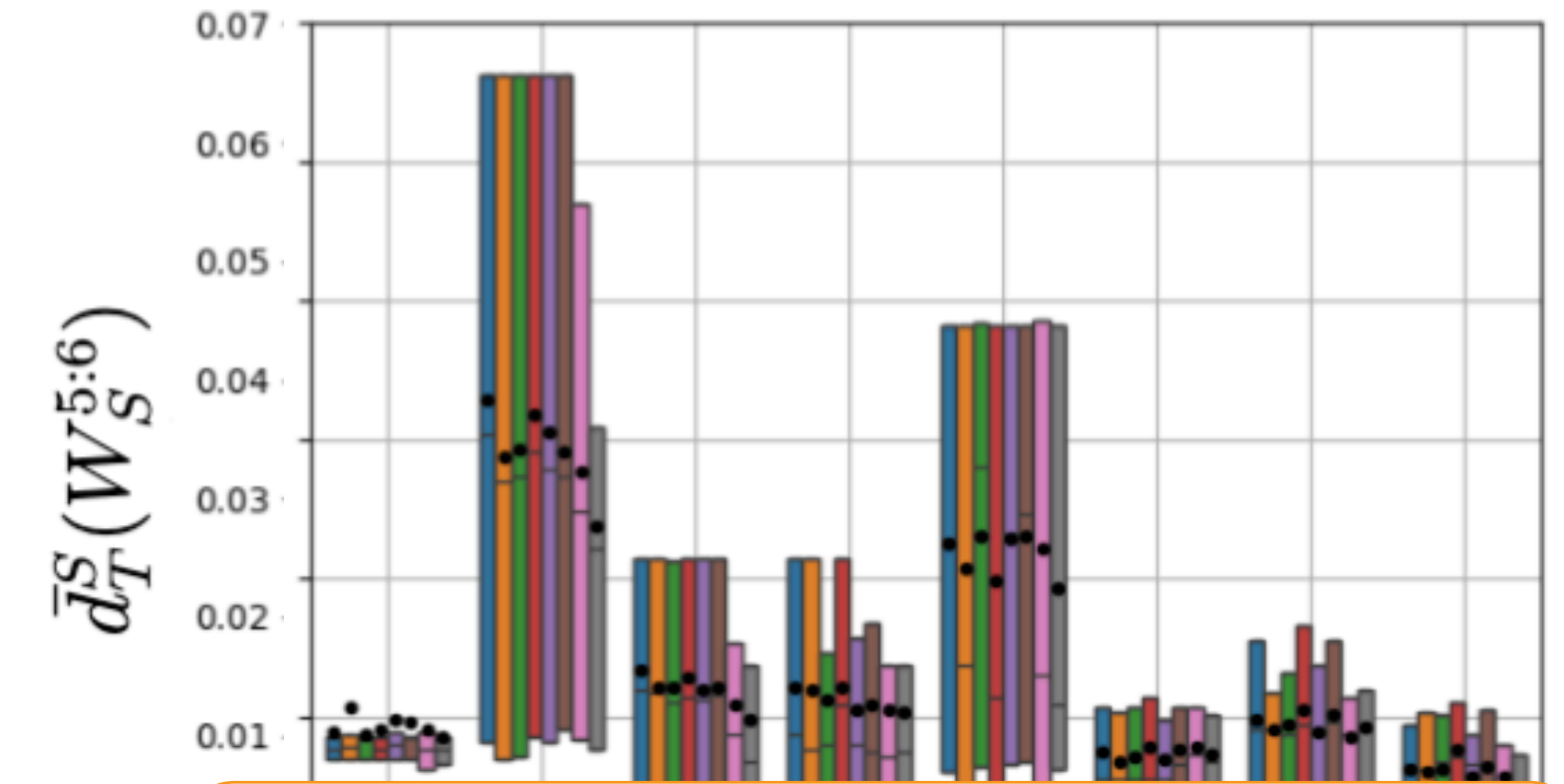
For more details:



Sinha and Beeson (JGCD, 2025)



Solutions leverage the 3:4 unstable manifold towards the beginning of the transfer

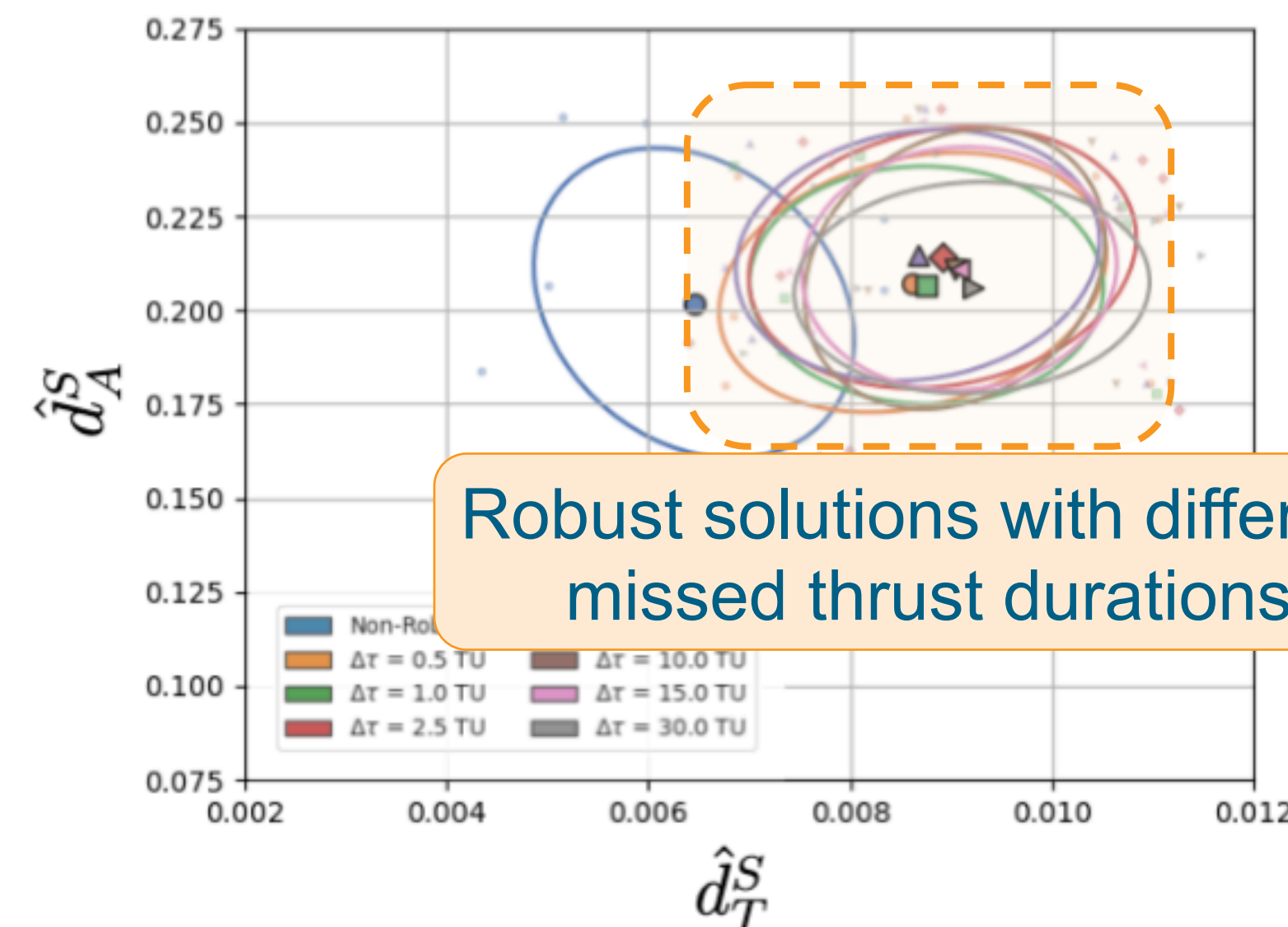


Solutions leverage the 5:6 stable manifold towards the end of the transfer

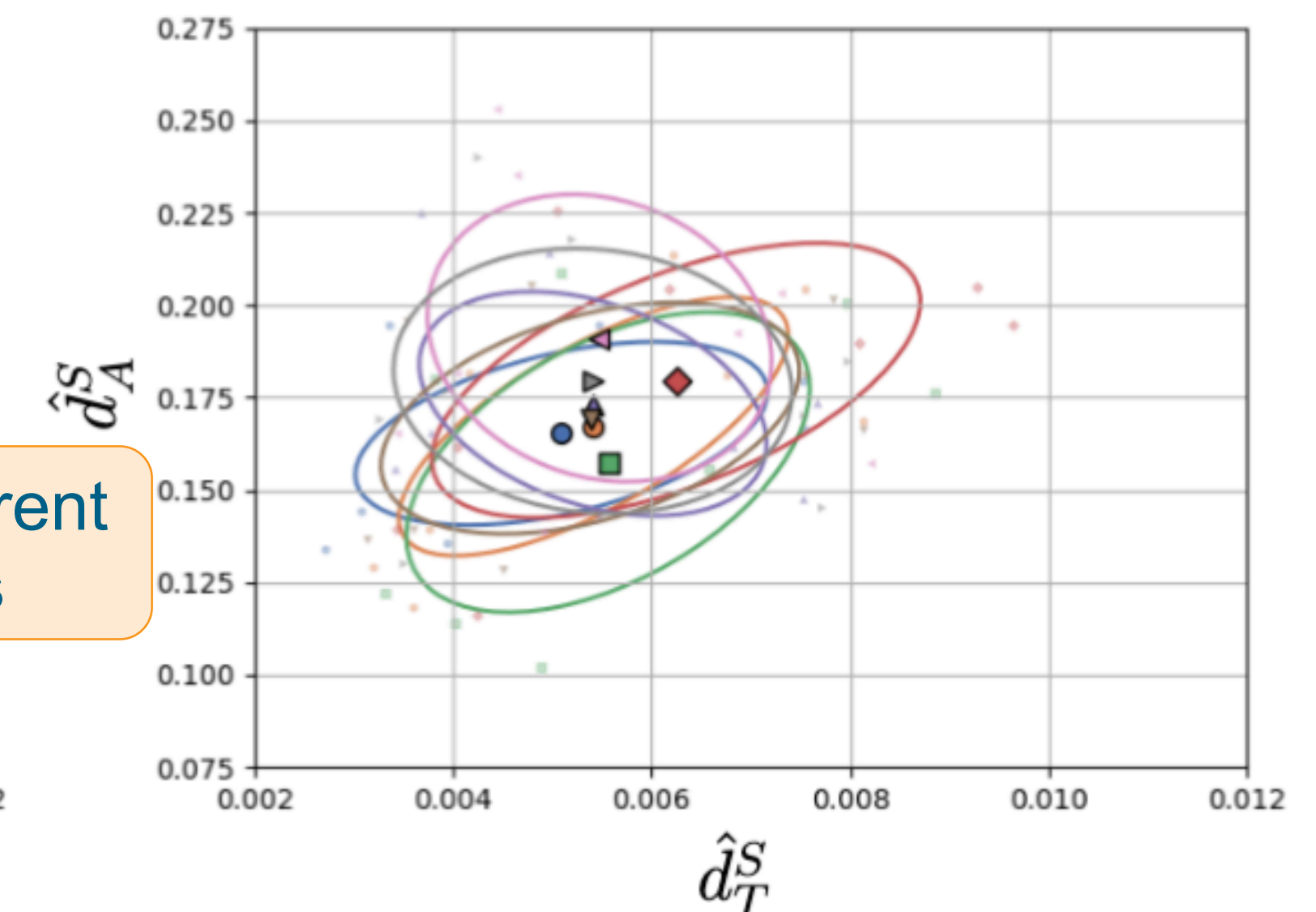
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Feasible

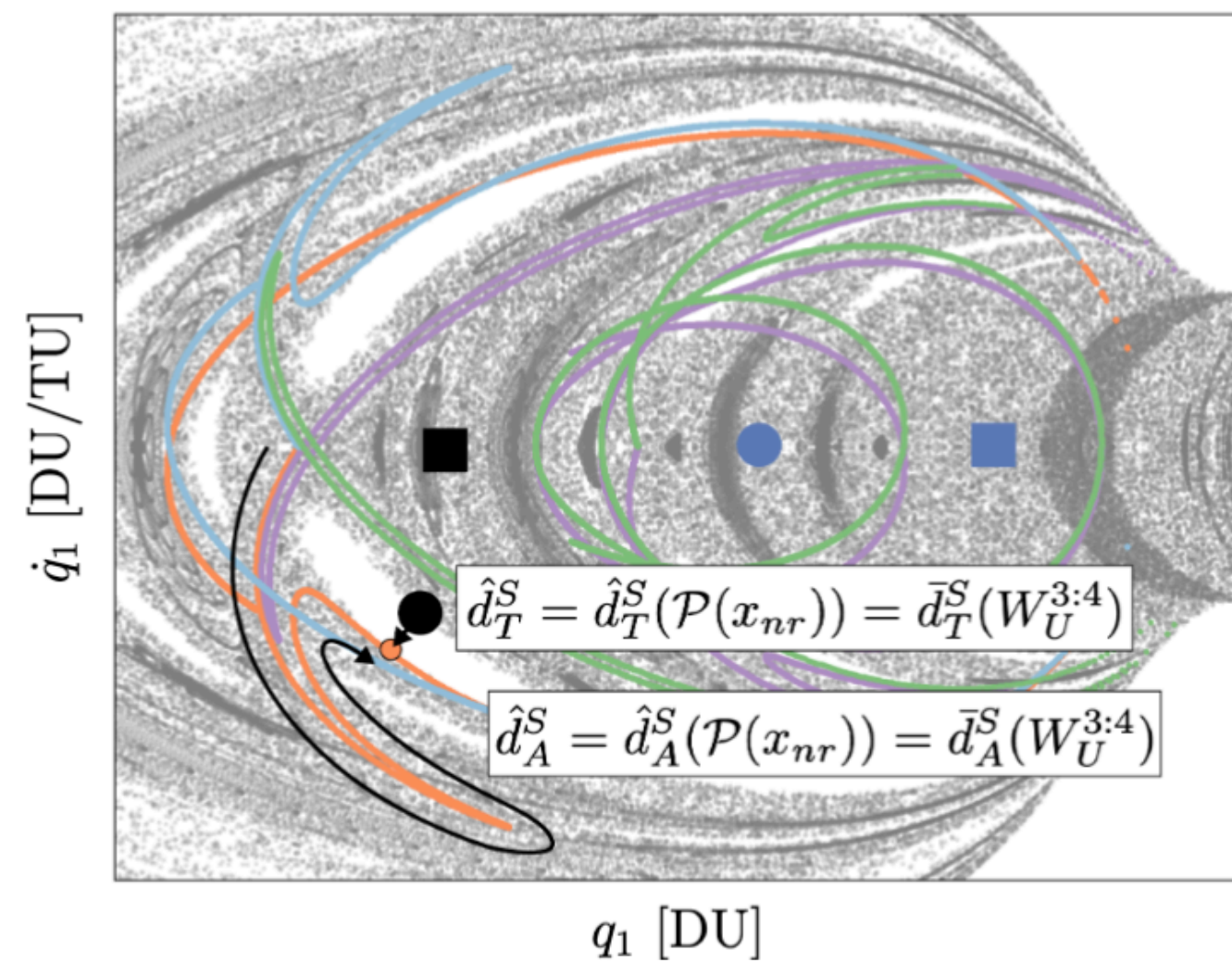
Optimal



Robust solutions with different missed thrust durations



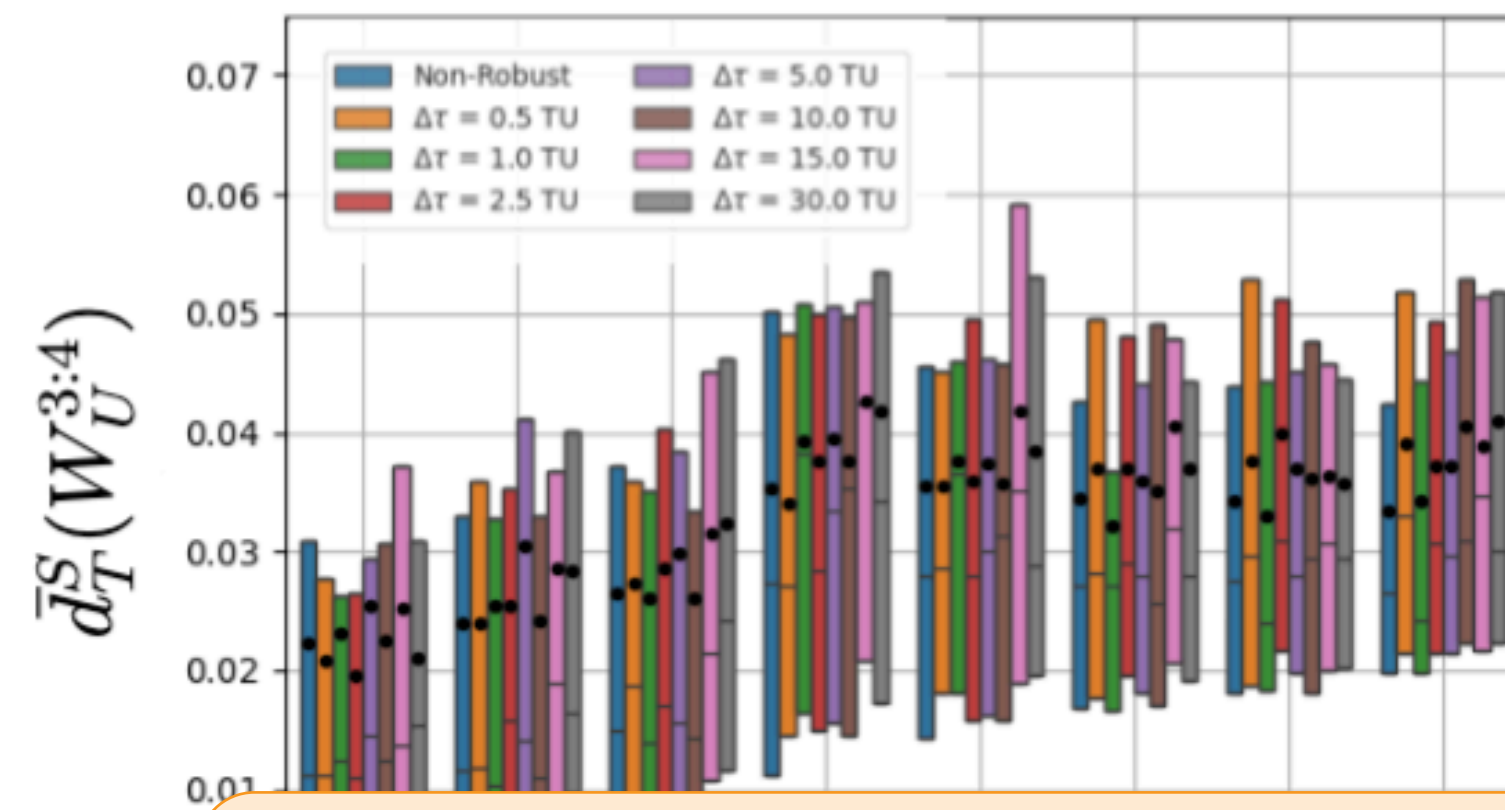
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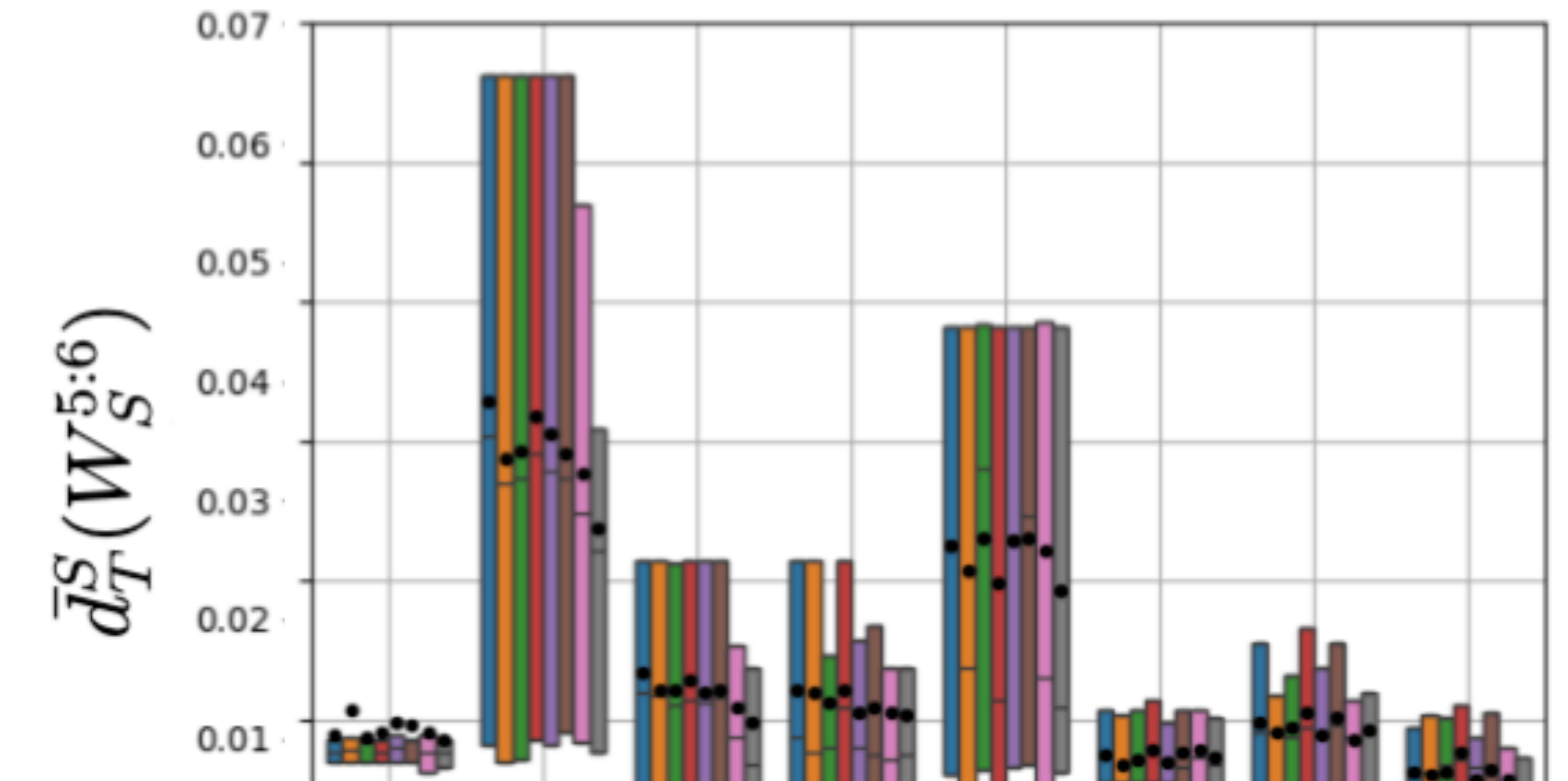
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Sinha and Beeson (JGCD, 2025)



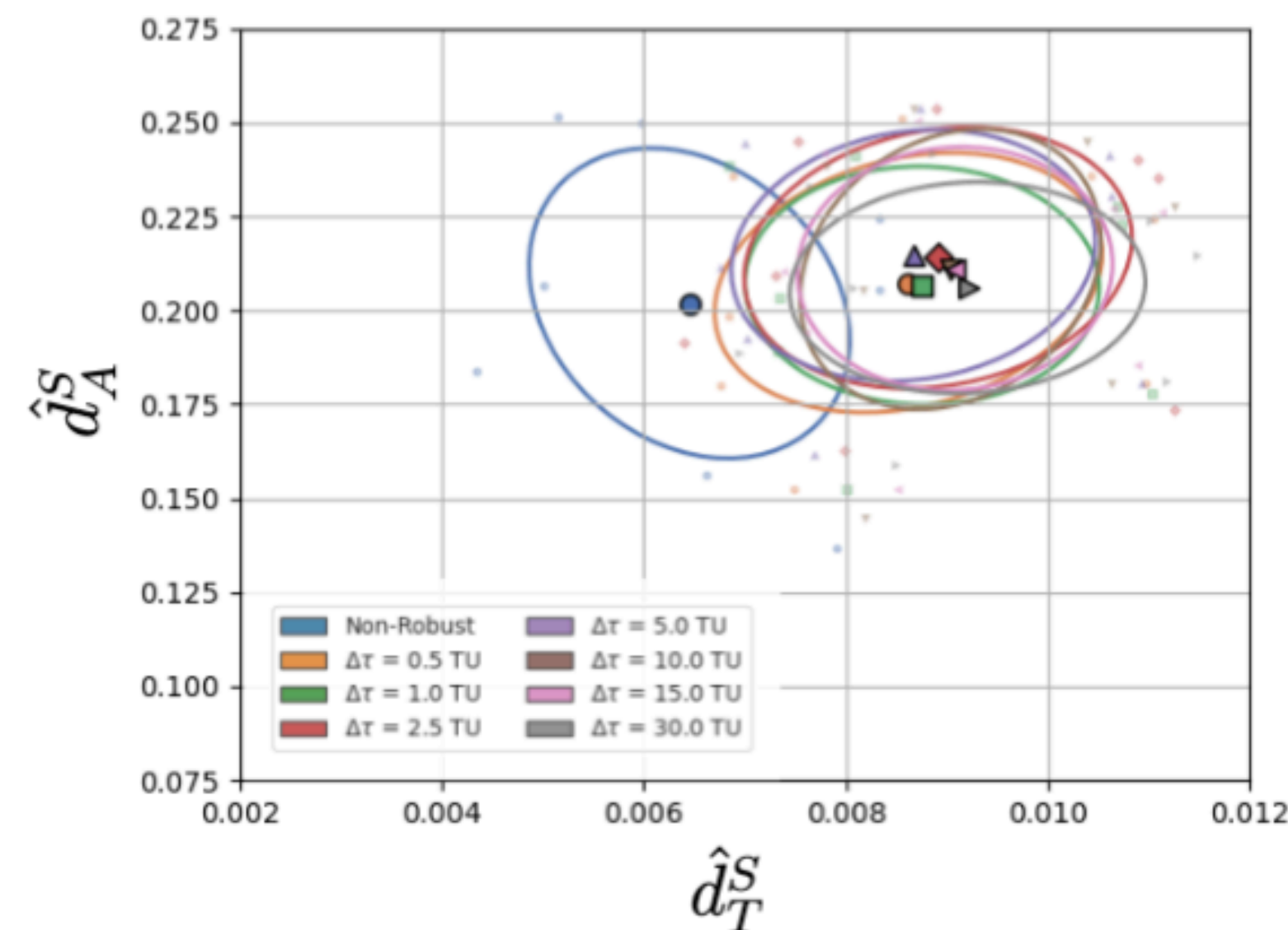
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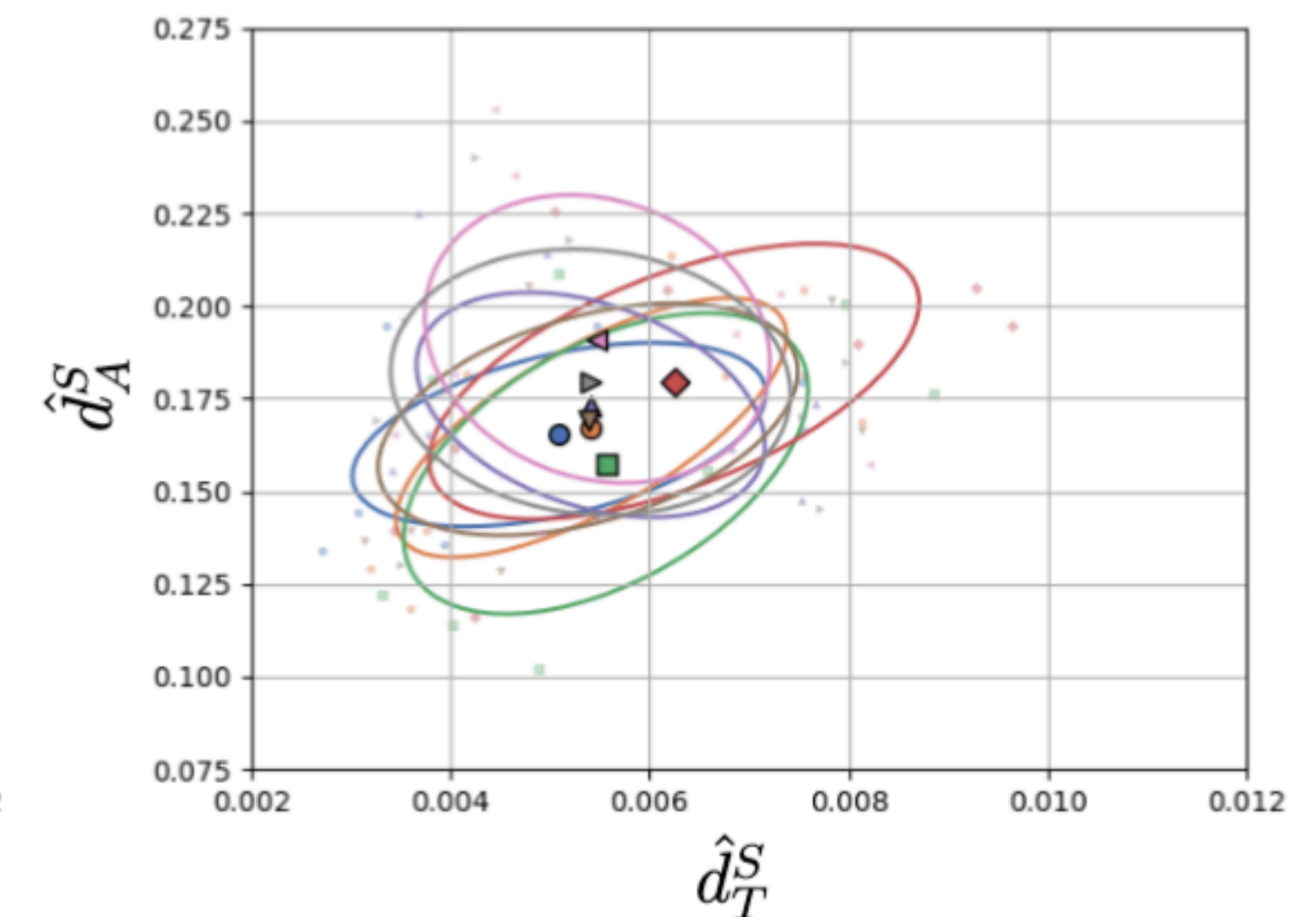
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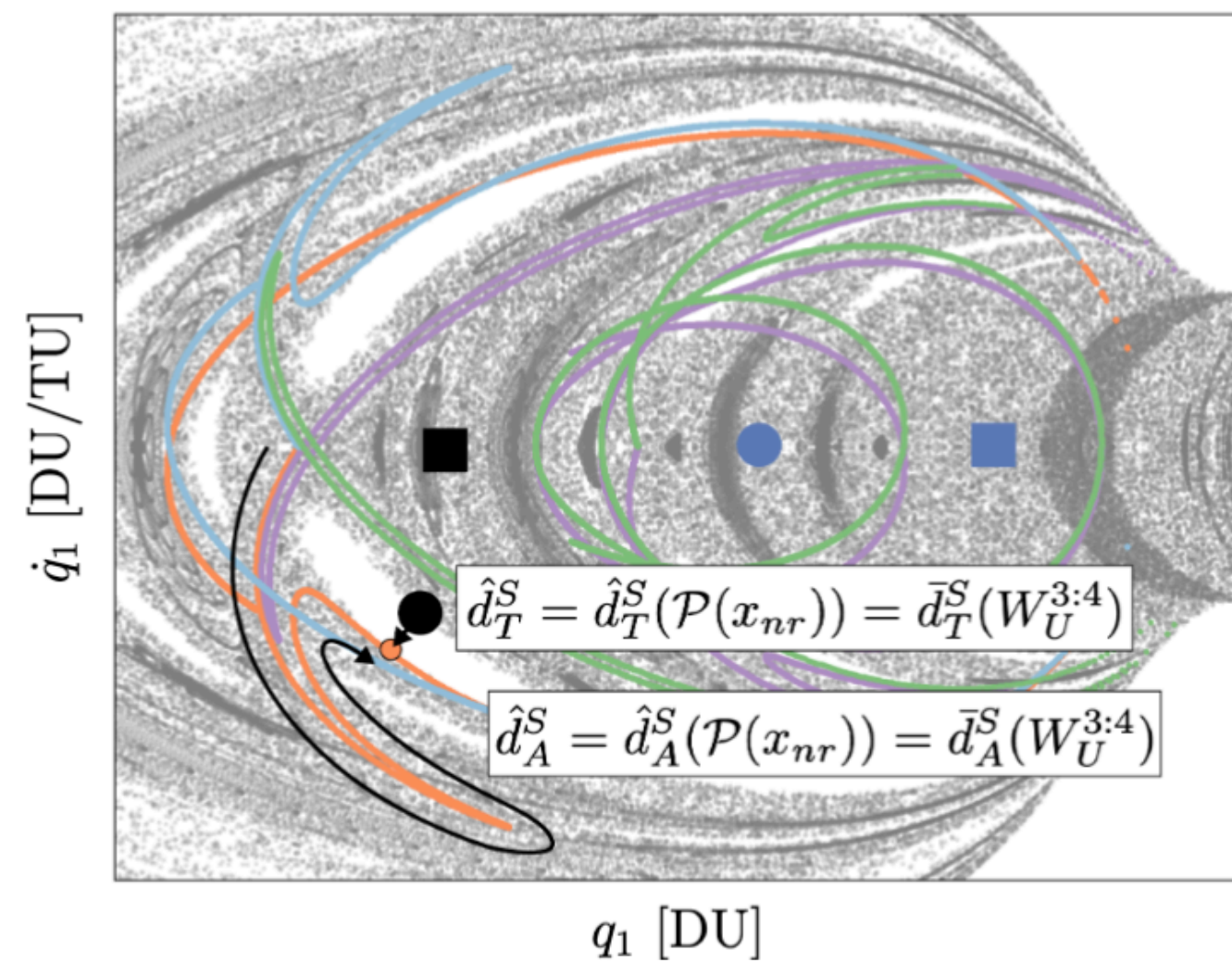
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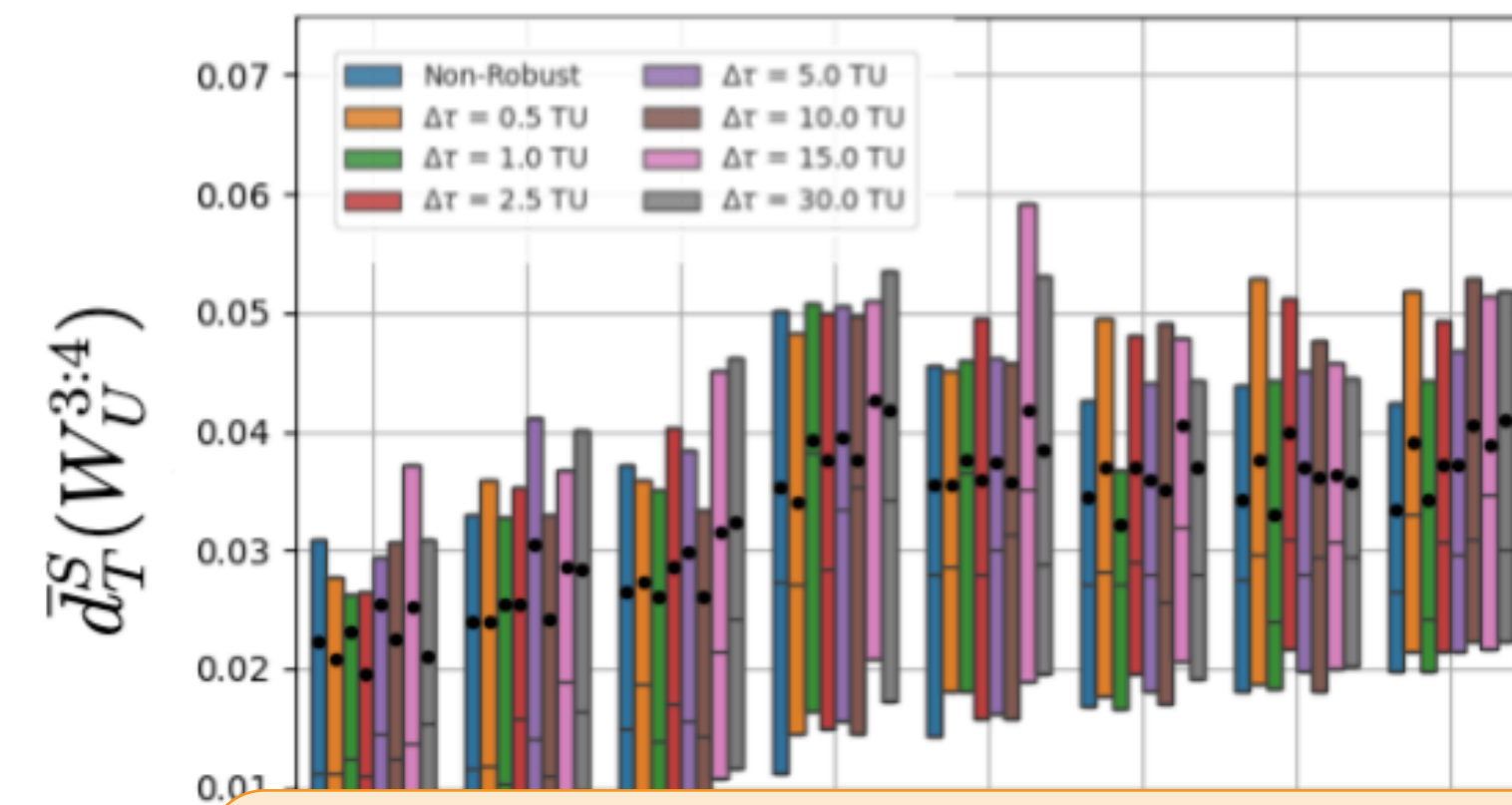
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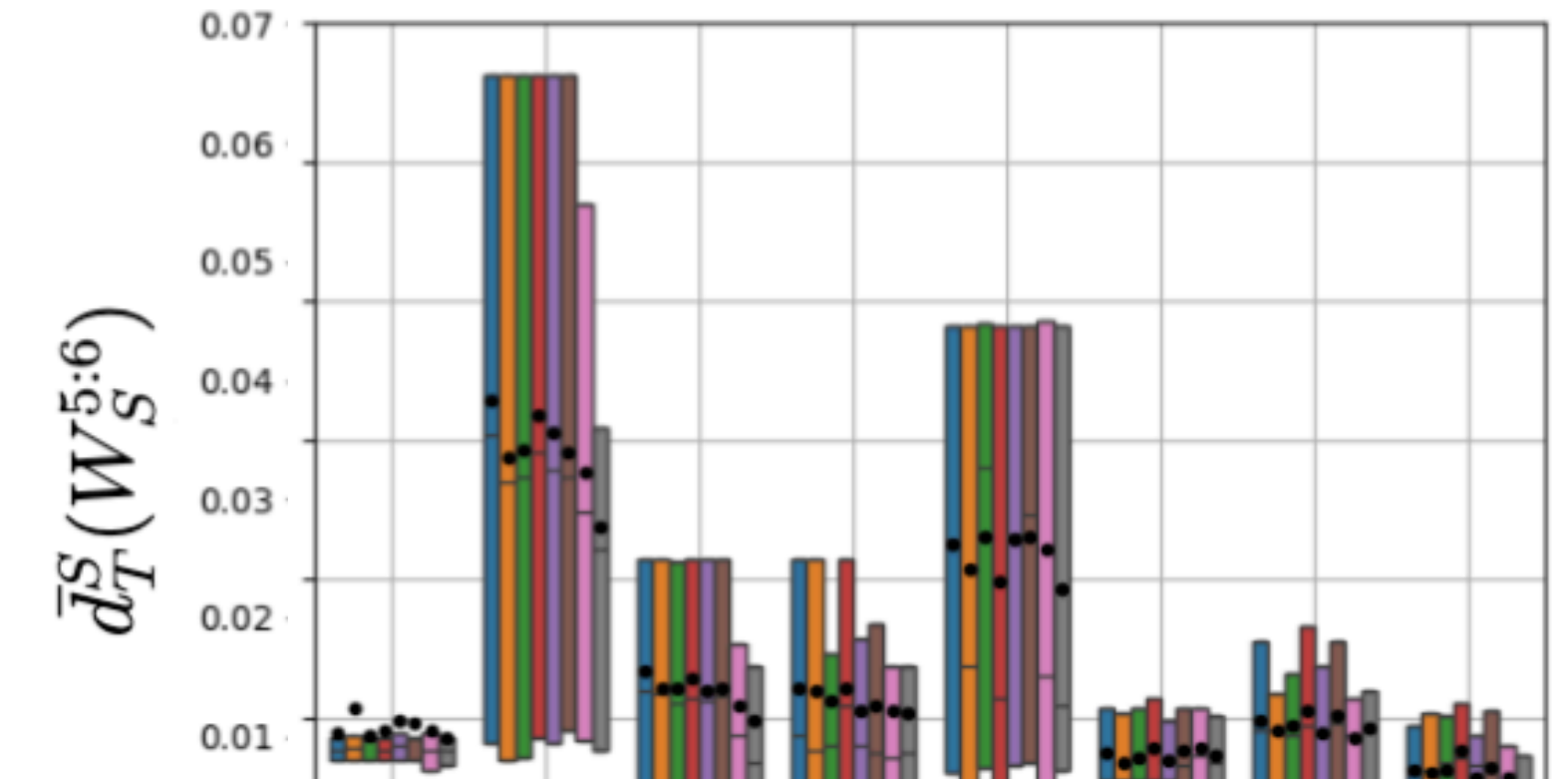
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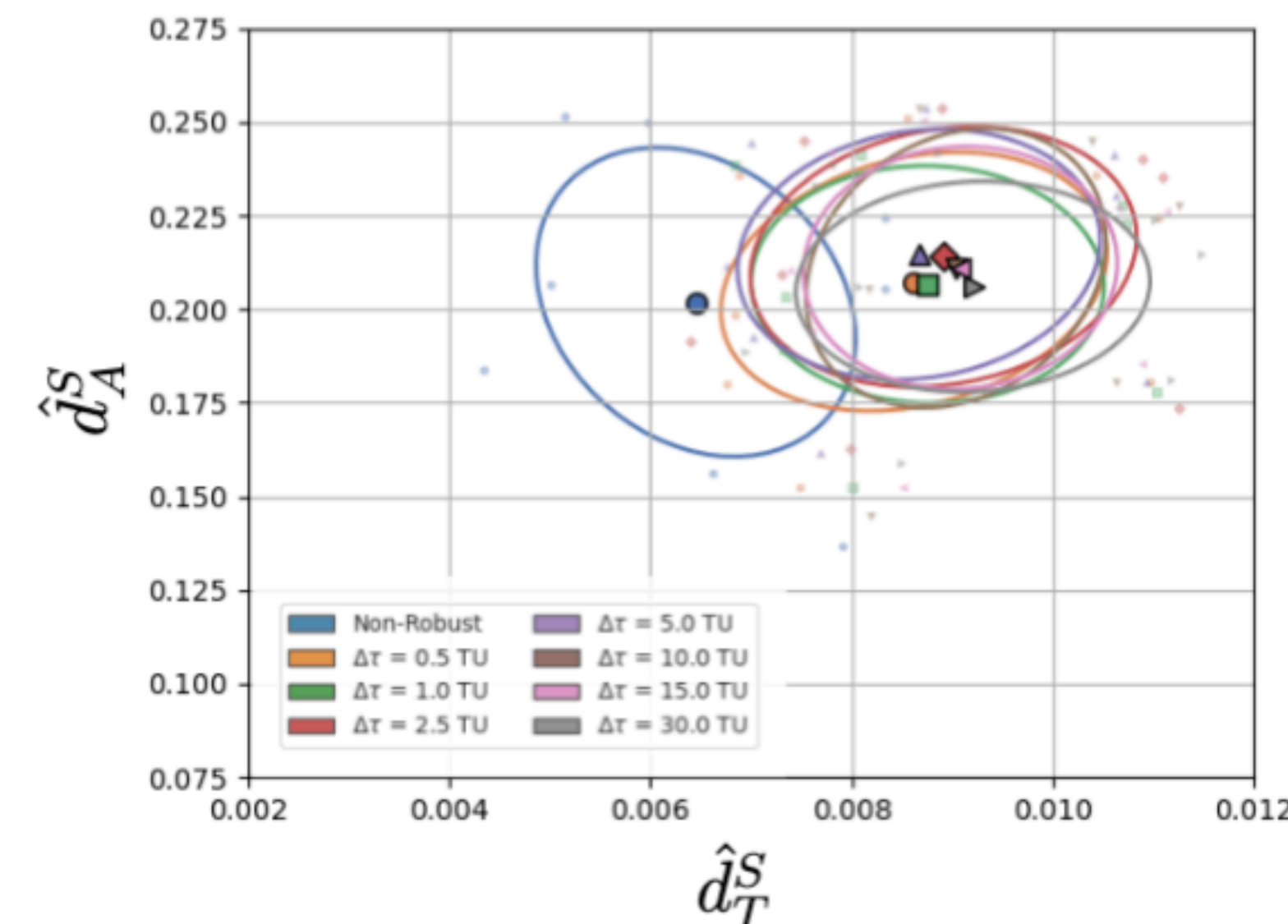
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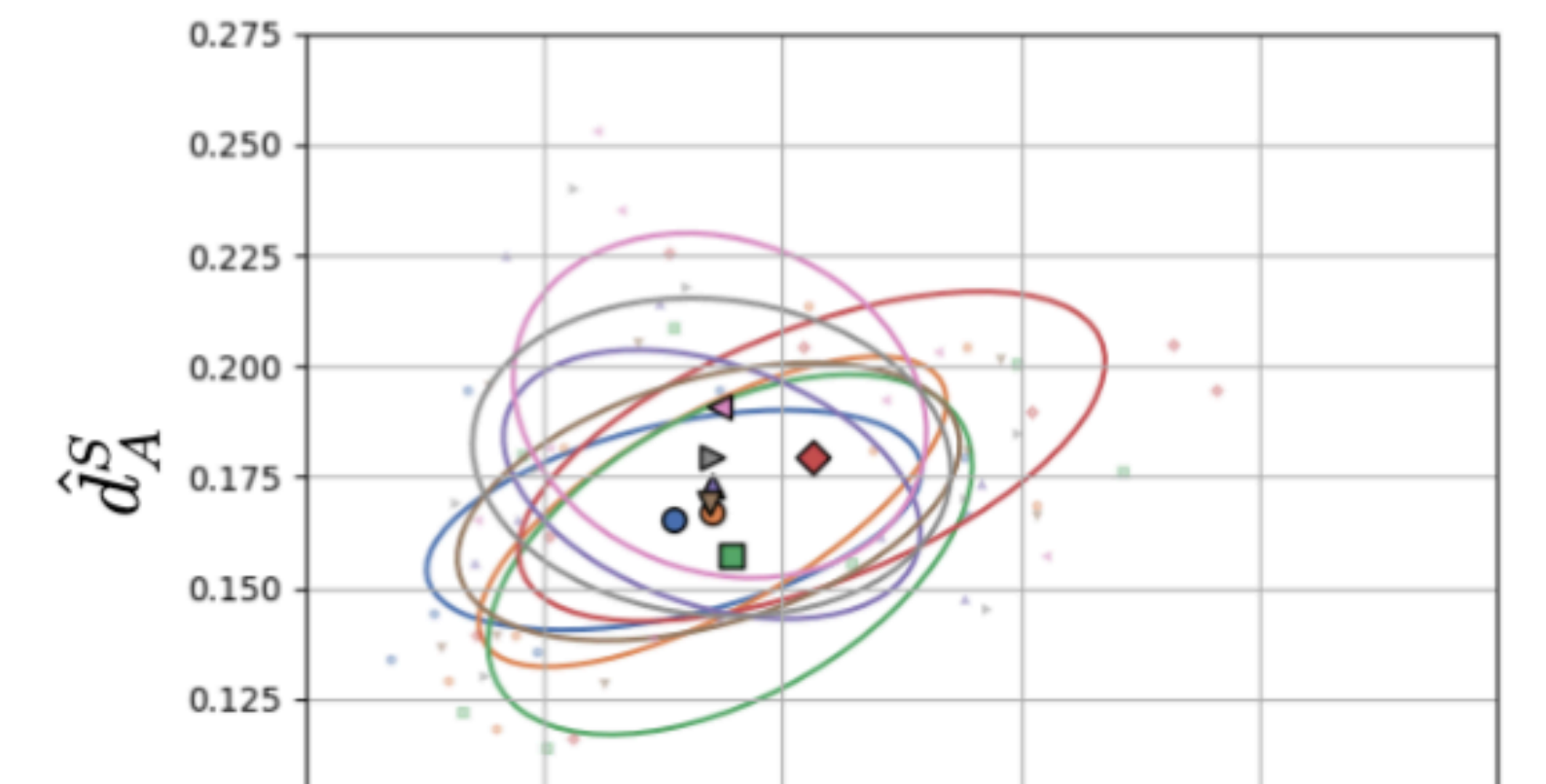
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Aggregate statistical comparison across all energy levels

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Optimal



Optimality can only be achieved by being in close proximity to the invariant manifolds

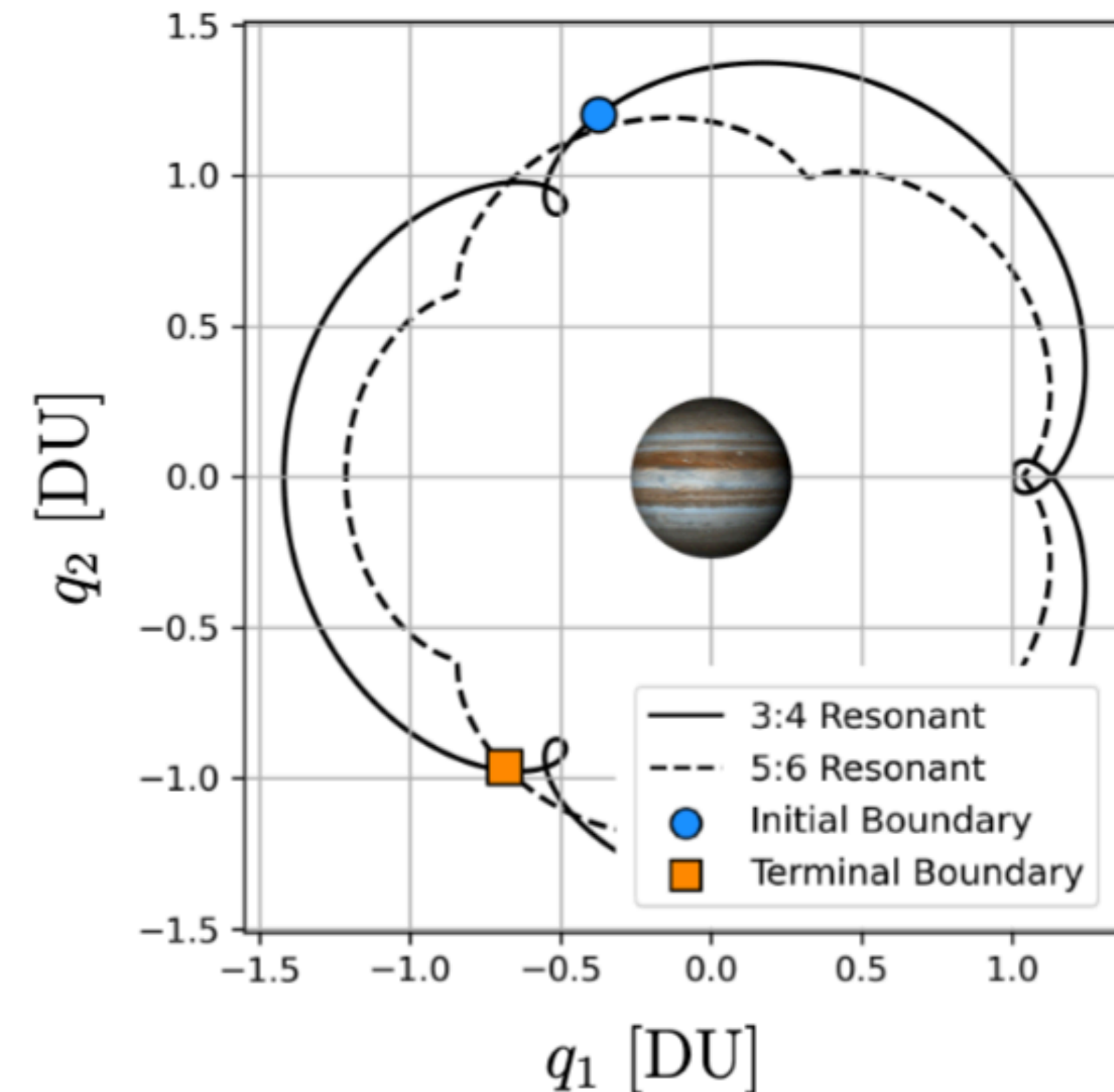


Part 2.2: Using Normal Forms For Understanding The Role Of Dynamical Structures On Missed Thrust Design

Normal forms reveal local geometry near periodic orbits

Robust solutions exhibit systematic relationships with invariant manifolds

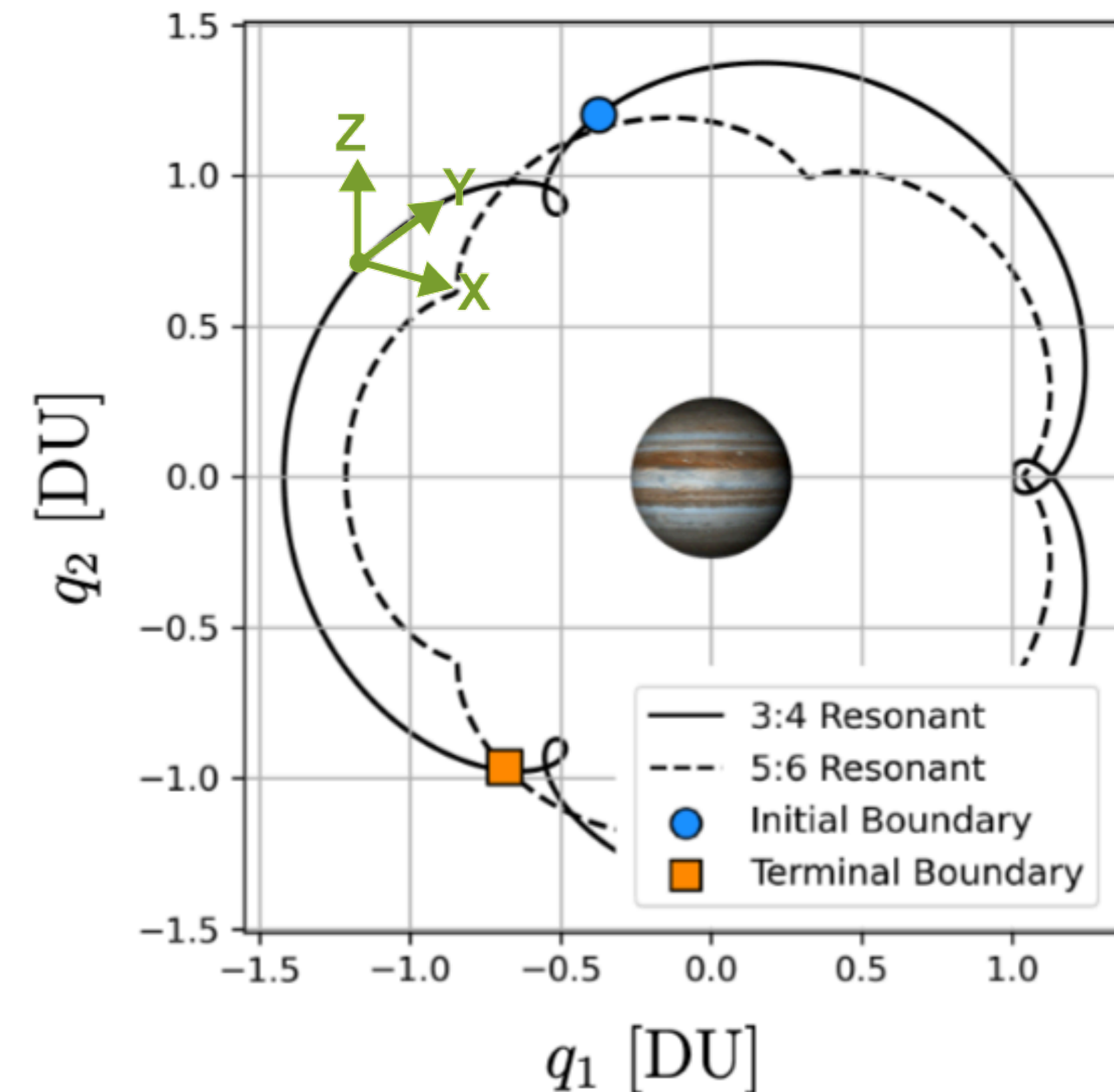
- We construct a **local chart on a periodic orbit** to enable a **direct comparison** between LT solutions and invariant manifolds
- At a high level, normal form construction involves
 - Near identity canonical transformation
 - A nonlinear coordinate change which separates motion along dynamically dominant directions



Normal forms reveal local geometry near periodic orbits

Robust solutions exhibit systematic relationships with invariant manifolds

- We construct a **local chart on a periodic orbit** to enable a **direct comparison** between LT solutions and invariant manifolds
- At a high level, normal form construction involves
 - Near identity canonical transformation
 - A nonlinear coordinate change which separates motion along dynamically dominant directions



The local normal form is constructed by

1. Coordinate translation,
2. Symplectic Floquet reduction,
3. Taylor-Fourier expansion, and
4. Degree-by-degree normalization

0 Original Hamiltonian Flow

$$\dot{\xi} = J\nabla H(\xi)$$

$$\dot{\xi}_{\gamma} = J\nabla H(\xi_{\gamma})$$

$$\gamma(t+T) = \gamma(t)$$

J symplectic

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1 Coordinate Translation

$$\xi' := \xi - \xi_\gamma$$

$$H'(t, \xi') := H(\xi_\gamma(t) + \xi') - \nabla H(\xi_\gamma(t))^\top \xi' - H(\xi_\gamma(t))$$

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2 Symplectic Floquet Reduction

$$\xi' = P(t)\eta \quad \Phi(t) = P(t)e^{tB}$$

$$\xi'(t) = \Phi(t)\xi'(0)$$

$$\dot{\Phi}(t) = A(t)\Phi(t), \quad \Phi(0) = I$$

$$A(t) = J\nabla^2 H(\xi_\gamma(t))$$

$$K(t, \eta) = H_2(\eta) + \sum_{d \geq 3} H_d^{(\eta)}(t, \eta),$$

$$H_2(\eta) = \frac{1}{2}\eta^\top (-JB)\eta$$

$$H_d^{(\eta)}(t, \eta) \text{ homogenous polynomial of degree } d \text{ in } \eta$$

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3 Taylor-Fourier Expansion

$$H_d^{(\eta)}(t, \eta) = \sum_{k \in \mathbb{Z}} H_{d,k}(\eta) e^{ik\omega t}, \quad \omega = 2\pi/T$$

2 Symplectic Floquet Reduction

$$\begin{aligned}\xi' &= P(t)\eta \quad \Phi(t) = P(t)e^{tB} \\ \xi'(t) &= \Phi(t)\xi'(0) \\ \dot{\Phi}(t) &= A(t)\Phi(t), \quad \Phi(0) = I \\ A(t) &= J\nabla^2 H(\xi_\gamma(t)) \\ K(t, \eta) &= H_2(\eta) + \sum_{d \geq 3} H_d^{(\eta)}(t, \eta), \\ H_2(\eta) &= \frac{1}{2}\eta^\top (-JB)\eta \\ H_d^{(\eta)}(t, \eta) &\text{ homogenous polynomial of degree } d \text{ in } \eta\end{aligned}$$

Jorba et al. (2020)

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4 Degree-by-degree normalization

At degree d

$$K^{\text{cur}}(t, \eta) = H_2(\eta) + \sum_{m=3}^{d-1} Z_m(t, \eta) + H_d^{\text{cur}}(t, \eta) + O(\|\eta\|^{d+1})$$

Pick generating function

$$\mathcal{L}_d W_d + Z_d = H_d^{\text{cur}}, \quad \mathcal{L}_d W_d := \partial_t W_d + \{W, H_2\}$$

Then, the transformed Hamiltonian becomes

$$K^{\text{new}}(t, \eta) = H_2(\eta) + \sum_{m=3}^d Z_m(t, \eta) + O(\|\eta\|^{d+1})$$

Jorba et al. (2020)

The local normal form is constructed by

1. Coordinate translation,
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4. Degree-by-degree normalization

At leading order, the chart separates unstable, stable, oscillatory and neutral directions

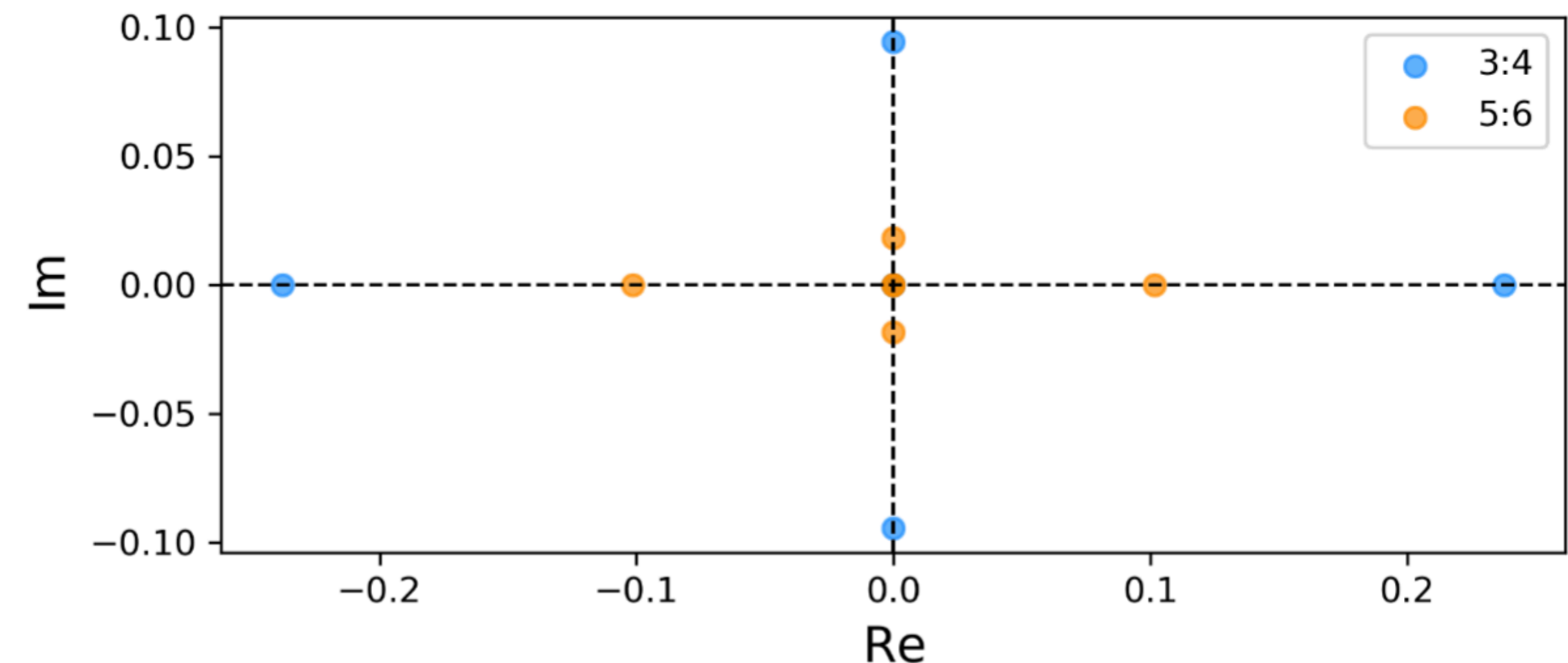
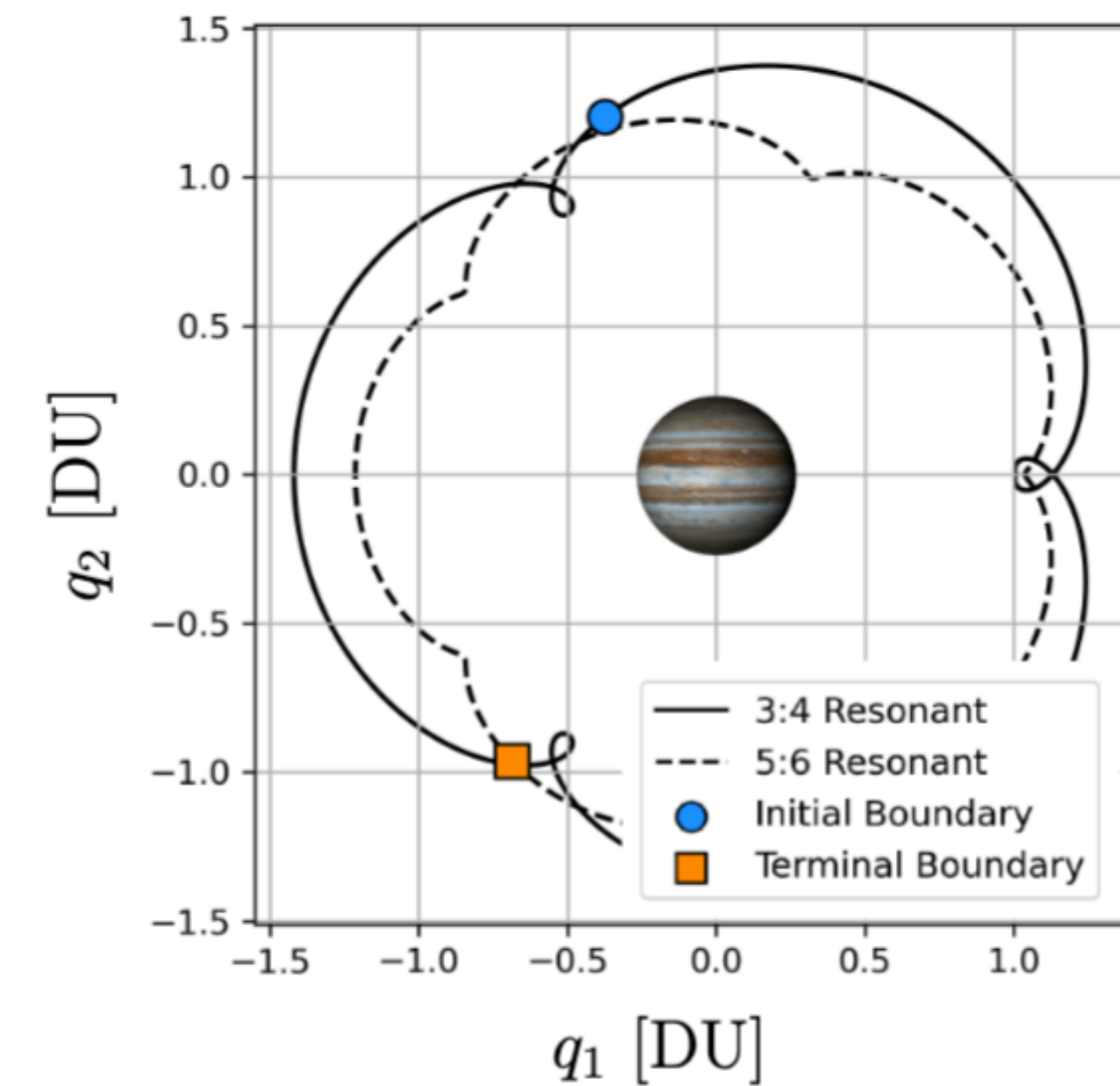
The quadratic Hamiltonian can be written in a suitable basis which reveals dominant local structure

- For the **saddle x center x neutral** case, the quadratic Hamiltonian takes the form

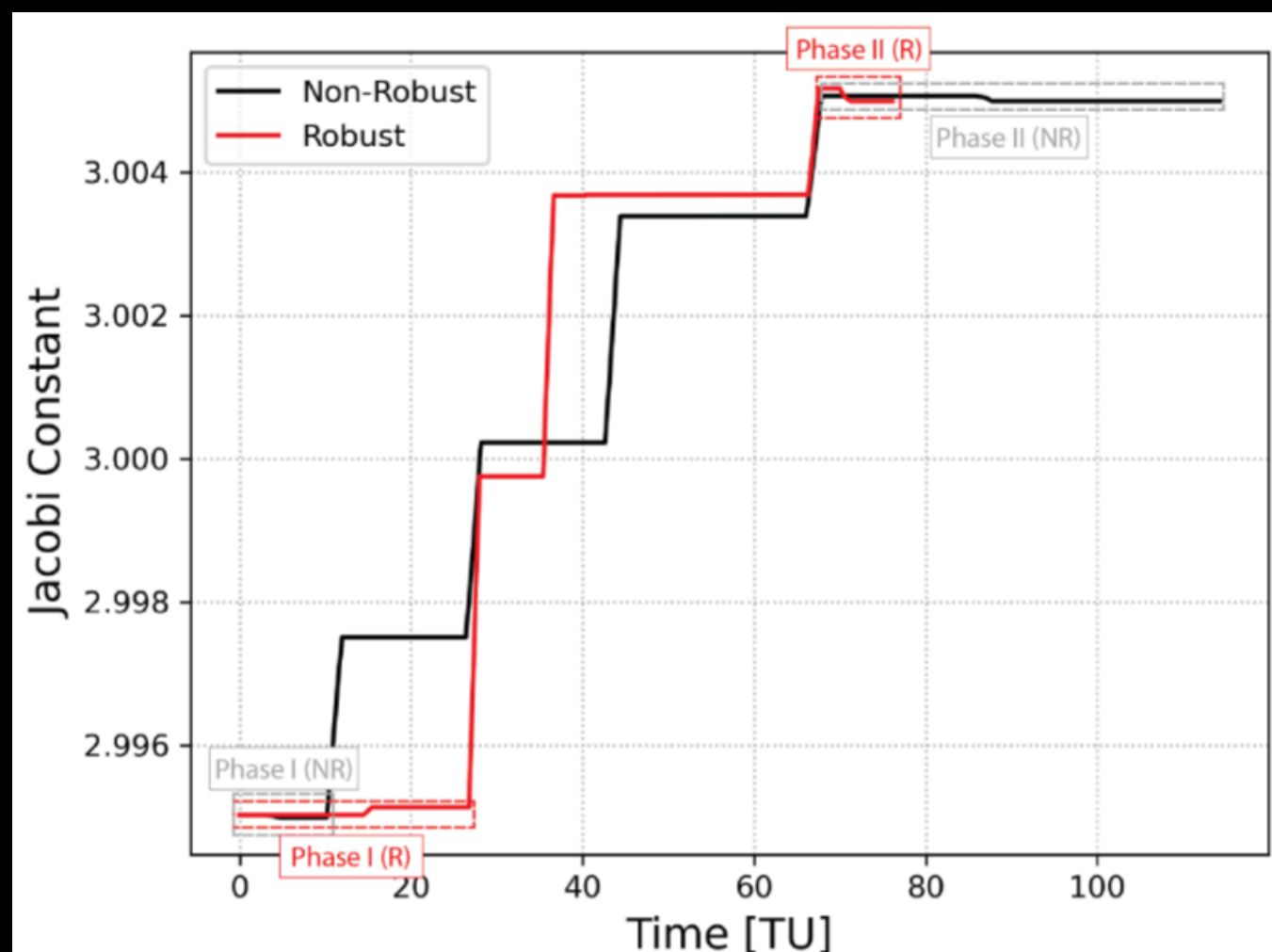
$$H_2 = \lambda q_h p_h + \frac{\nu}{2}(Q^2 + P^2)$$

after a symplectic coordinate change $\eta = Mu$, where M is a constant symplectic basis change adapted to the spectral splitting of B

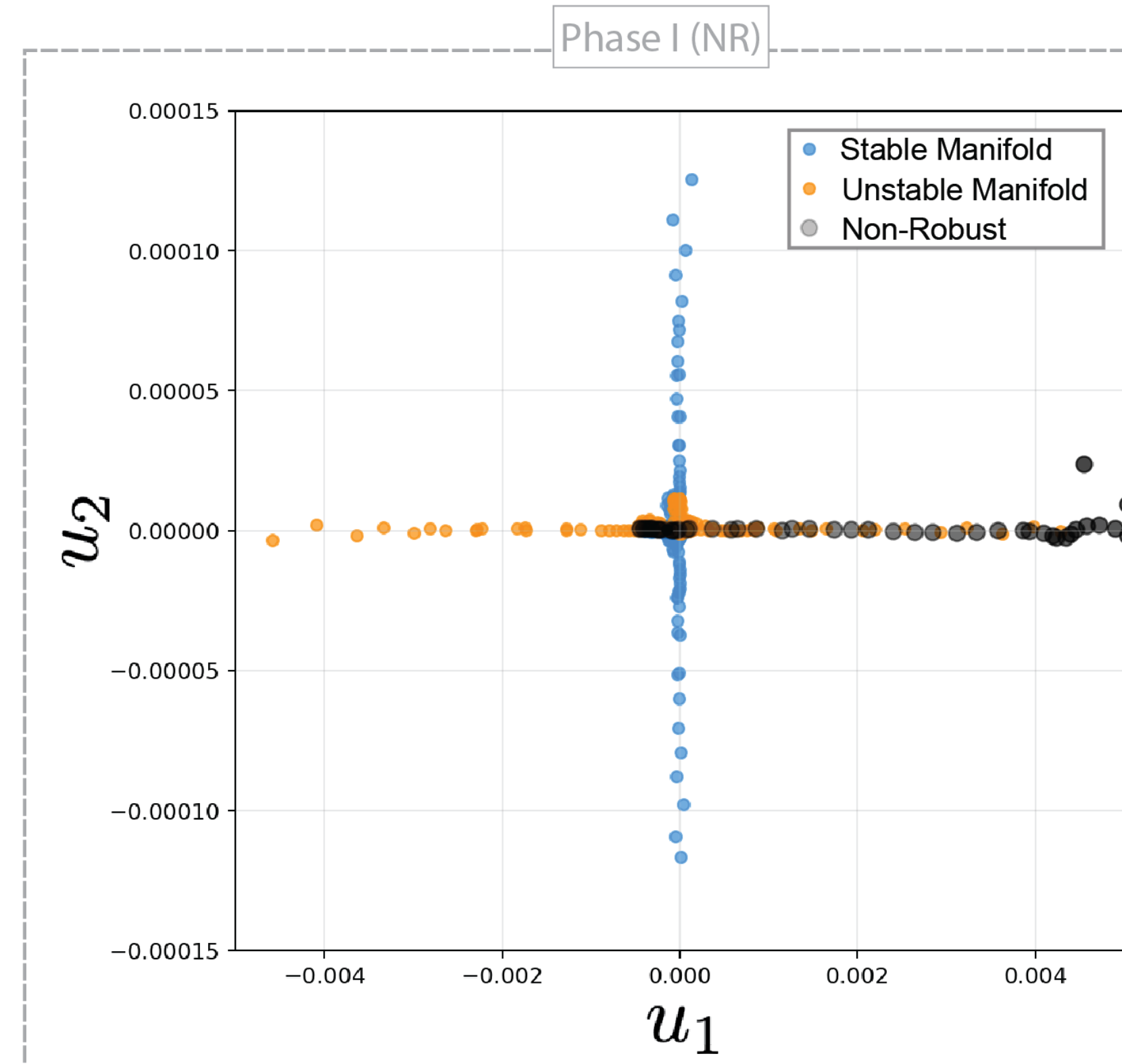
- Generating functions** W_d defines the coordinate change
- $d_{s(t)} = |q_{h(t)}|$ and $d_{u(t)} = |p_{h(t)}|$ describes the distance from the stable and unstable manifolds



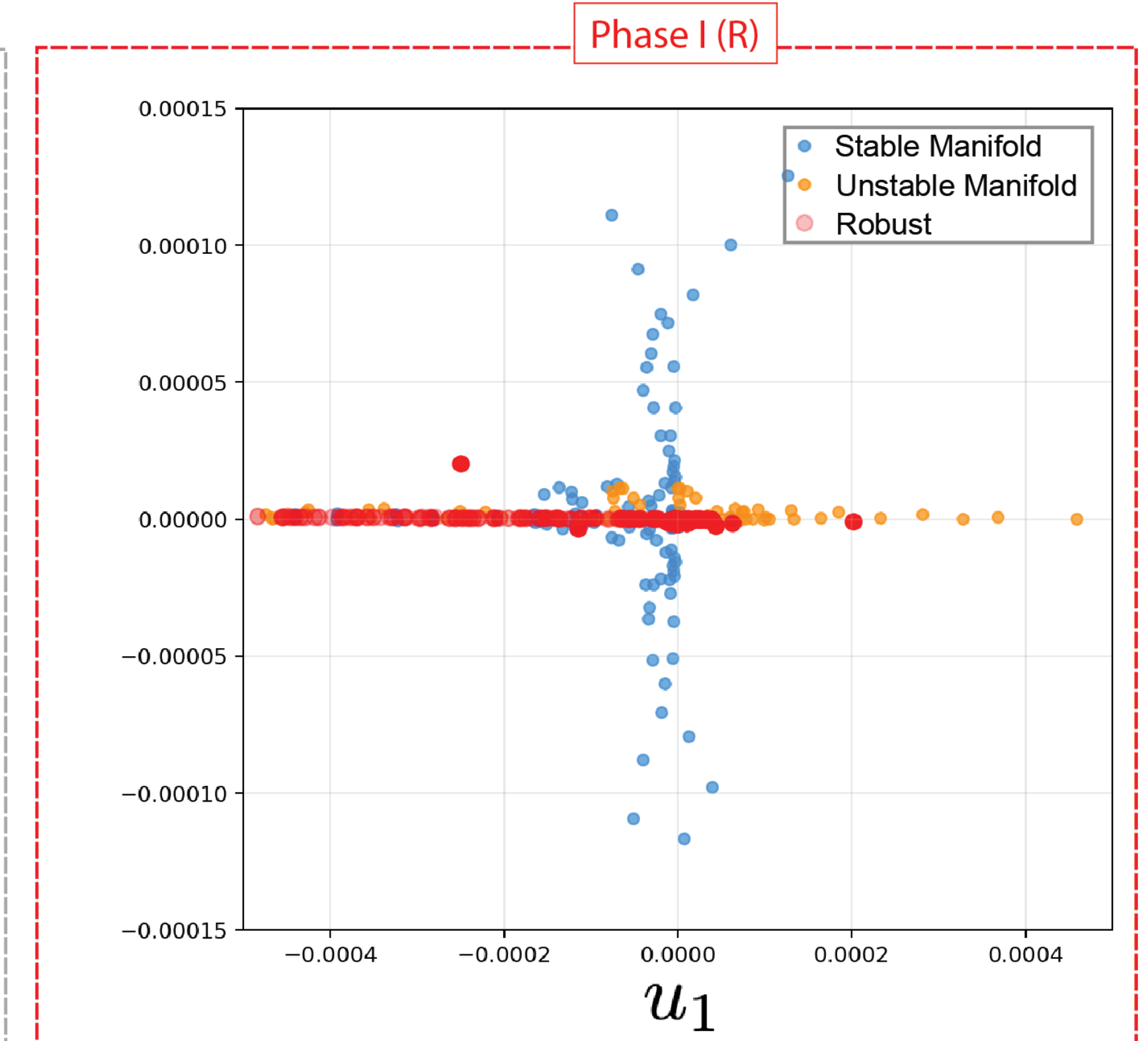
The robust low-thrust solutions leverage the invariant manifolds closely, just like the robust solutions.



Non-Robust



Robust



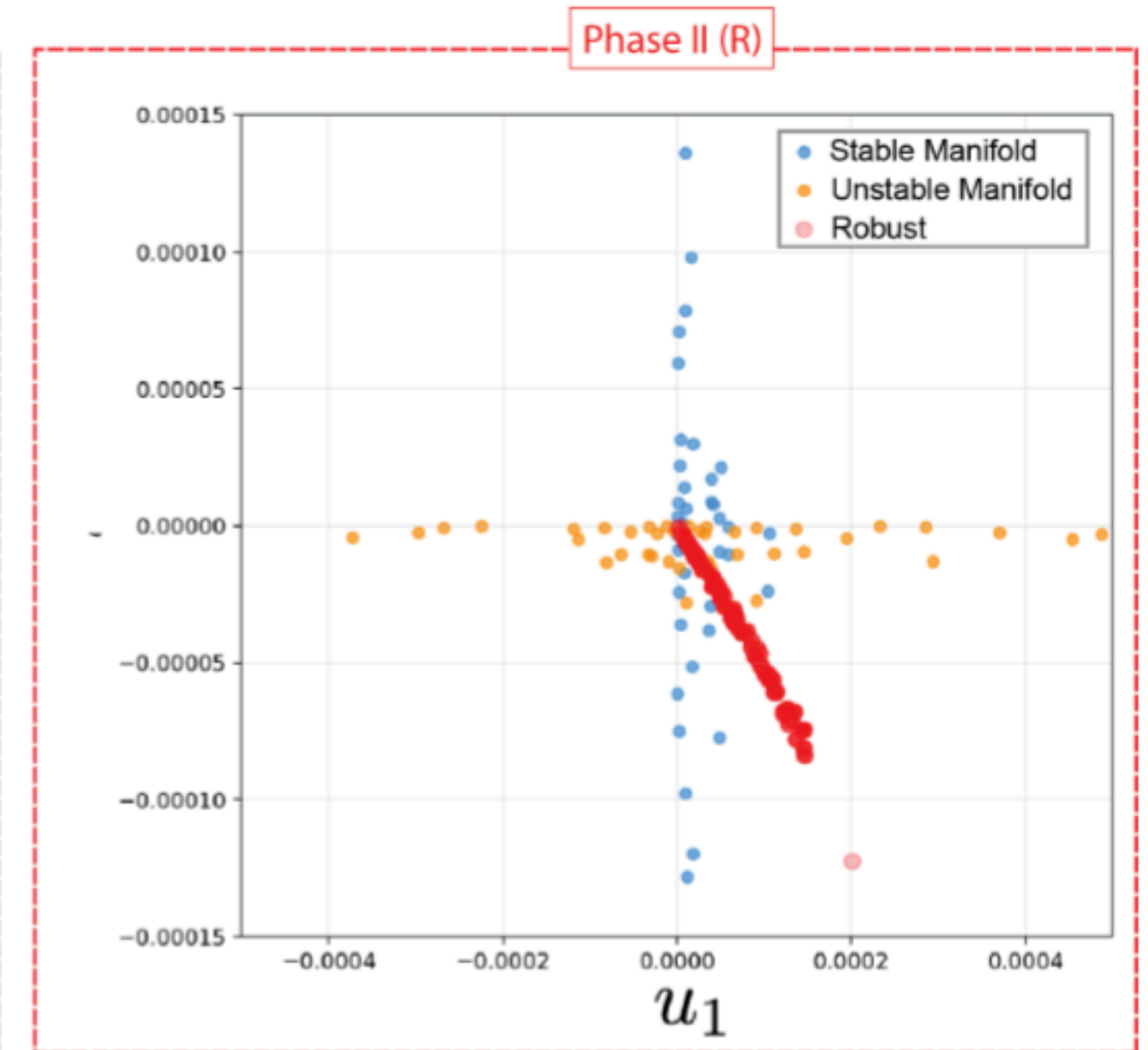
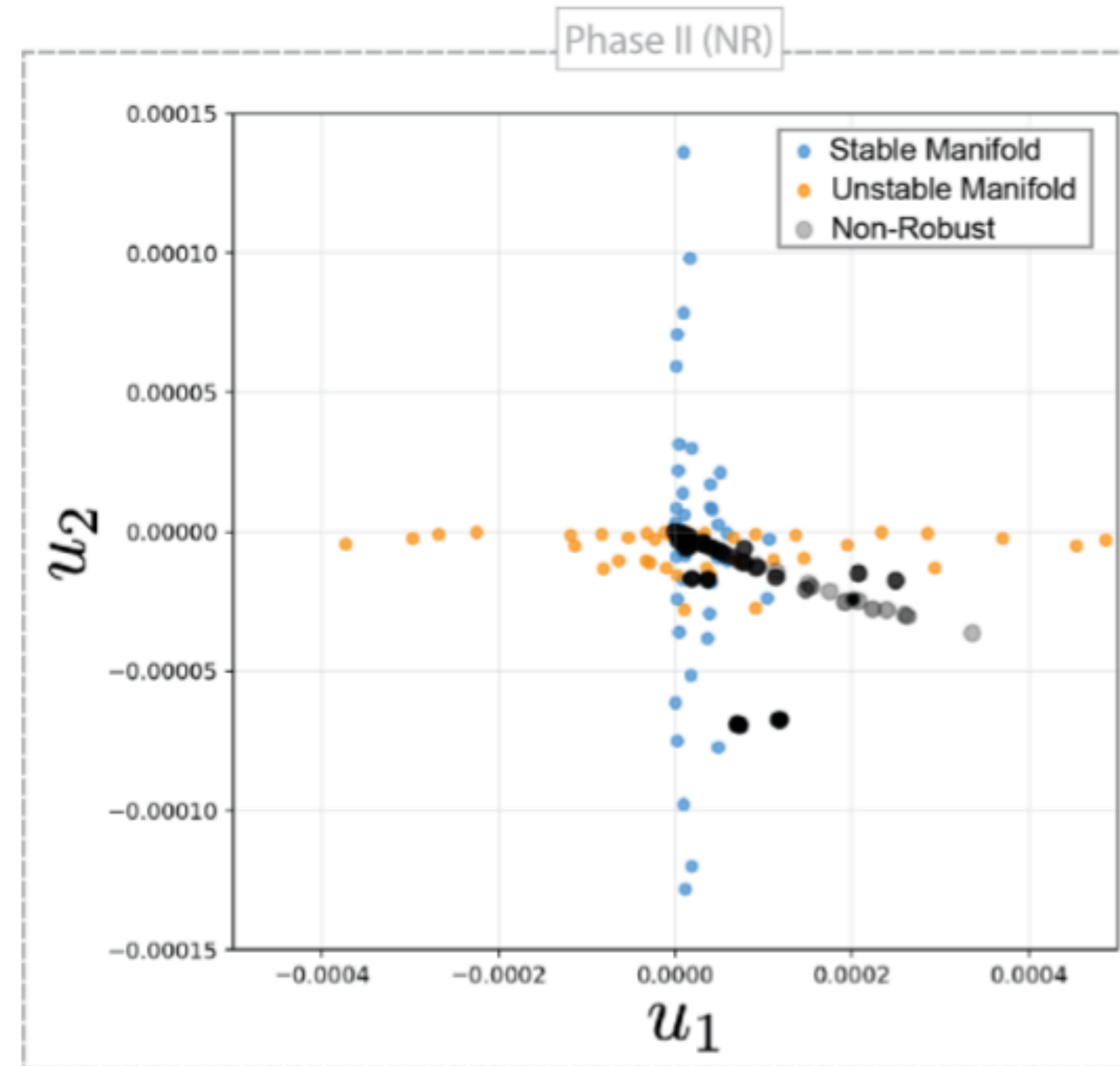
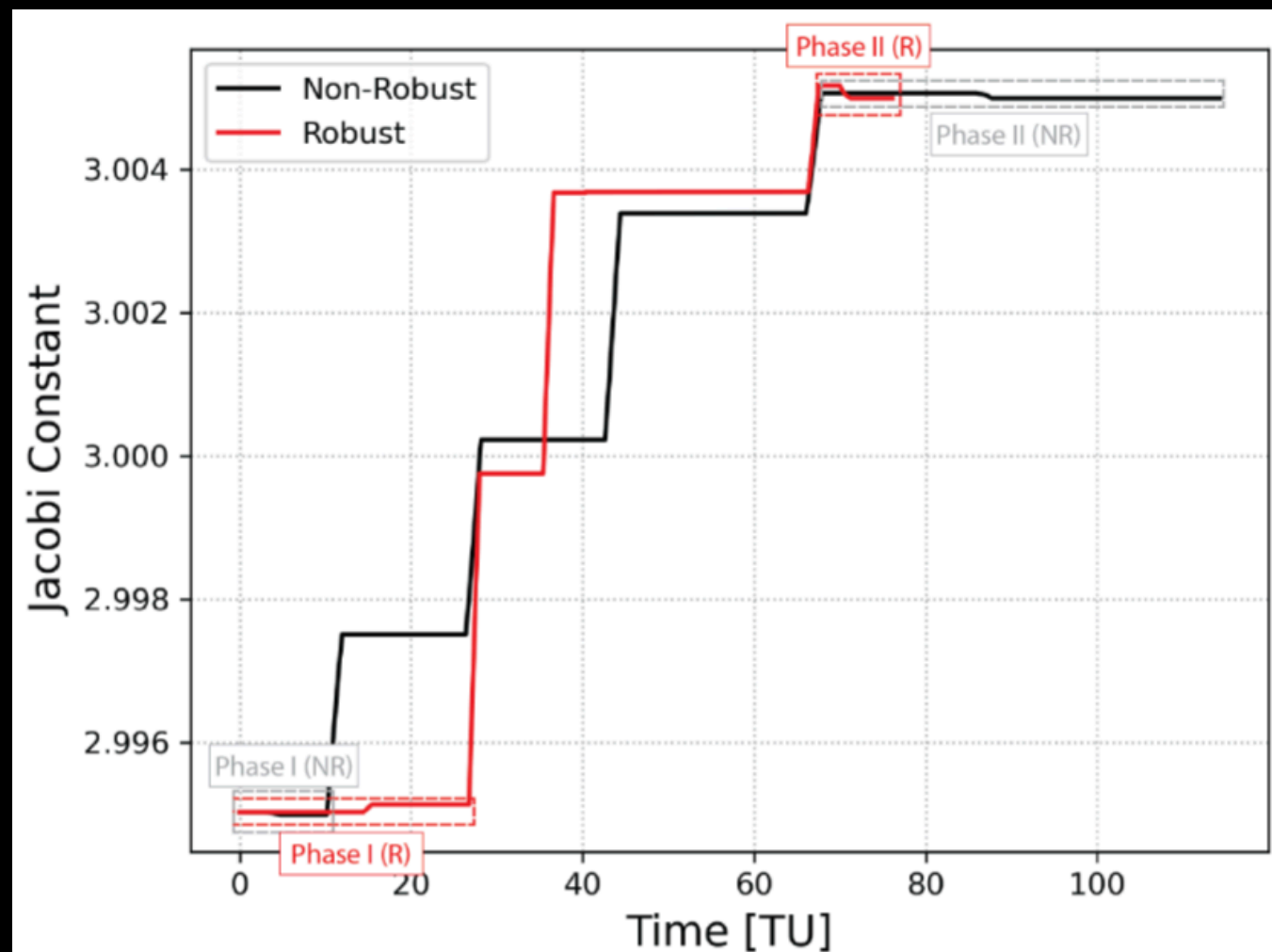
$$u = (q_h, p_h, Q, P, \sigma, \rho)$$

$$\xi \rightleftharpoons \xi' \rightleftharpoons \eta \rightleftharpoons u$$

The robust low-thrust solutions leverage the invariant manifolds closely, just like the robust solutions.

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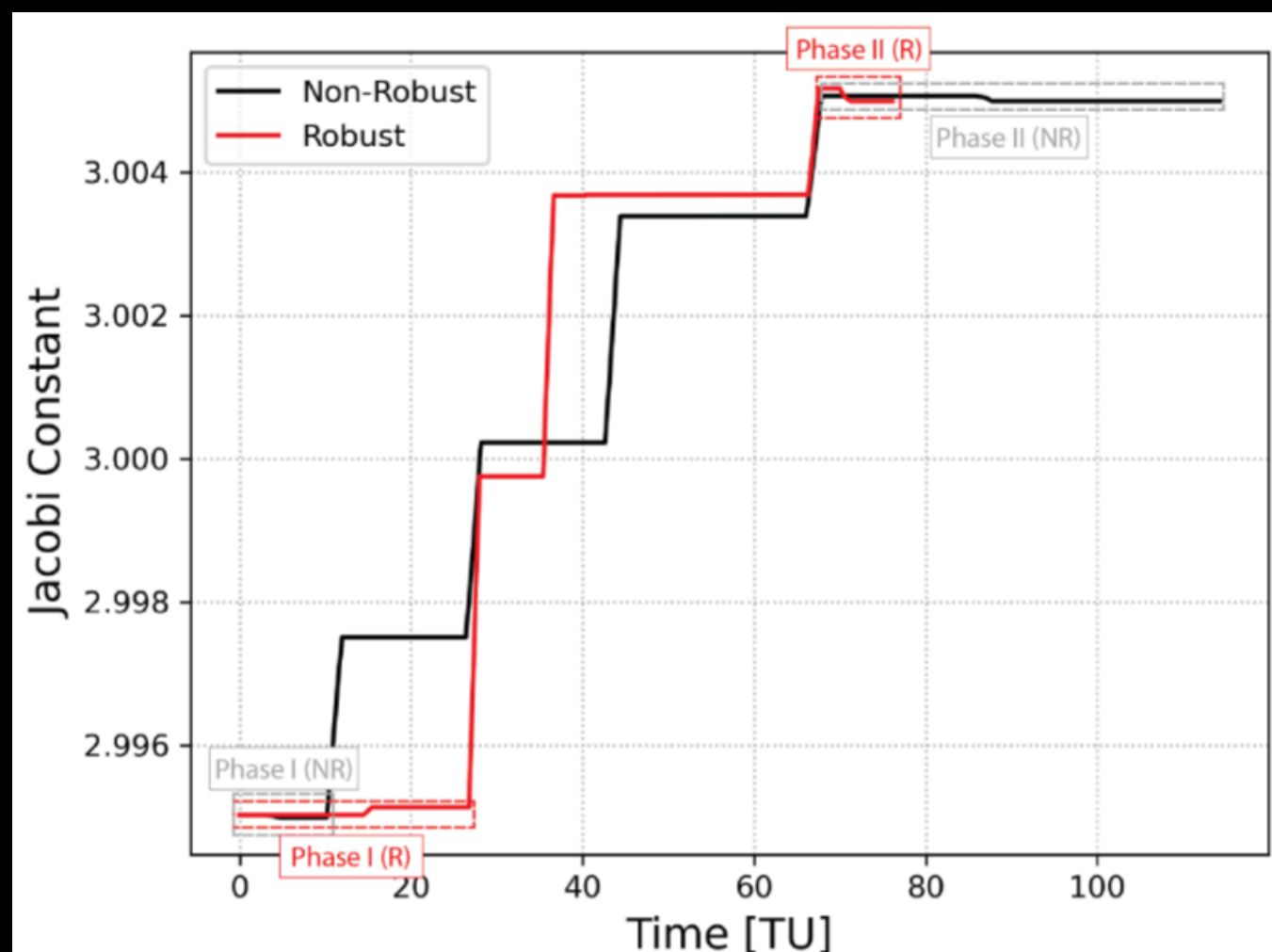
Robust



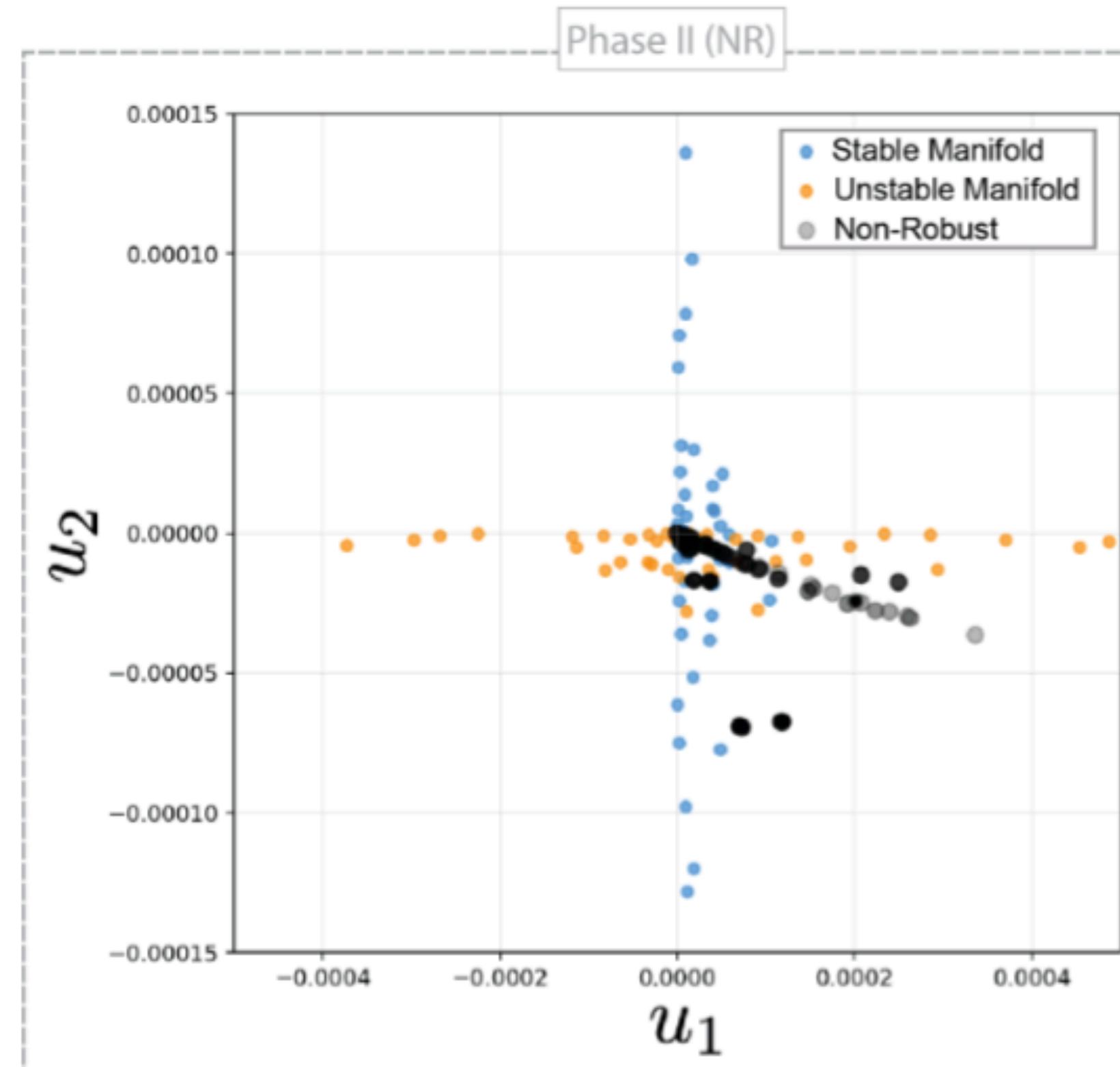
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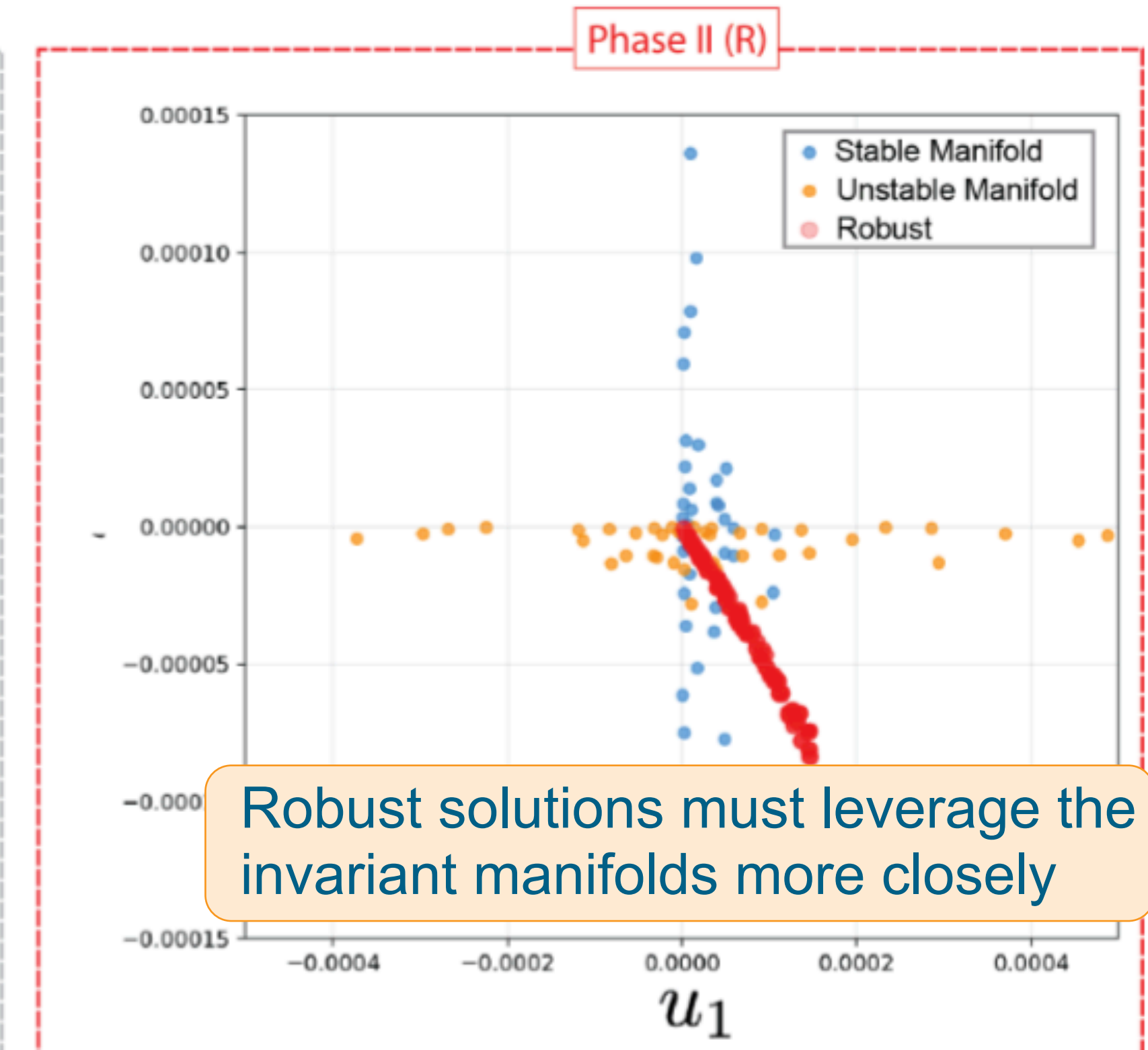
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Non-Robust



Robust



Robust solutions must leverage the invariant manifolds more closely

$$u = (q_h, p_h, Q, P, \sigma, \rho)$$

$$\xi \rightleftharpoons \xi' \rightleftharpoons \eta \rightleftharpoons u$$

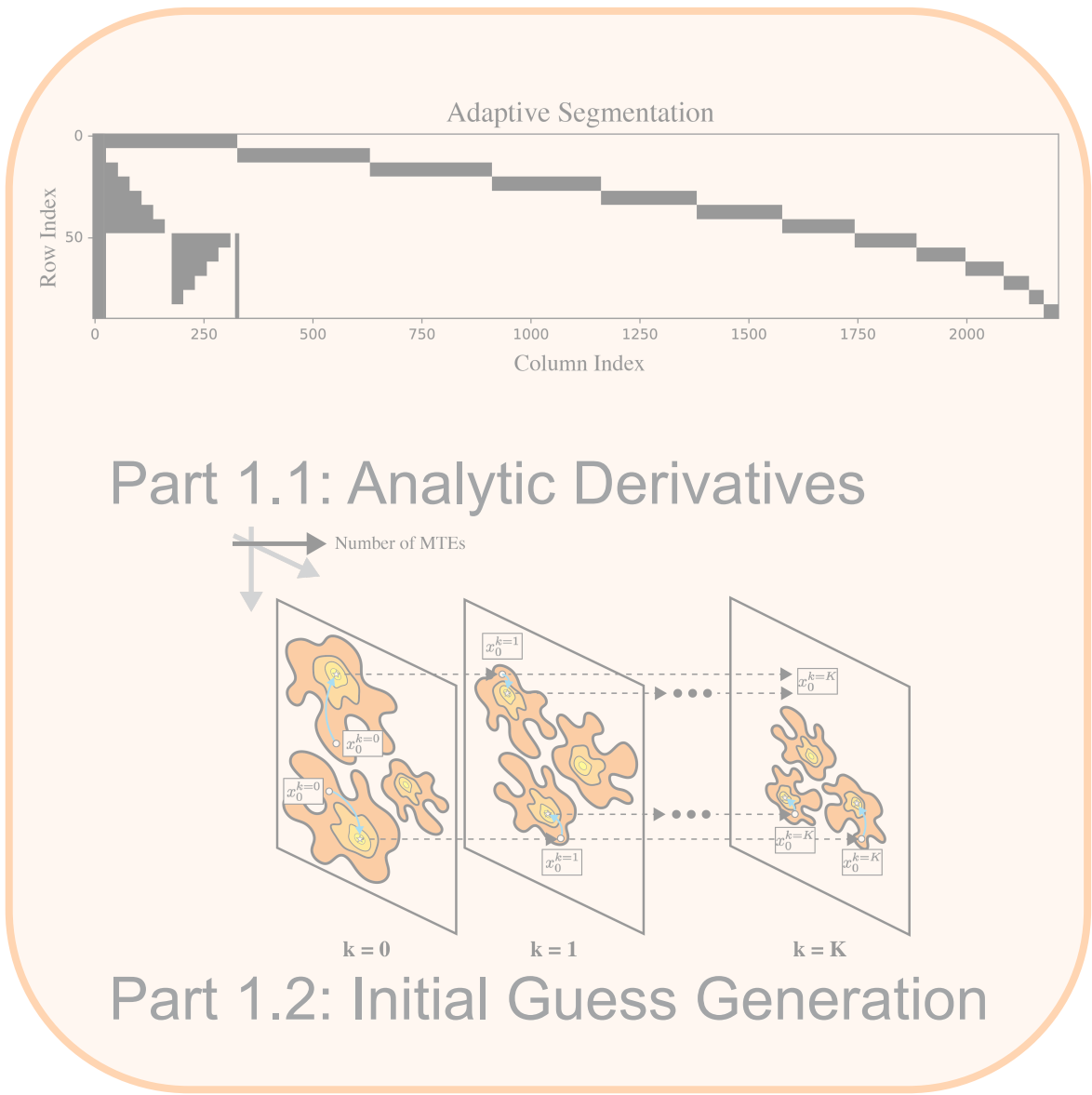


Part 3: A Bilevel Optimal Control Framework For Missed Thrust Design

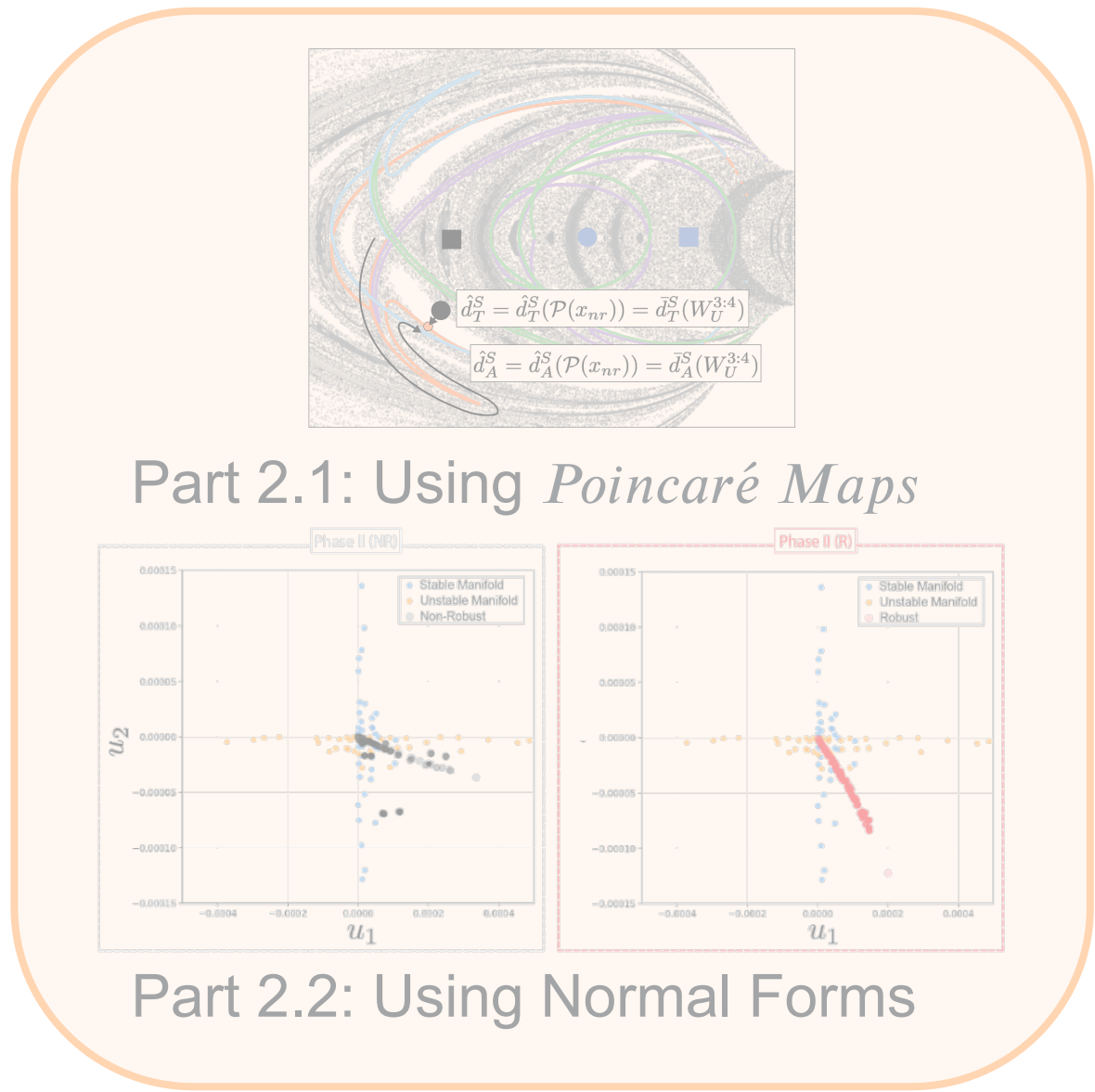
Amlan Sinha and Ryne Beeson, “A Bi-Level Optimal Control Framework For Low-Thrust Trajectory Design Under Missed-Thrust-Events”, Submitted to Journal of Guidance, Control, and Dynamics (in review).



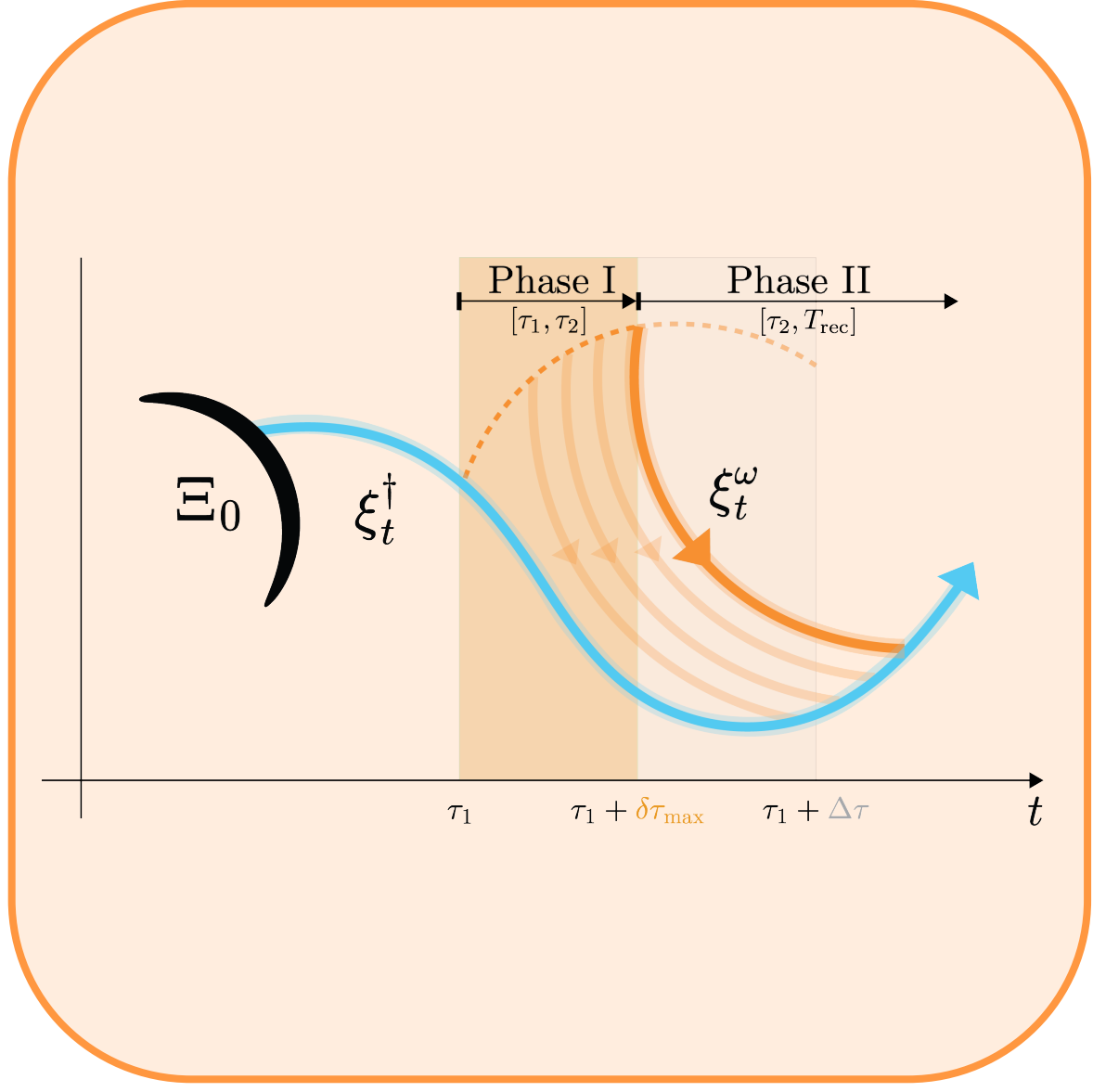
Part 1: Scalable Computational Methods



Part 2: Dynamical Systems Analysis



Part 3: Bilevel Optimal Control Framework

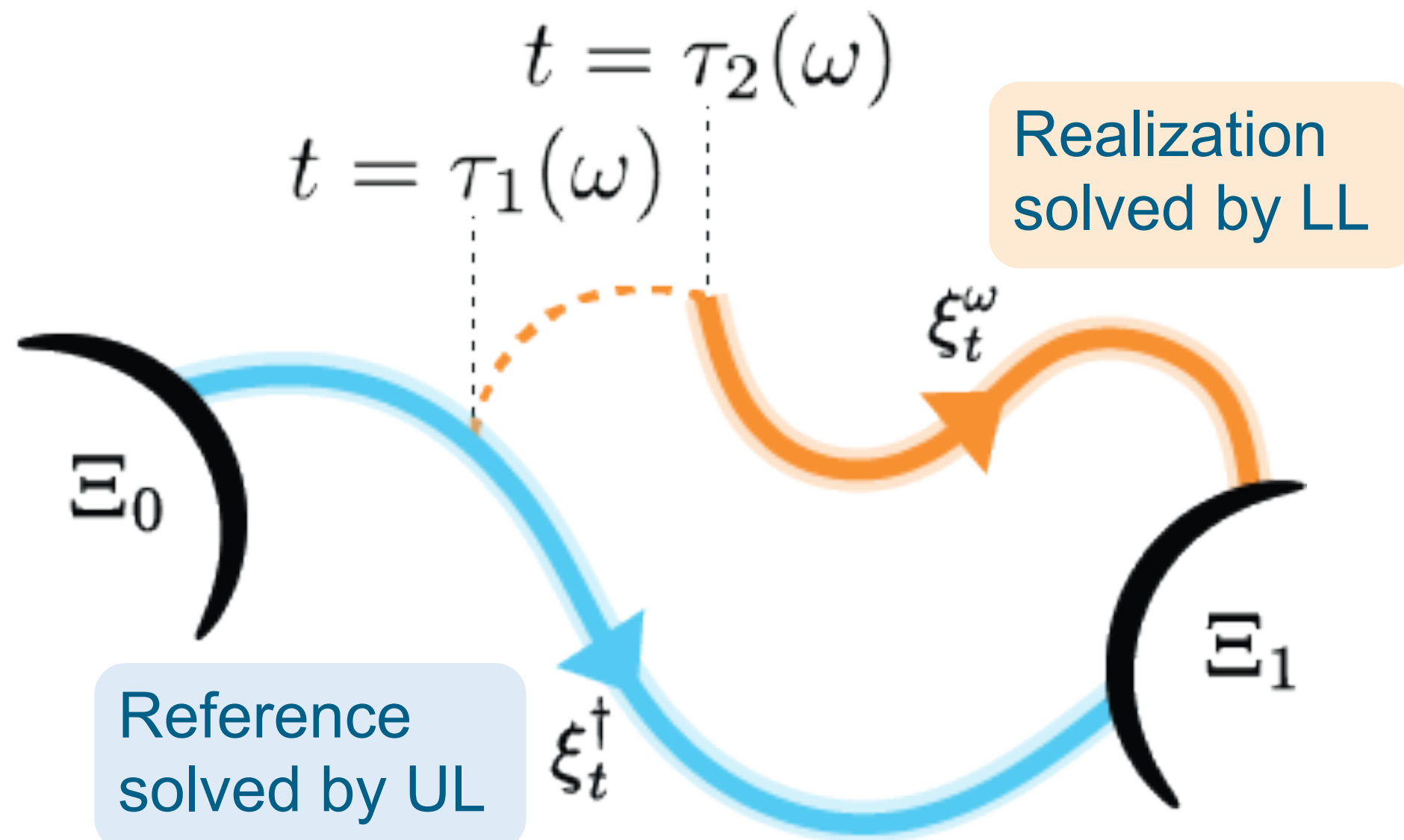
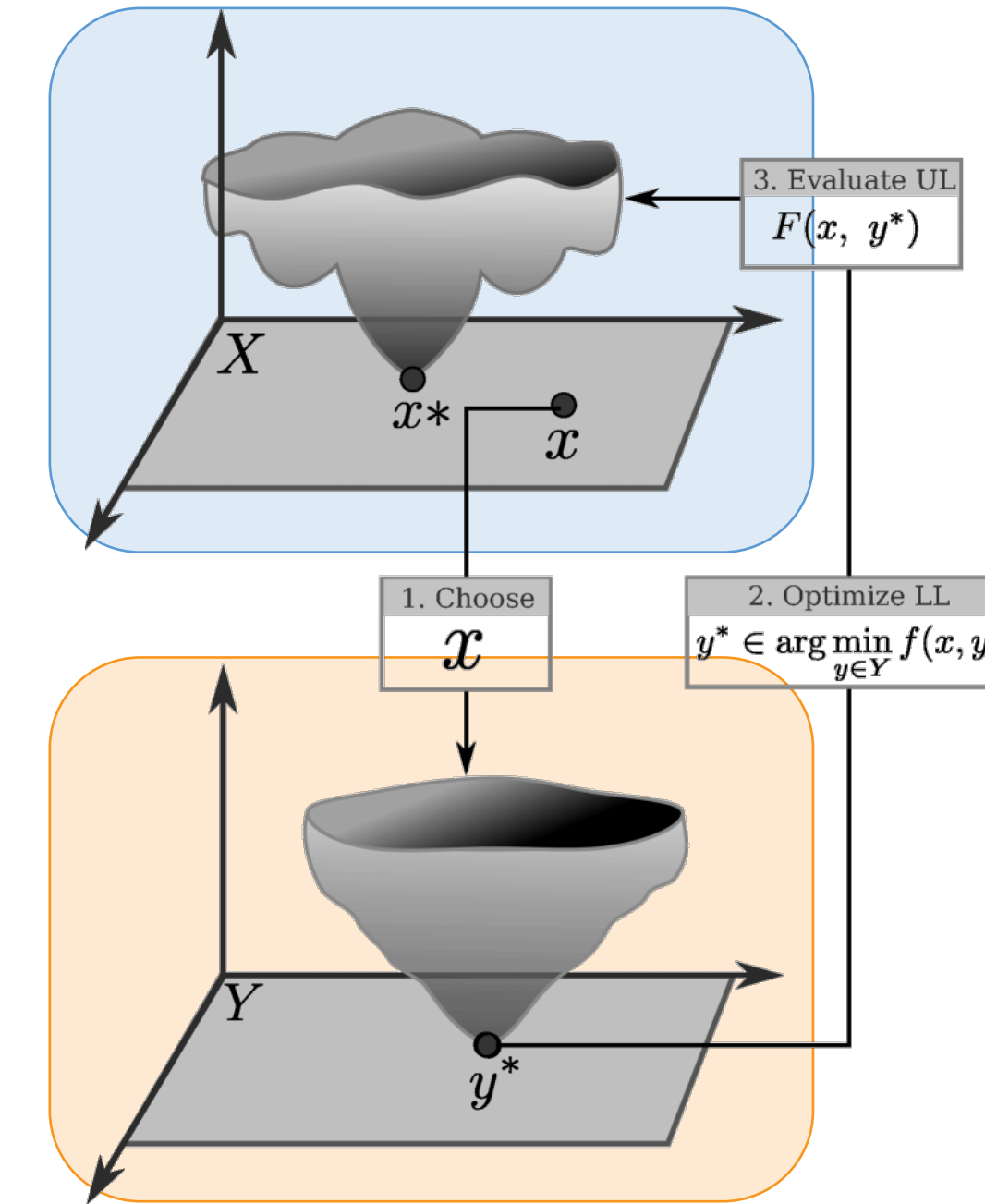


Part 0: Missed Thrust Design Problem Formulation

Bilevel optimal control is an alternative way to pose the missed thrust design problem

The missed thrust event naturally admits a bilevel optimal control formulation

The **upper level** chooses the nominal reference solution assuming full control authority, while the **lower level** solves the recovery problem



$$\begin{aligned} \min_{x \in \mathcal{X}, y \in \mathcal{Y}} \quad & \{ \mathcal{F}^{\text{UL}}(x, y) \} \\ \text{s.t.} \quad & \mathcal{G}_i^{\text{UL}}(x, y) \leq 0, \quad i = 1, \dots, I^{\text{UL}}, \\ & \mathcal{H}_j^{\text{UL}}(x, y) = 0, \quad j = 1, \dots, J^{\text{UL}}, \end{aligned}$$

Upper-Level

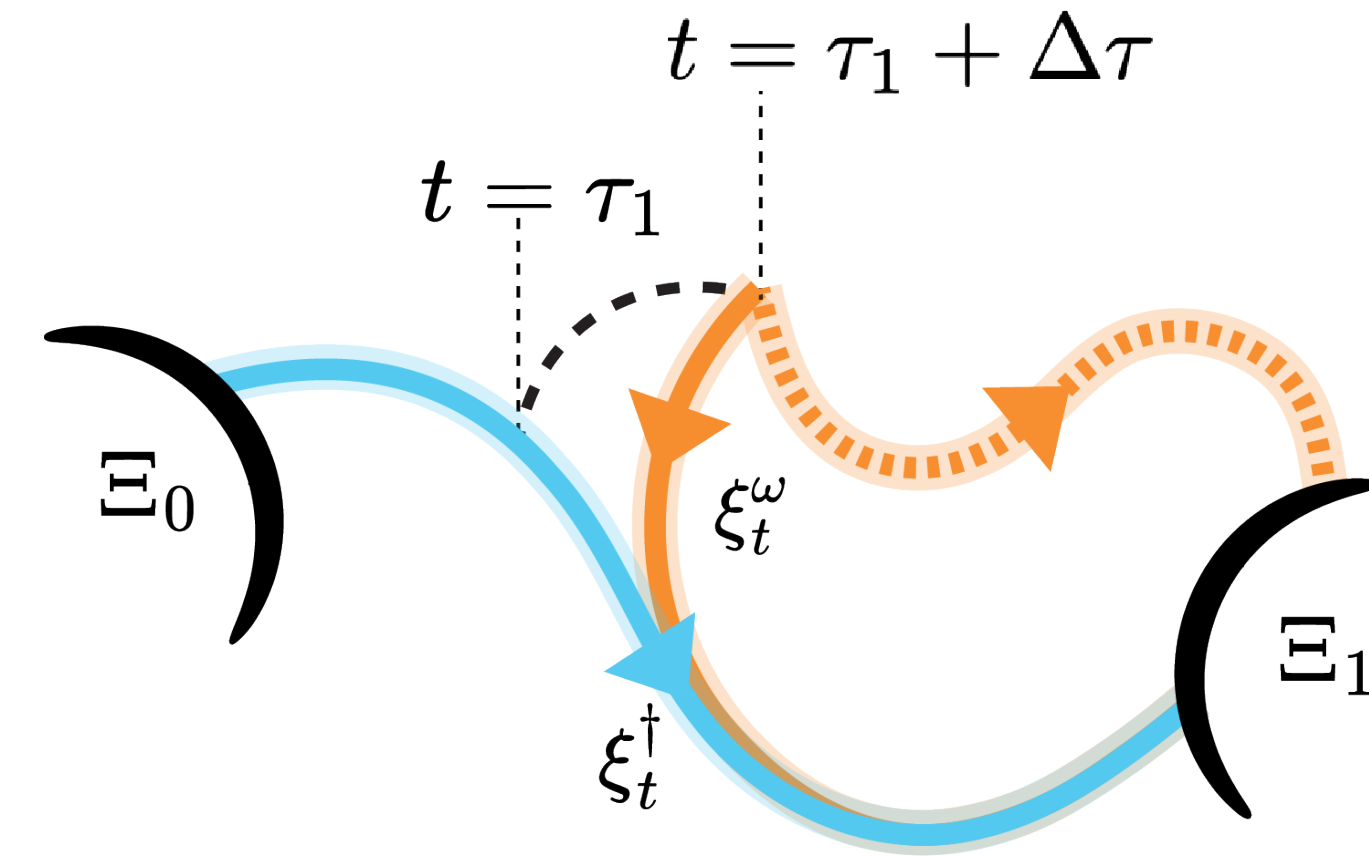
Lower-Level

$$y \in \left\{ \begin{aligned} & \arg \min_{\tilde{y} \in \mathcal{Y}(x)} \mathcal{F}^{\text{LL}}(x, \tilde{y}) \\ \text{s.t.} \quad & \mathcal{G}_{i'}^{\text{LL}}(x, \tilde{y}) \leq 0, \quad i' = 1, \dots, I^{\text{LL}}, \\ & \mathcal{H}_{j'}^{\text{LL}}(x, \tilde{y}) = 0, \quad j' = 1, \dots, J^{\text{LL}}. \end{aligned} \right\}.$$

The bilevel structure becomes tractable when the recovery problem is modelled as an unconstrained finite horizon LQR

Bilevel programs are difficult to solve

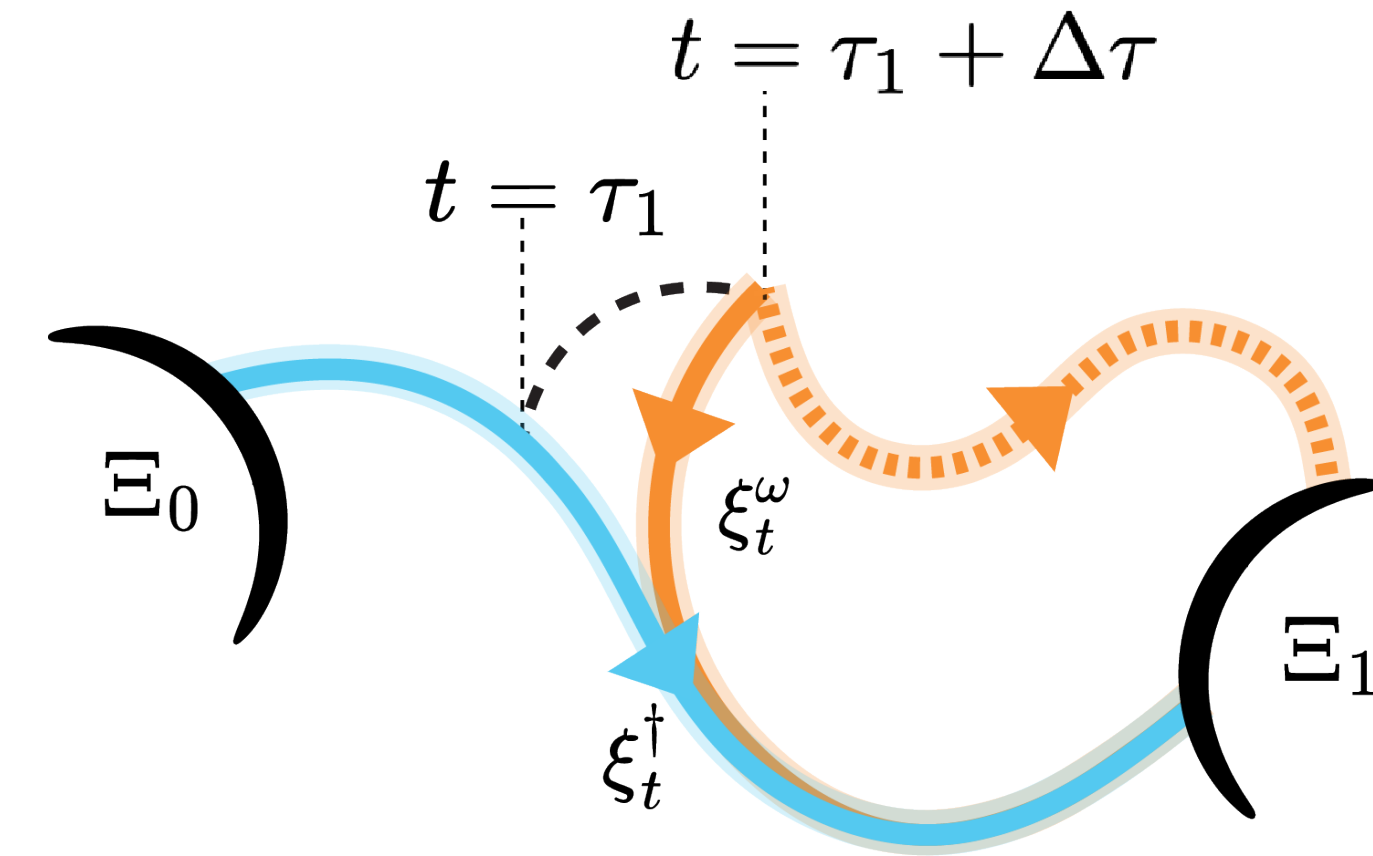
- Difficulty arises due to:
 - Non-convexity
 - Non-smoothness
- LL recovery problem can be modeled as an **unconstrained finite-horizon LQR**
 - **LICQ**: no need for complementarity constraints
 - **FONC/FOSC**: LL first-order conditions necessary and sufficient
 - Allows for **single-level reformulation**



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$$J^\omega(\mathbf{u}^\dagger, \mathbf{u}^\omega) = \left\{ \tilde{\xi}_1^T Q_f \tilde{\xi}_1 + \int_0^1 \left(\tilde{\xi}_t^T Q \tilde{\xi}_t + \tilde{\mathbf{u}}_t^T R \tilde{\mathbf{u}}_t \right) dt \right\},$$

$$\text{LL primal } \mathbf{u}_t^\omega = \begin{cases} \mathbf{0} & \forall t \in \mathcal{O}^\omega \equiv \bigcup_{i \in \mathbb{Z}_+} [\tau_{2i+1}(\omega), \tau_{2i+2}(\omega)), \\ -R^{-1} B_t^\top \boldsymbol{\mu}_t^\omega & \forall t \in \mathcal{A}^\omega \equiv [0, 1] \setminus \mathcal{O}^\omega, \end{cases}$$

$$\text{LL state } \dot{\tilde{\xi}}_t^\omega = A_t \tilde{\xi}_t + B_t \tilde{\mathbf{u}} \quad \forall t \in [0, 1],$$

$$\text{LL dual } -\dot{\boldsymbol{\mu}}_t^\omega = A_t^\top \boldsymbol{\mu}_t^\omega + Q \tilde{\xi}_t \quad \forall t \in [0, 1],$$

$$\boldsymbol{\mu}_1^\omega = Q_f \tilde{\xi}_1.$$

Standard LQR Setup

The bilevel framework yields a two-phase robustness certification for missed thrust recovery

The two phases differentiate between *model validity* and *recoverability*

- In **Phase I**, the goal is to bound the maximum missed thrust duration $\delta\tau_{\max}$ for which **the linear model remains appropriate** during the outage
- In **Phase II**, the goal is to assess whether **enough control authority and time horizon** remains to **recover** after the outage

Let ϵ be the acceptable relative error

$$\frac{\|r(e, t)\|}{\|A(t)e(t) + B(t)u_e(t)\|} \leq \epsilon,$$

$\forall t \in [\tau_1, \tau_2]$, where:

- τ_1 represents the time where the MTE begins,
- $\tau_2 = \tau_1 + \Delta\tau$ represents the time where the MTE ends,
- r represents the residual in the first order expansion of the nonlinear dynamics in the LL subproblem.

The bilevel framework yields a two-phase robustness certification for missed thrust recovery

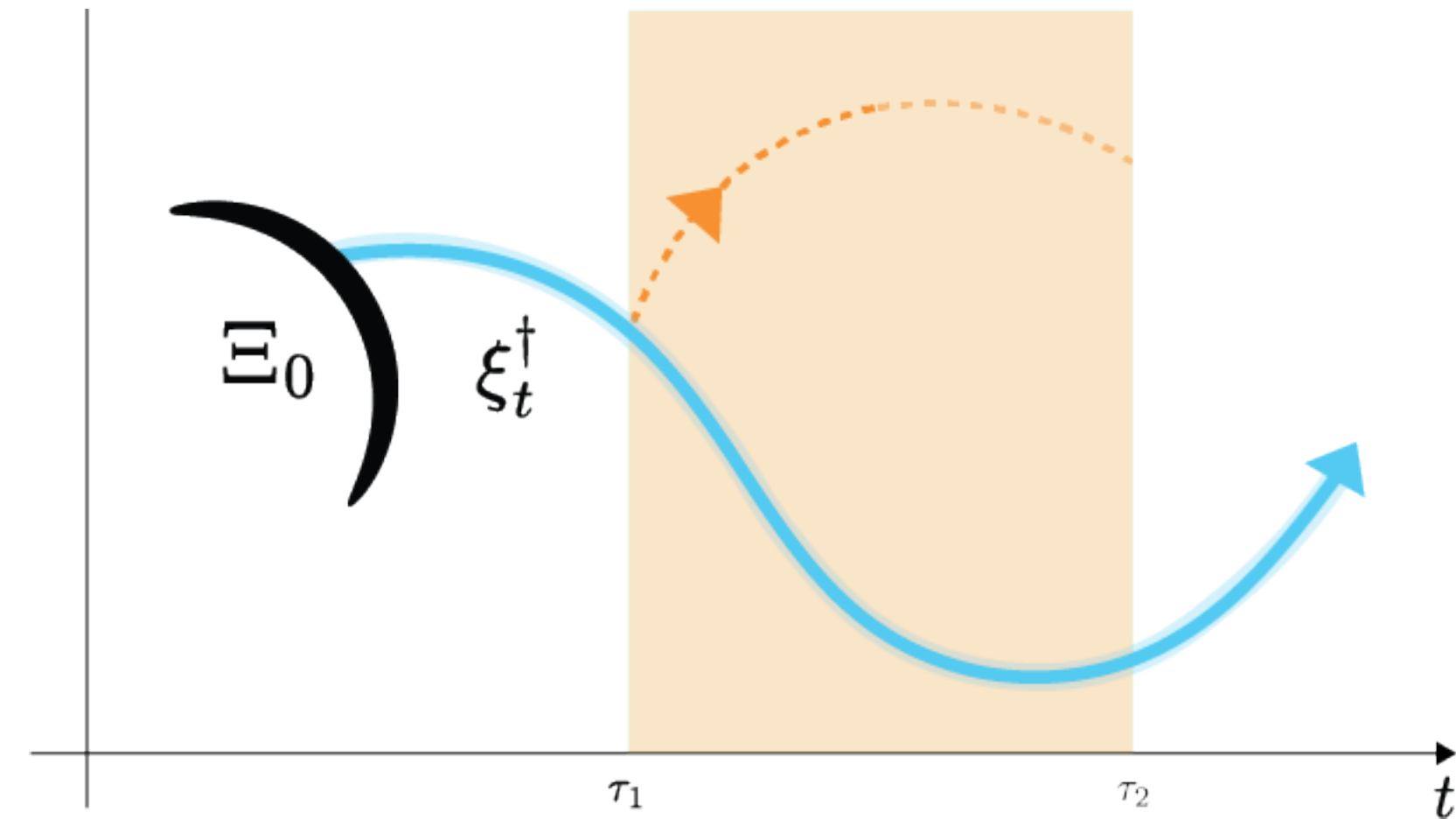
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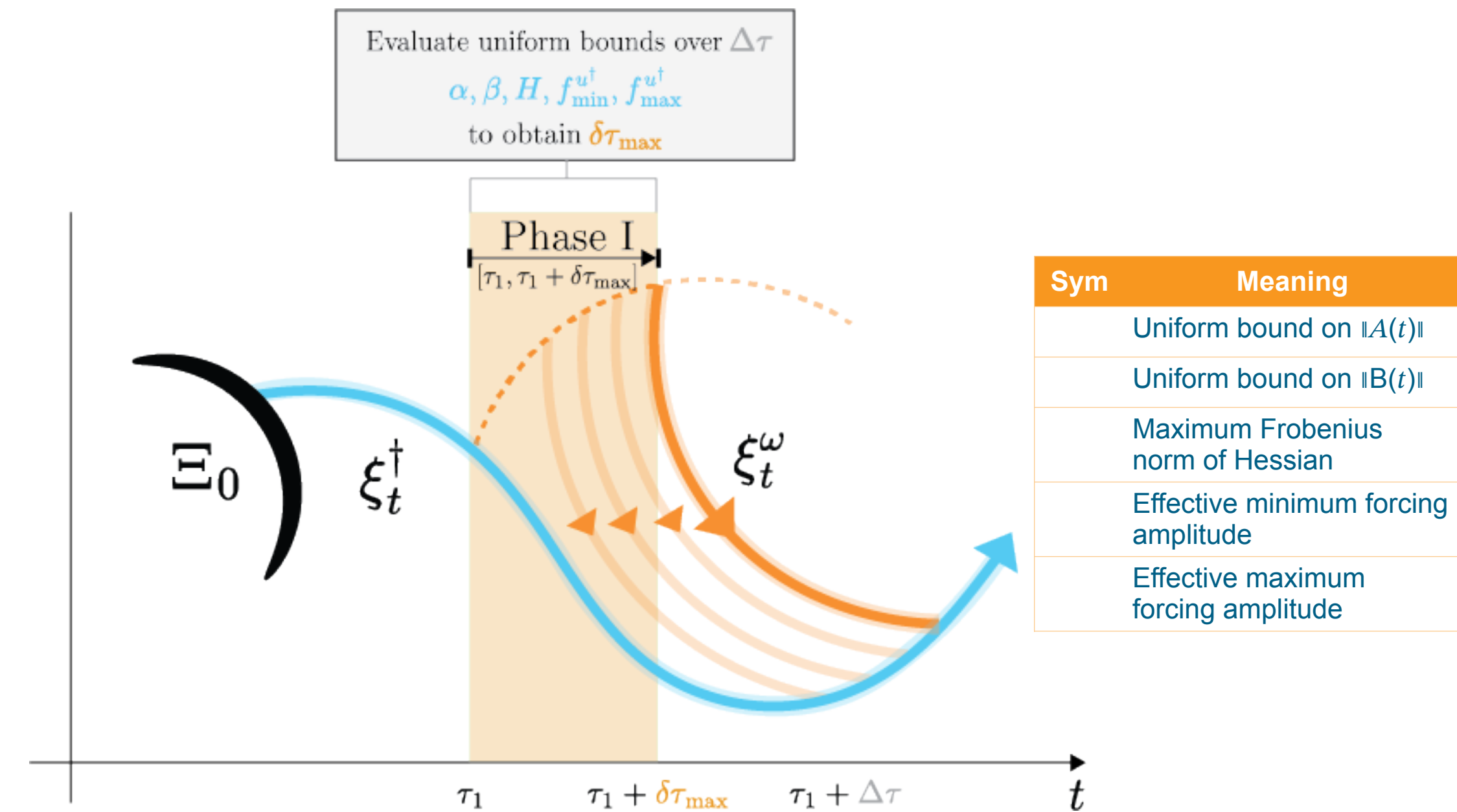
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$$\forall t \in [\tau_1, \tau_2], \text{ where: } \frac{\|r(e, t)\|}{\|A(t)e(t) + B(t)u_e(t)\|} \leq \epsilon,$$

- τ_1 represents the time where the MTE begins,
- $\tau_2 = \tau_1 + \Delta\tau$ represents the time where the MTE ends,
- r represents the residual in the first order expansion of the nonlinear dynamics in the LL subproblem.



The bilevel framework yields a two-phase robustness certification for missed thrust recovery

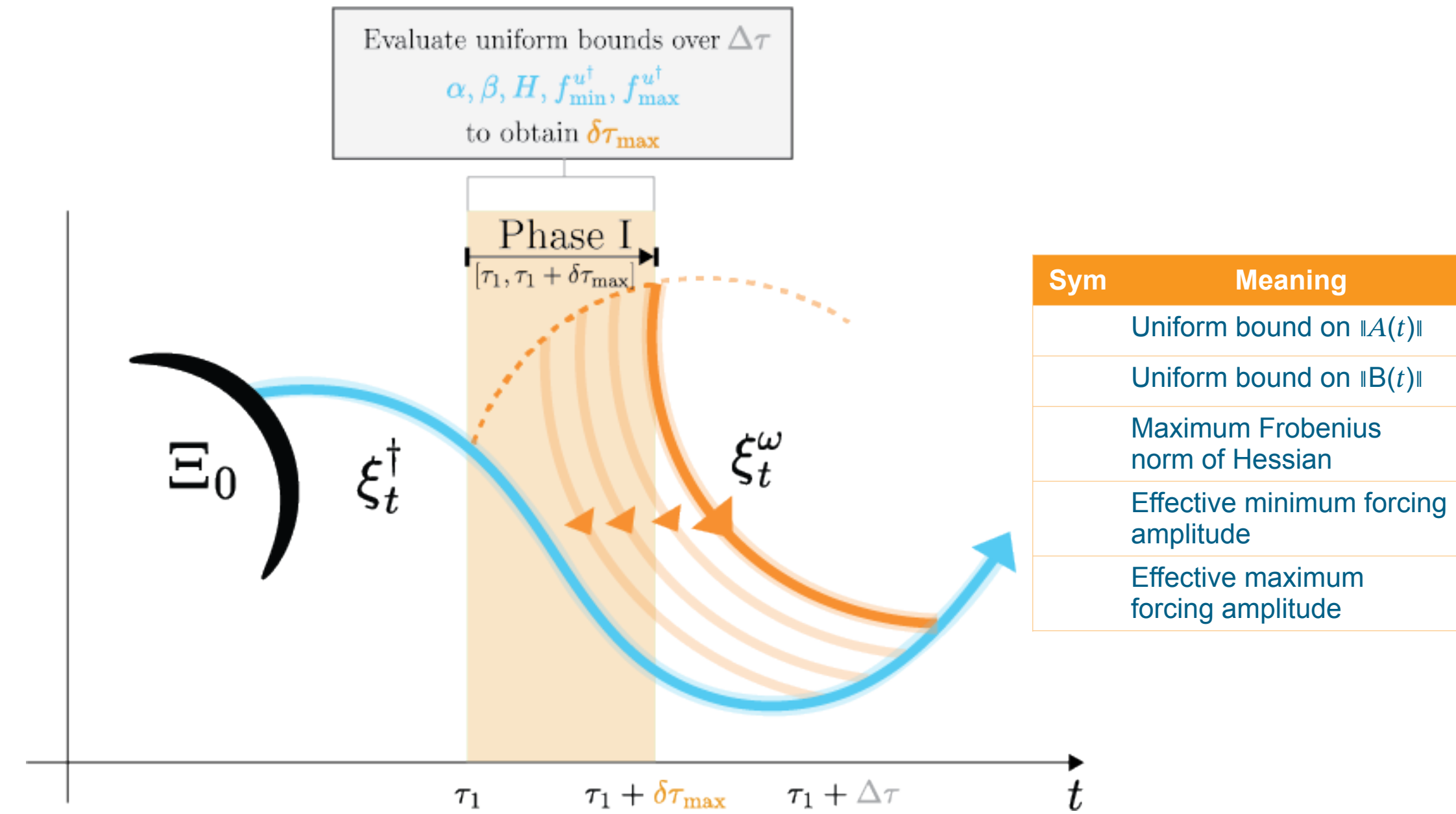
The two phases differentiate between *model validity* and *recoverability*

- In **Phase I**, the goal is to bound the maximum missed thrust duration $\delta\tau_{\max}$ for which **the linear model remains appropriate** during the outage
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Phase I

$$\delta\tau_{\max} = \begin{cases} \frac{2}{H(r_1 - r_2)} \ln \left(\frac{r_2(\delta - r_1)}{r_1(\delta - r_2)} \right), & \Delta > 0, \\ \frac{\delta}{\frac{\alpha}{2} \delta + \frac{\alpha^2}{2H}}, & \Delta = 0, \\ \frac{1}{\gamma} \left(\tan^{-1} \left(\frac{\delta + \frac{\alpha}{H}}{\gamma} \right) - \phi \right), & \Delta < 0, \end{cases}$$

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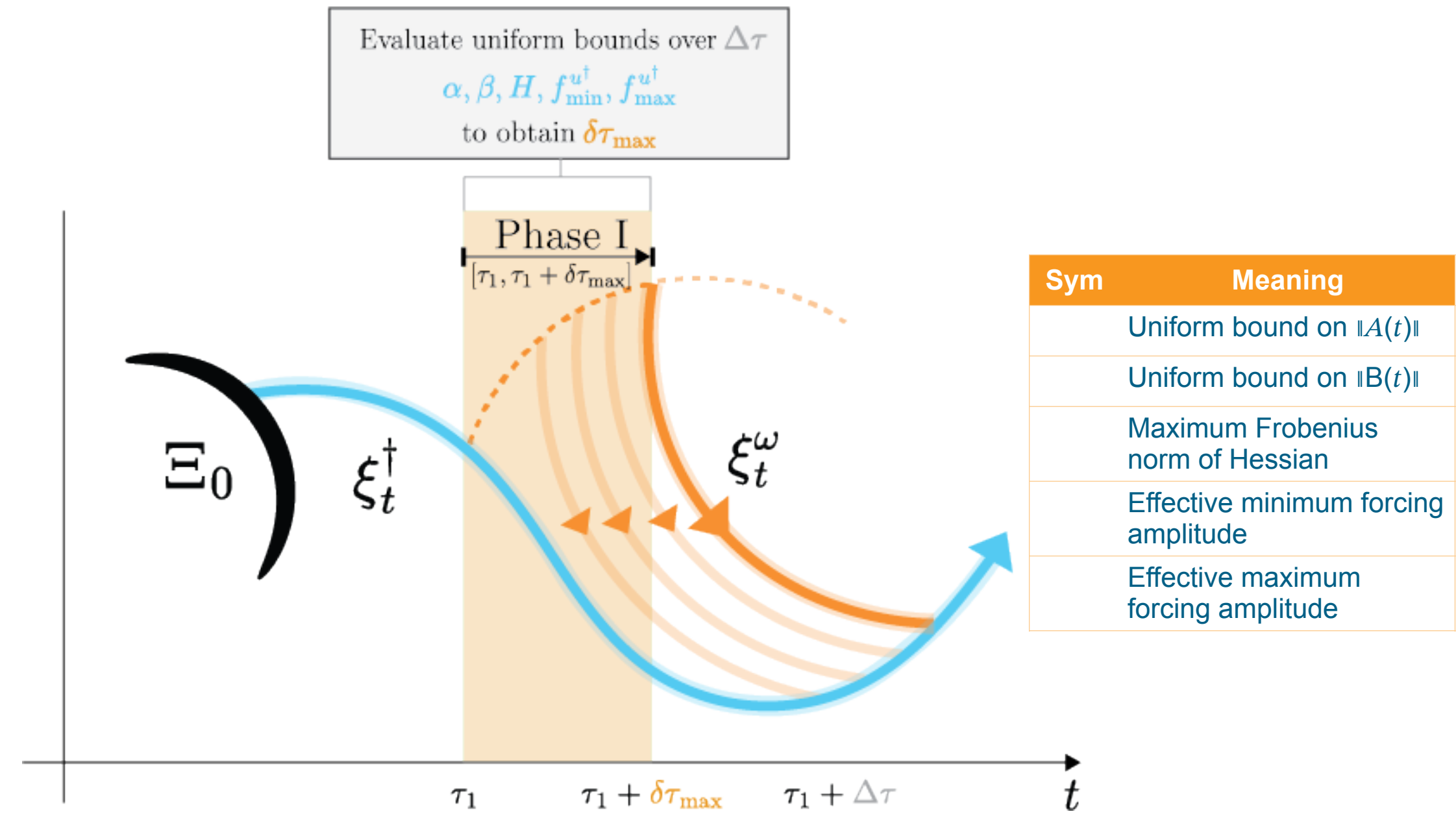
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Phase I

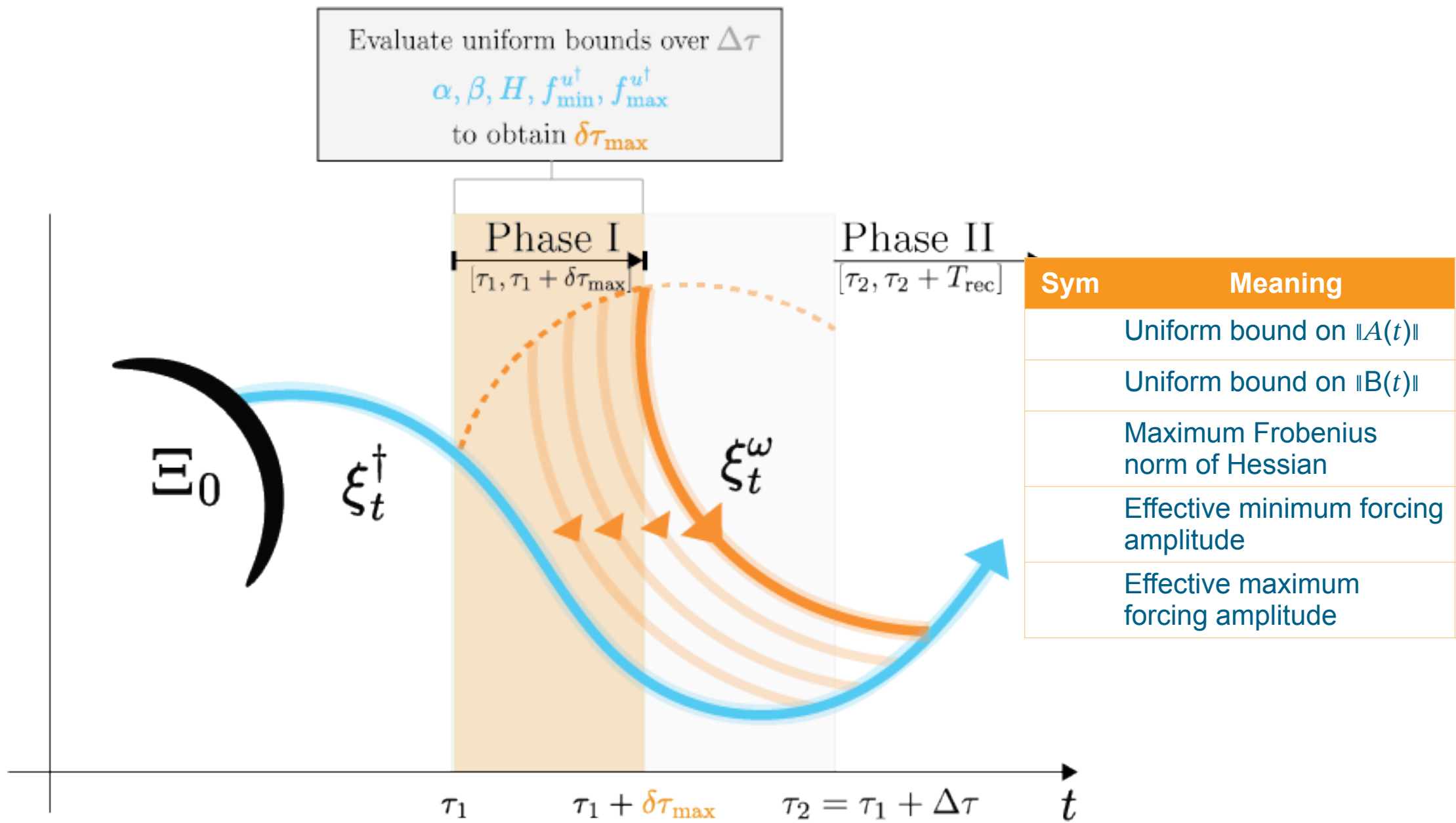
$$\delta\tau_{\max} = \begin{cases} \frac{2}{H(r_1 - r_2)} \ln \left(\frac{r_2(\delta - r_1)}{r_1(\delta - r_2)} \right), & \Delta > 0, \\ \frac{\delta}{\alpha}, & \Delta = 0, \\ \frac{1}{\gamma} \left(\tan^{-1} \left(\frac{\delta + \frac{\alpha}{H}}{\gamma} \right) - \phi \right), & \Delta < 0, \end{cases}$$

Recovery guaranteed
if $\Delta\tau < \delta\tau_{\max}$

The bilevel framework yields a two-phase robustness certification for missed thrust recovery

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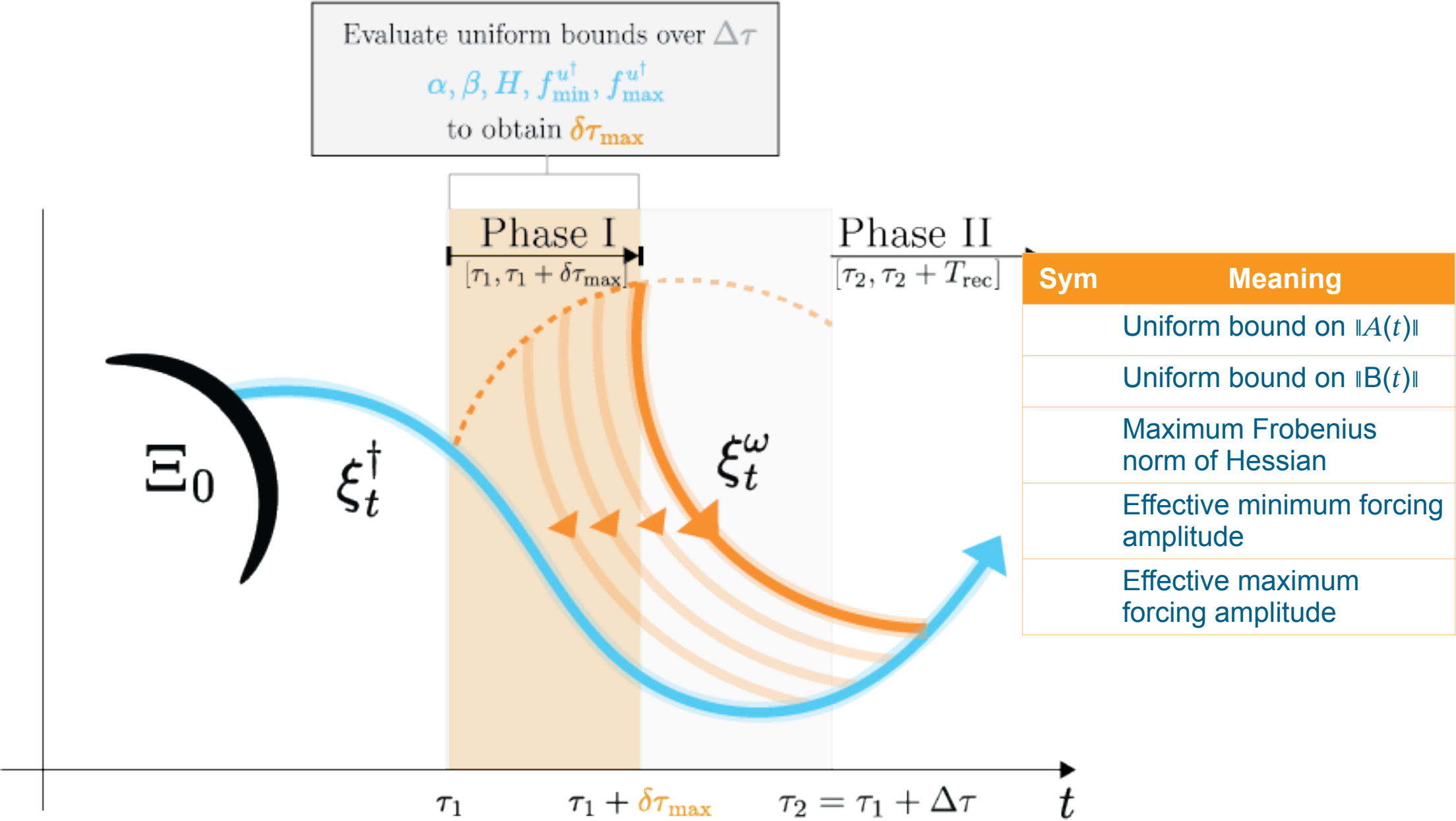
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Phase II

$$E_{\min}(\tilde{\xi}^+; \tau_2, T_{\text{rec}}) = \tilde{\xi}^{+\top} W(\tau_2, T_{\text{rec}})^{-1} \tilde{\xi}^+$$
$$E_{\text{ava}}(T_{\text{rec}}) = T_{\text{rec}} \|\bar{u}\|_2^2$$
$$r_e := \frac{E_{\text{ava}}(T_{\text{rec}})}{E_{\min}(\tilde{\xi}^+; \tau_2, T_{\text{rec}})}$$

Recovery guaranteed
if $\Delta\tau < \delta\tau_{\max}$

The bilevel framework yields a two-phase robustness certification for missed thrust recovery

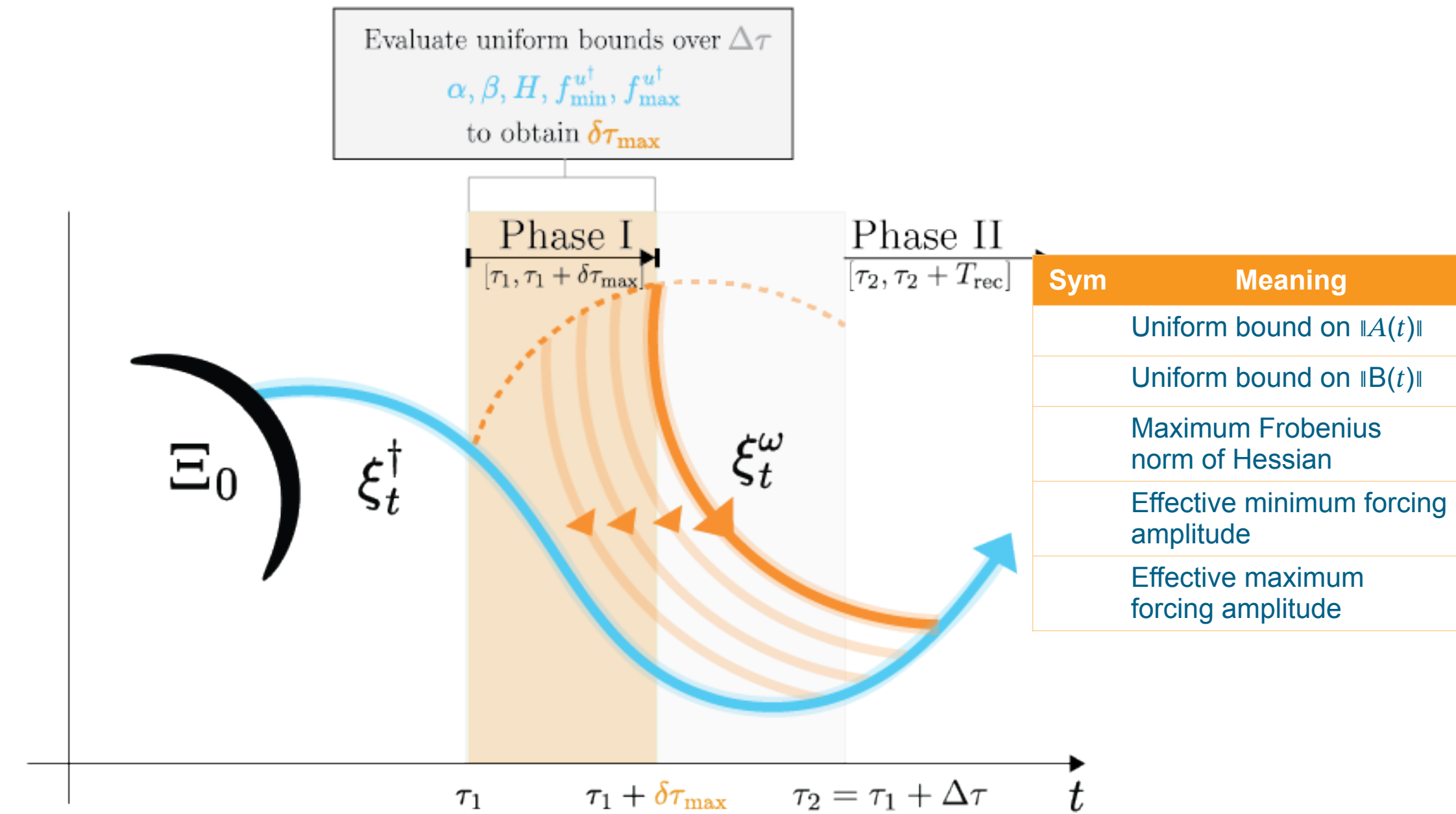
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Recovery guaranteed
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Phase II

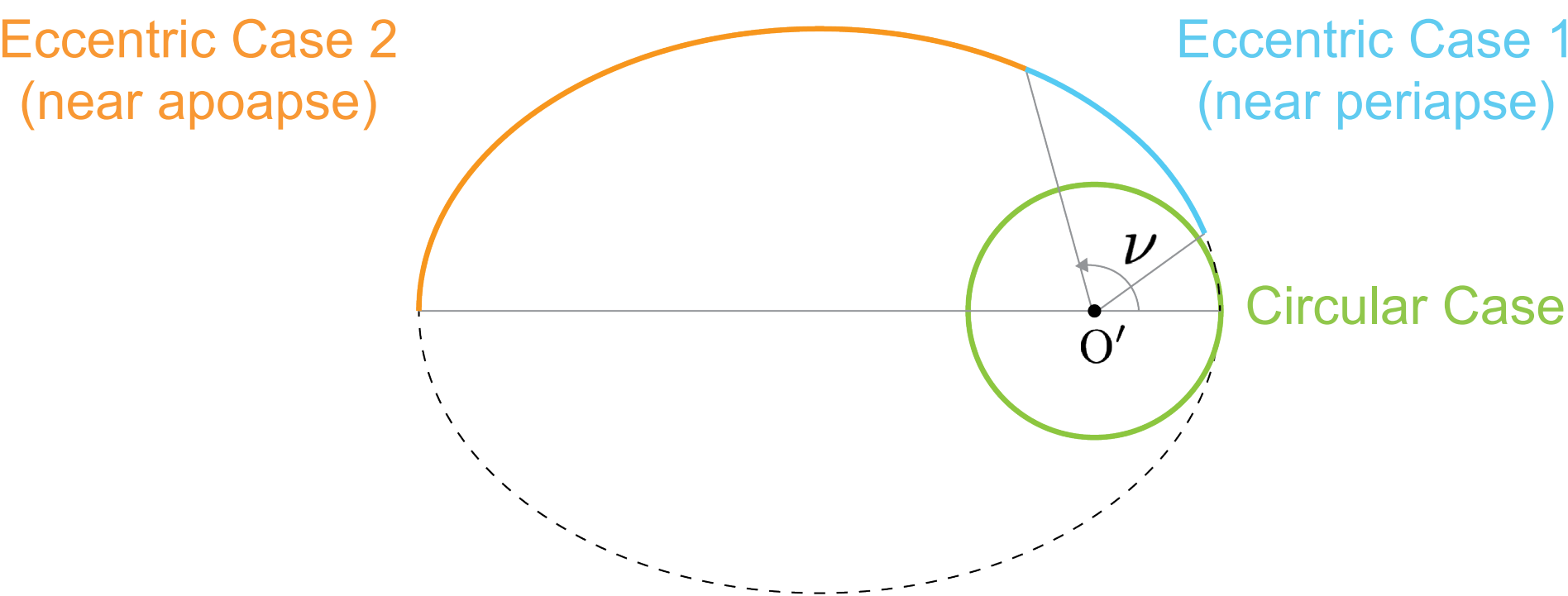
$$\begin{aligned} E_{\min}(\tilde{\xi}^+; \tau_2, T_{\text{rec}}) &= \tilde{\xi}^{+\top} W(\tau_2, T_{\text{rec}})^{-1} \tilde{\xi}^+ \\ E_{\text{ava}}(T_{\text{rec}}) &= T_{\text{rec}} \|\bar{u}\|_2^2 \\ r_e &:= \frac{E_{\text{ava}}(T_{\text{rec}})}{E_{\min}(\tilde{\xi}^+; \tau_2, T_{\text{rec}})} \end{aligned}$$

If $\Delta\tau > \delta\tau_{\max}$, recovery
is still possible if $r_e > 1$

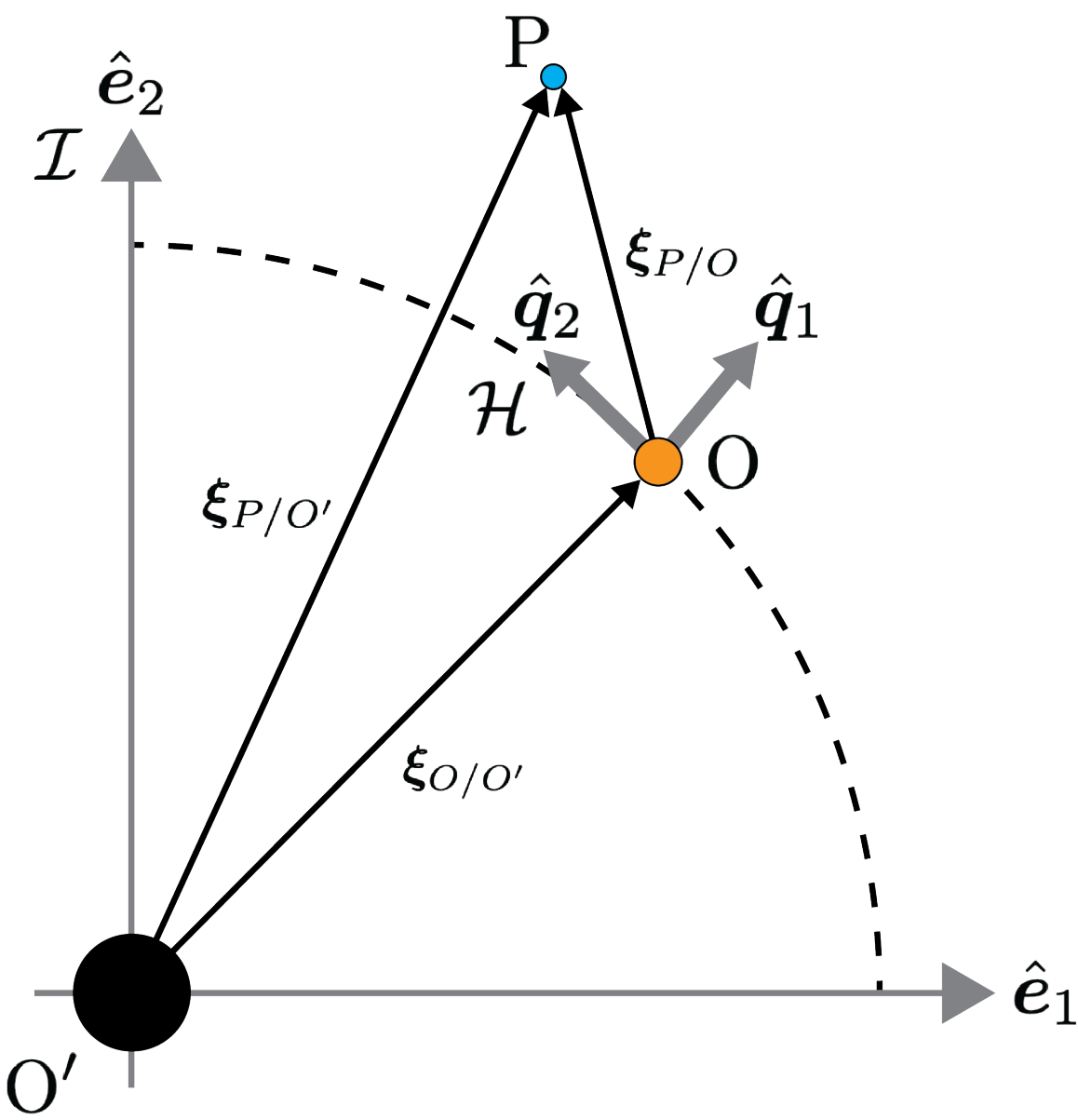
Case study: Spacecraft rendezvous problem near Earth

The problem considers a motion of a chaser spacecraft pursuing a target spacecraft in a two-body gravity field

Three representative target orbits are considered



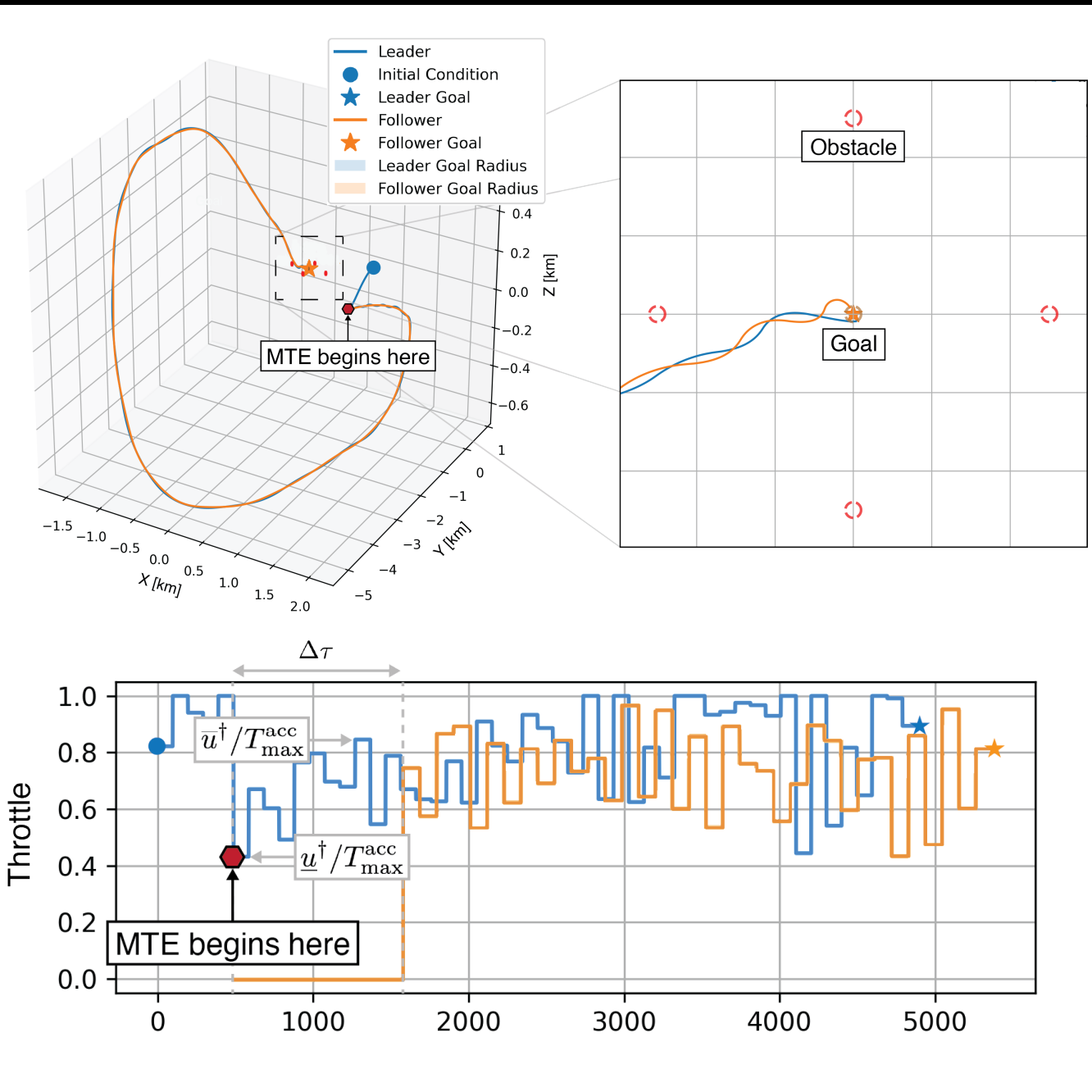
Parameter	Circular	Eccentric Case 1	Eccentric Case 2
n [s ⁻¹]	1.109×10^{-3}	—	—
a [km]	6,871.0	22,903.3	22,903.3
e	0	0.7	0.7
ν [deg]	[0°, 360°]	[60°, 120°]	[120°, 180°]
$\hat{\xi}_{P/O}^H$ [km, km/s]	$[\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}, 0, 0, 0]^T$	$[\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}, 0, 0, 0]^T$	$[\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}, 0, 0, 0]^T$



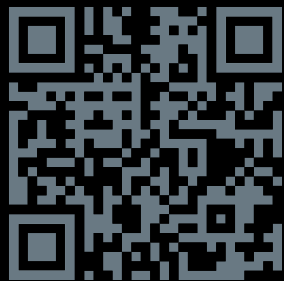
$$\begin{bmatrix} \dot{\xi}_{P/O} \\ \ddot{\xi}_{P/O} \end{bmatrix}' = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} \\ \Xi_e & \Omega_e \end{bmatrix} \begin{bmatrix} \xi_{P/O} \\ \dot{\xi}_{P/O} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ \dot{\nu}^{-2} \mathbf{u} \end{bmatrix} + \frac{\mu}{\dot{\nu}^2 r^4} \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ \mathbf{g}^{(2)}(\xi_{P/O}) \end{bmatrix} + \frac{\mu}{\dot{\nu}^2 r^5} \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ \mathbf{g}^{(3)}(\xi_{P/O}) \end{bmatrix},$$

$$\mathbf{g}^{(2)}(\xi_{P/O}) = \begin{bmatrix} -2q_1^2 + q_2^2 + q_3^2 \\ 2q_1q_2 \\ 2q_1q_3 \end{bmatrix}, \quad \mathbf{g}^{(3)}(\xi_{P/O}) = \begin{bmatrix} 4q_1^3 - 6q_1q_2^2 - 6q_1q_3^2 \\ -6q_1^2q_2 + \frac{3}{2}(q_2^3 + q_2q_3^2) \\ -6q_1^2q_3 + \frac{3}{2}(q_2^2q_3 + q_3^3) \end{bmatrix},$$
$$\Xi_e := \begin{bmatrix} -\frac{3+e \cos \nu}{1+e \cos \nu} & 2\frac{e \sin \nu}{1+e \cos \nu} & 0 \\ -2\frac{e \sin \nu}{1+e \cos \nu} & -\frac{e \cos \nu}{1+e \cos \nu} & 0 \\ 0 & 0 & 1+e \cos \nu \end{bmatrix}, \quad \Omega_e := \begin{bmatrix} -2\frac{e \sin \nu}{1+e \cos \nu} & -2 & 0 \\ 2 & -2\frac{e \sin \nu}{1+e \cos \nu} & 0 \\ 0 & 0 & -2\frac{e \sin \nu}{1+e \cos \nu} \end{bmatrix}.$$

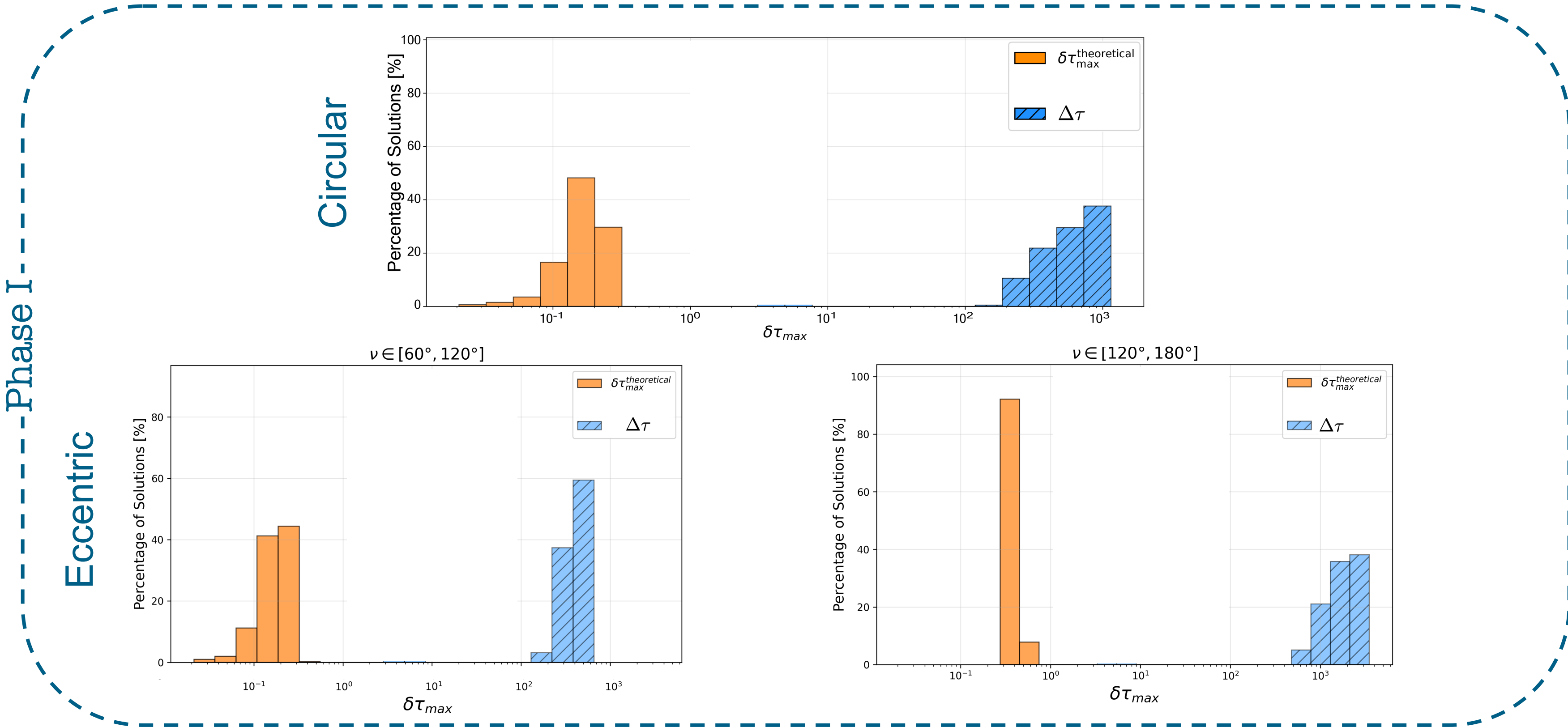
Feasible solutions may fail the Phase I certification, only if they pass the Phase II certification.



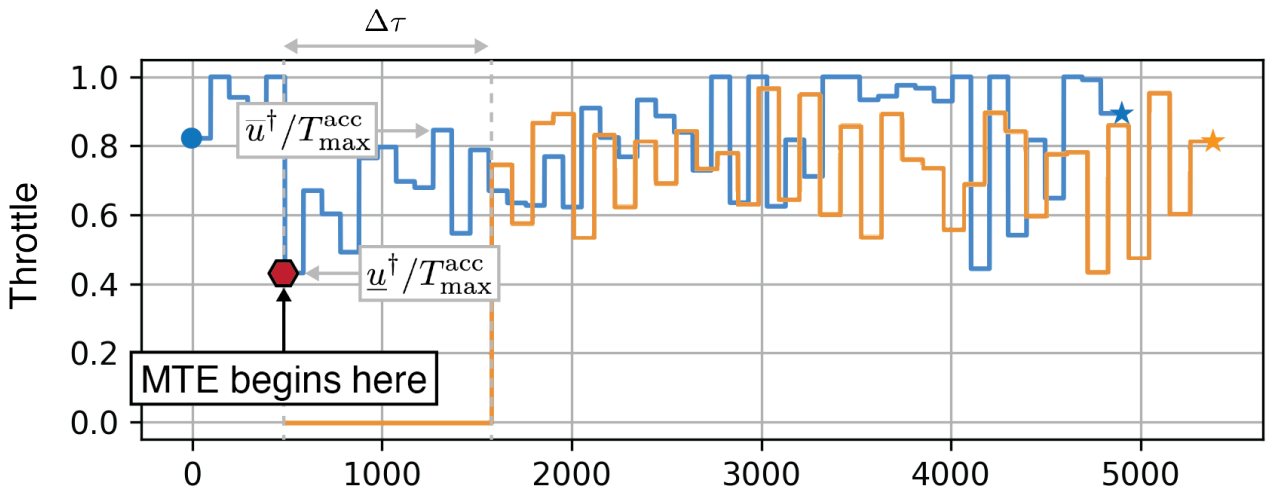
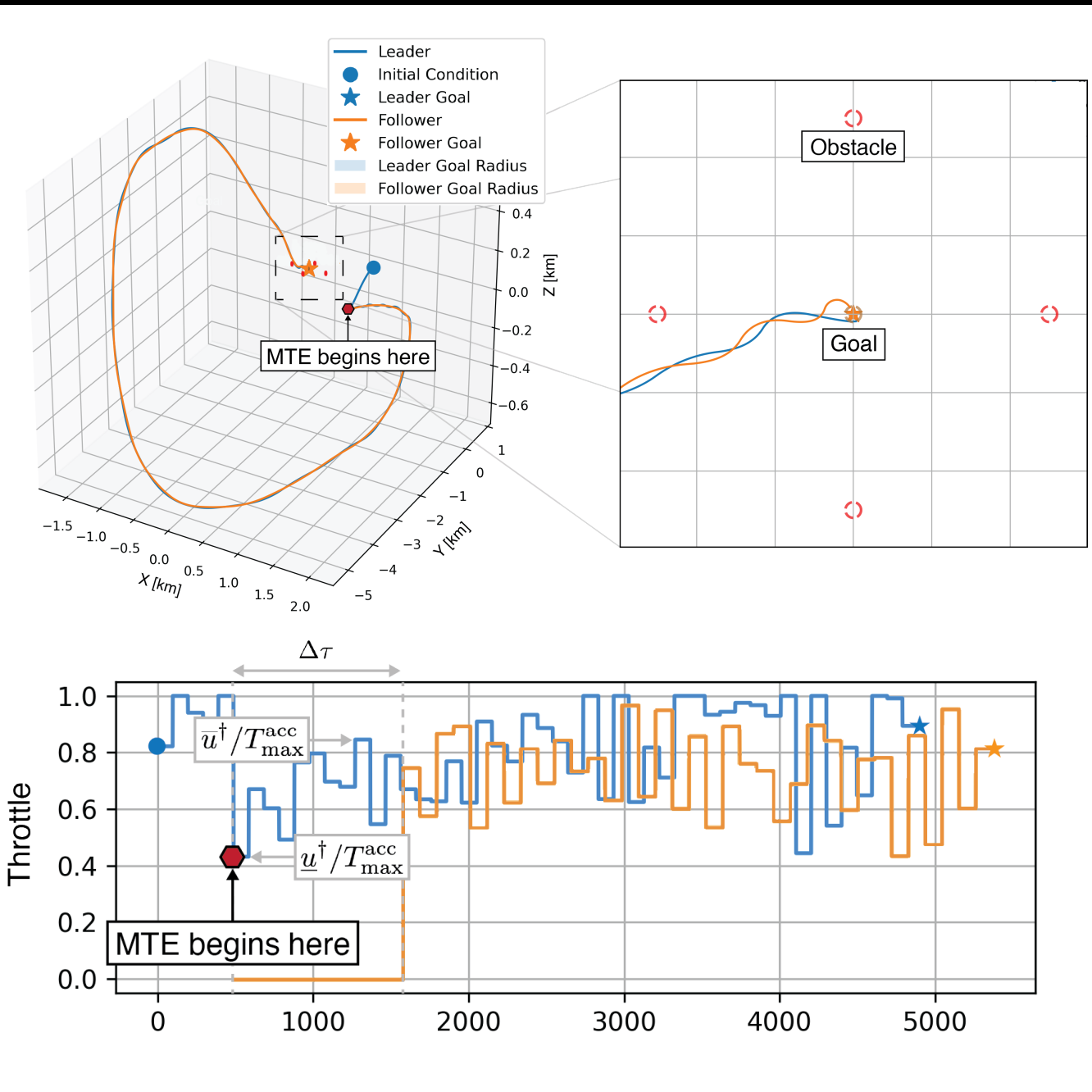
For more details:



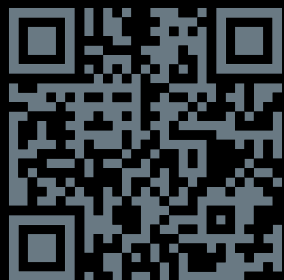
Sinha and Beeson (JGCD, 2025)



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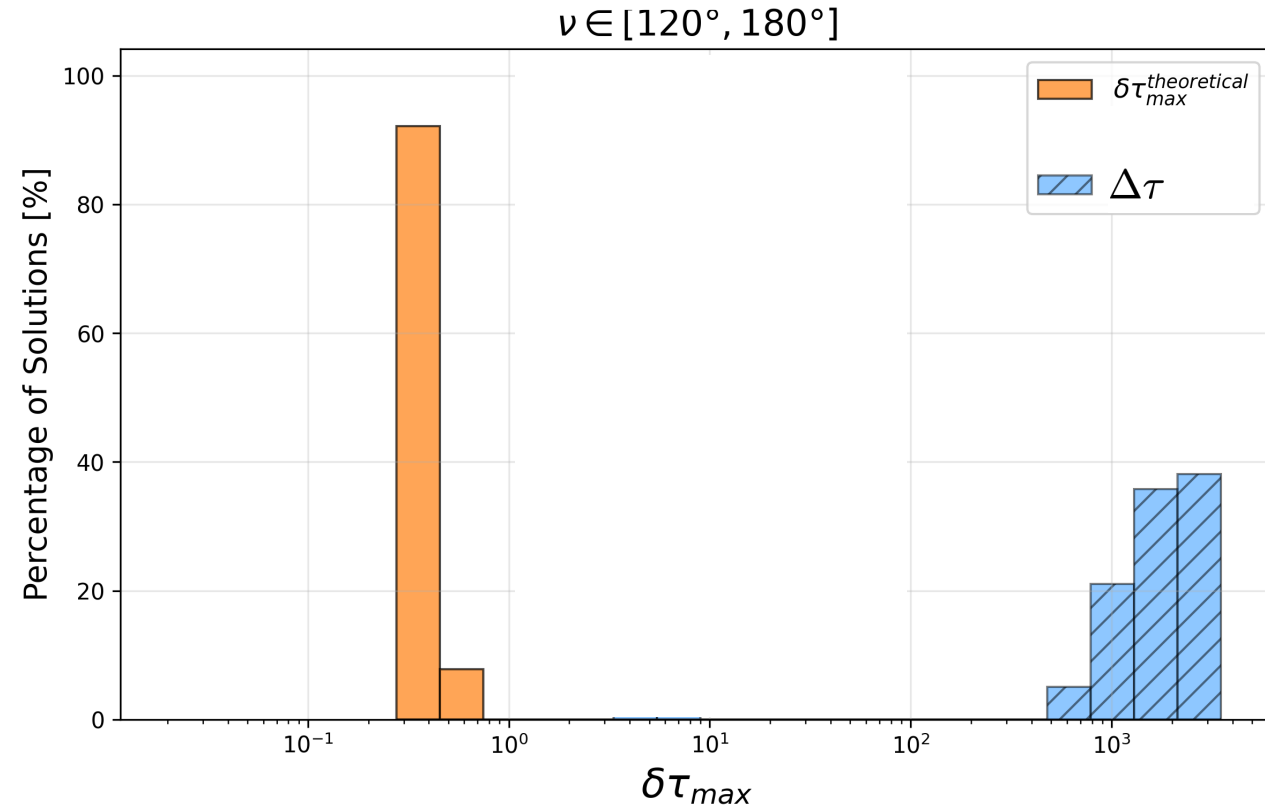
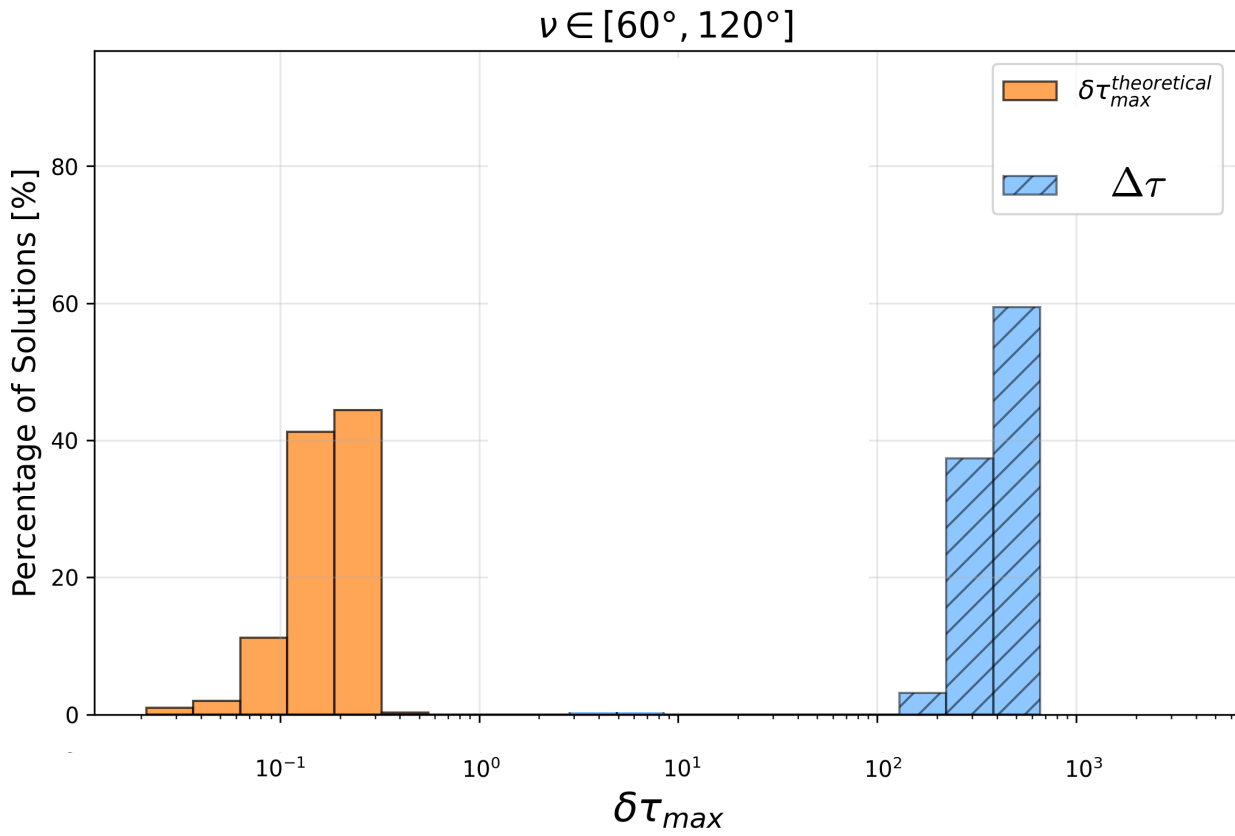
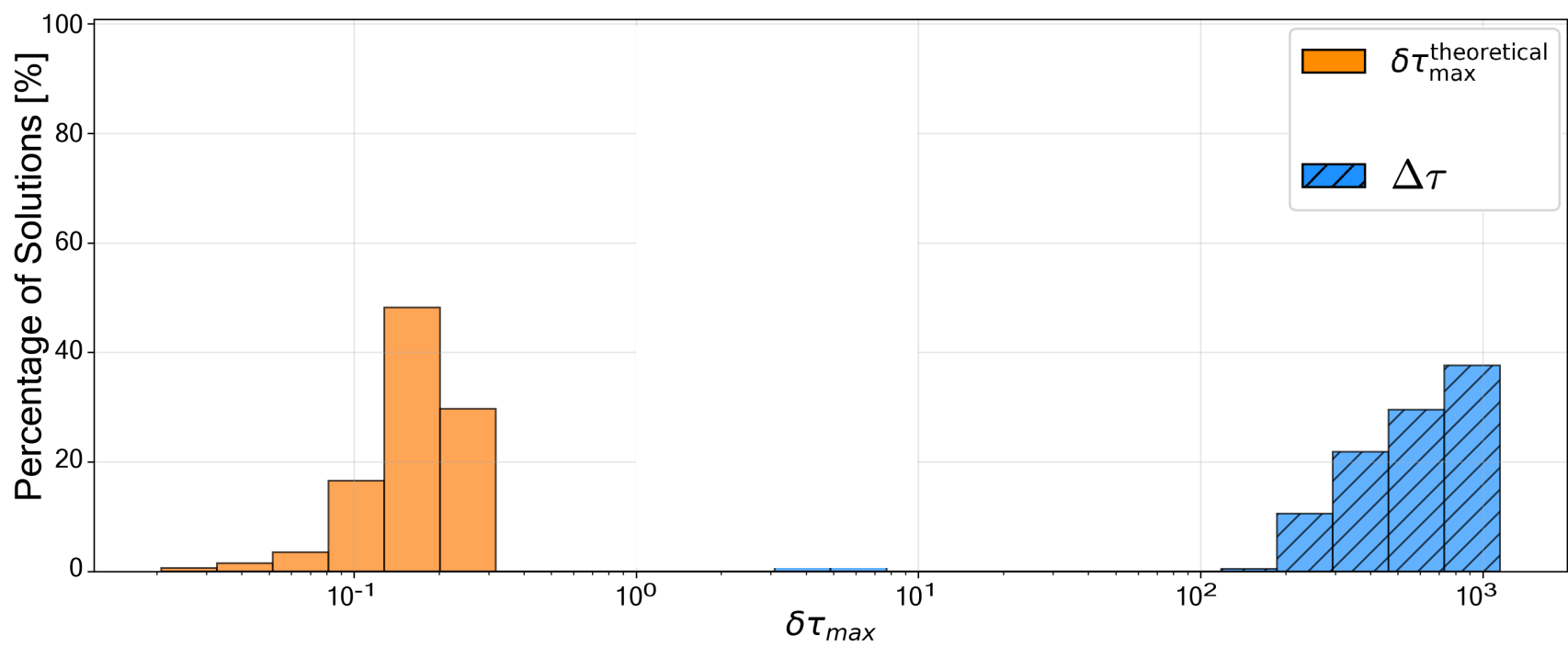
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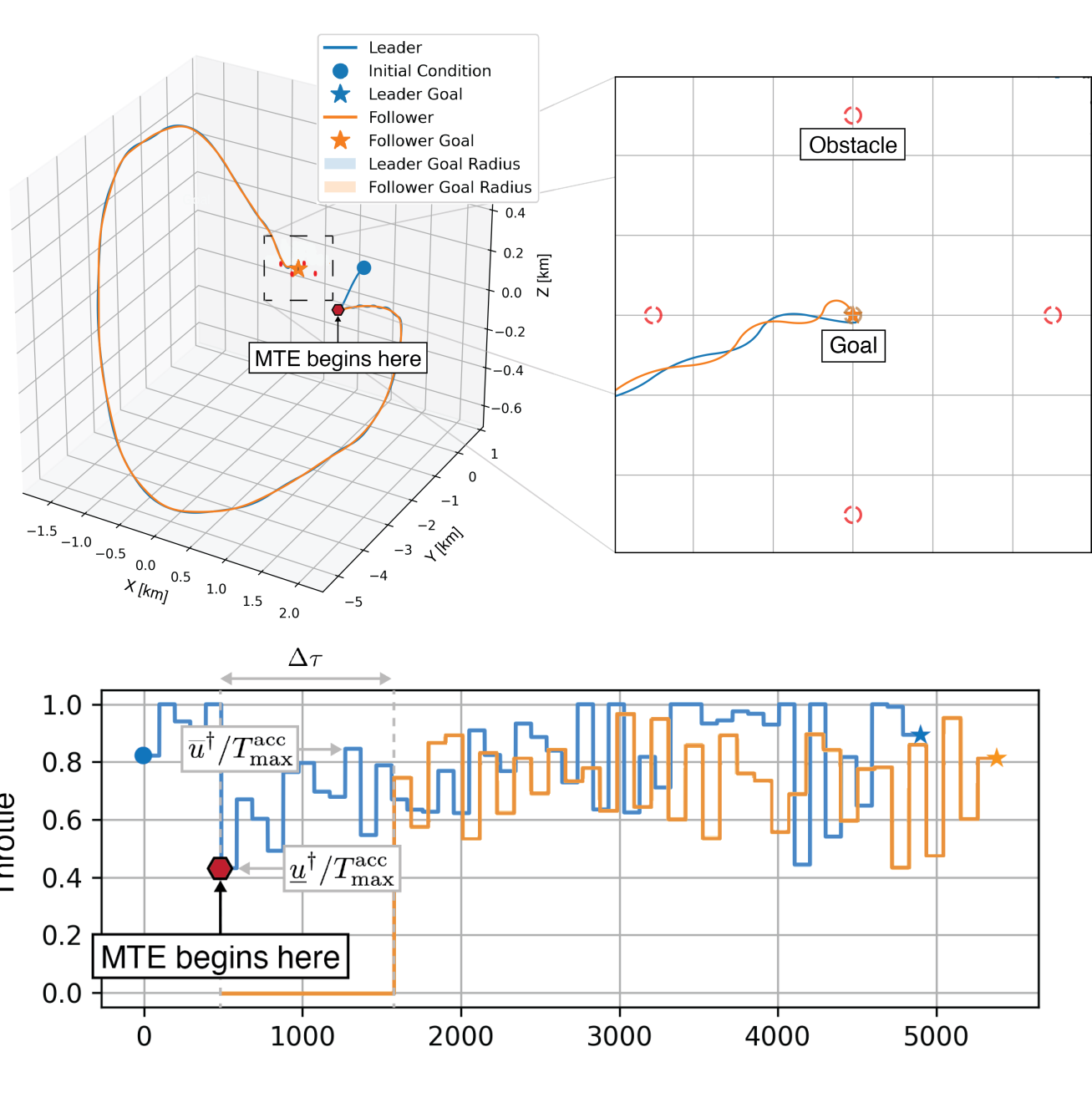
Sinha and Beeson (JGCD, 2025)

Phase I certification failed: $\delta\tau_{\max}^{\text{theoretical}} \ll \Delta\tau$

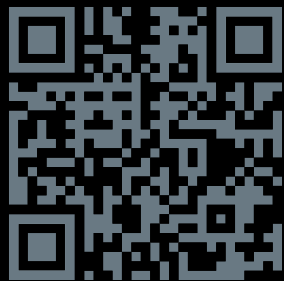
Phase I
Circular
Eccentric



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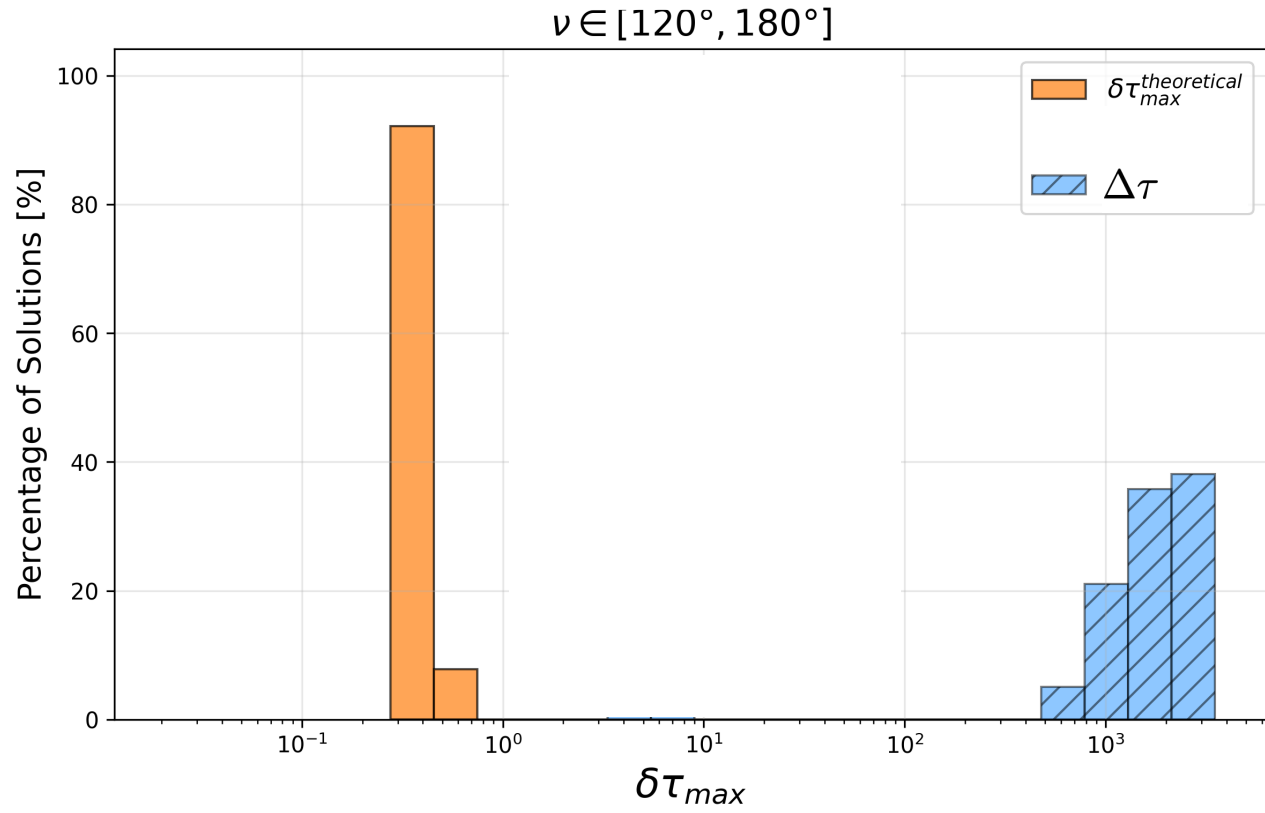
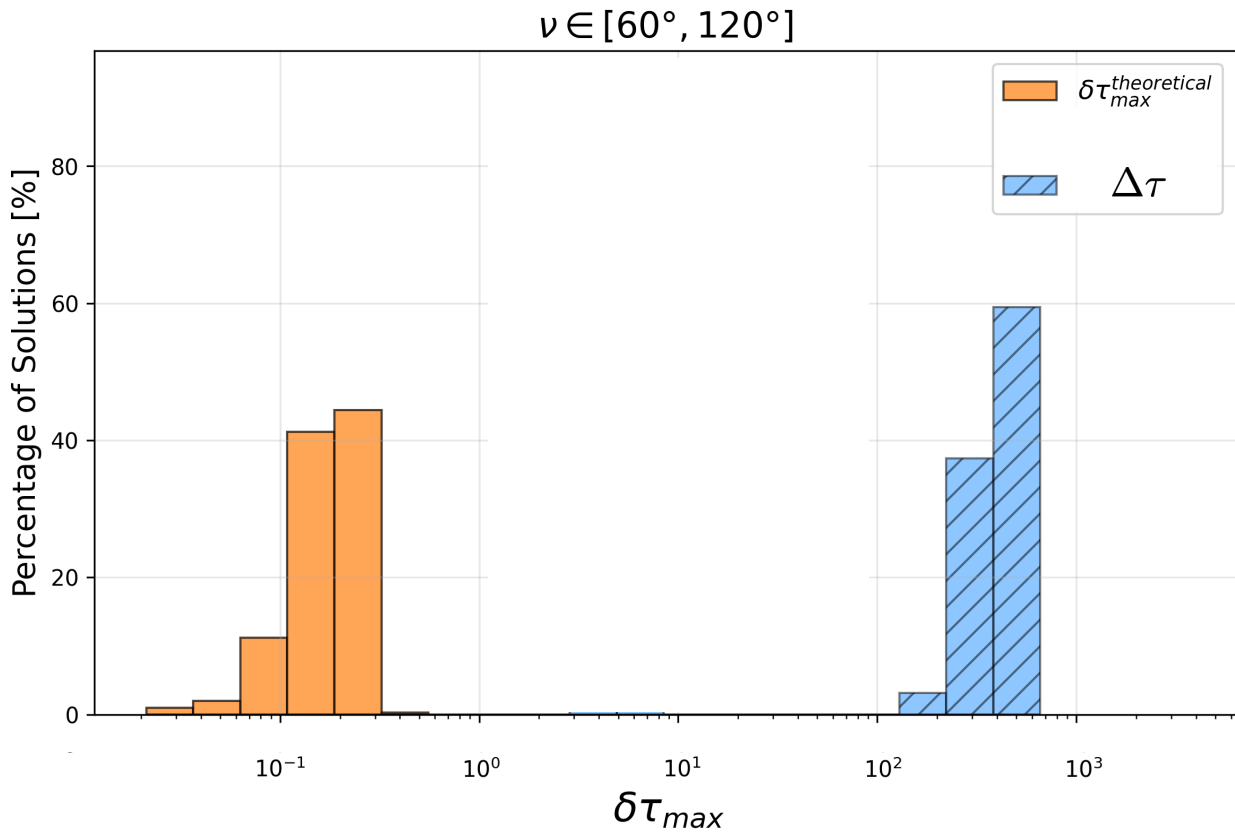
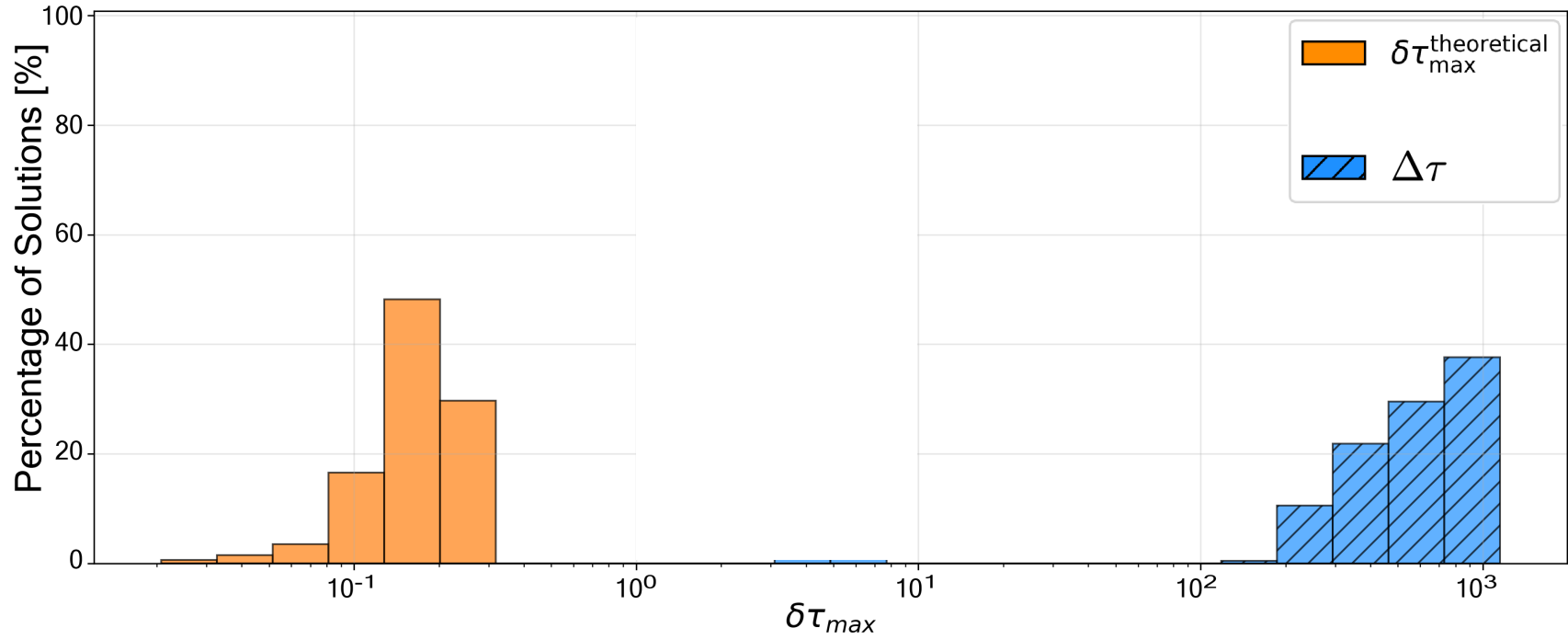
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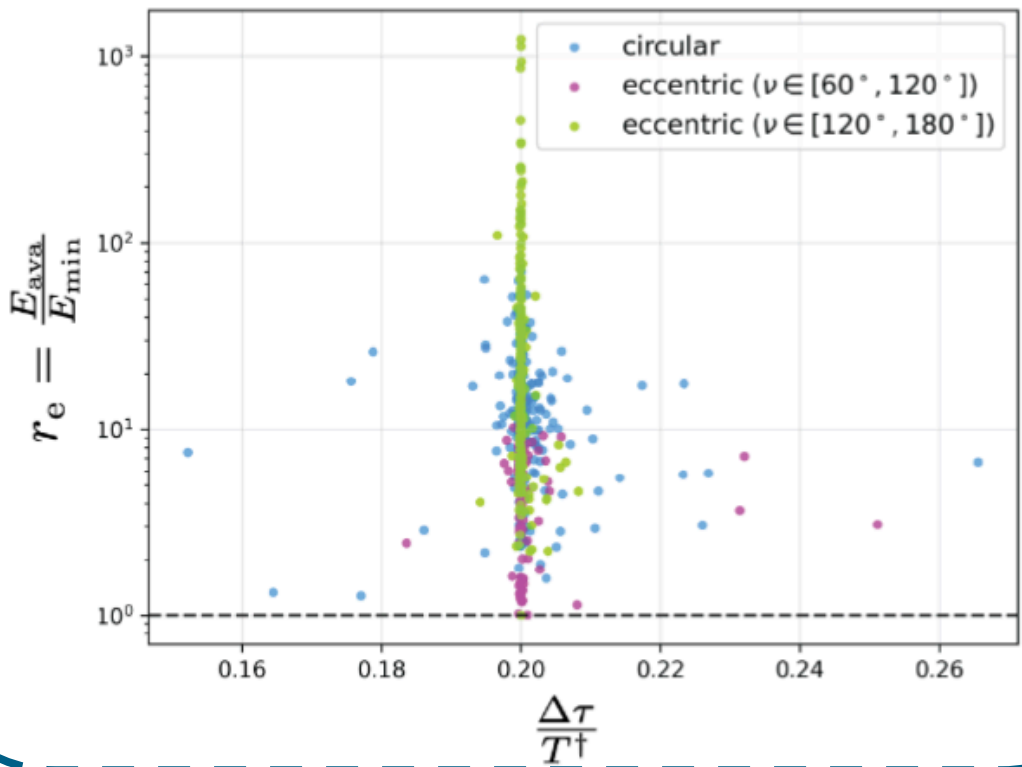
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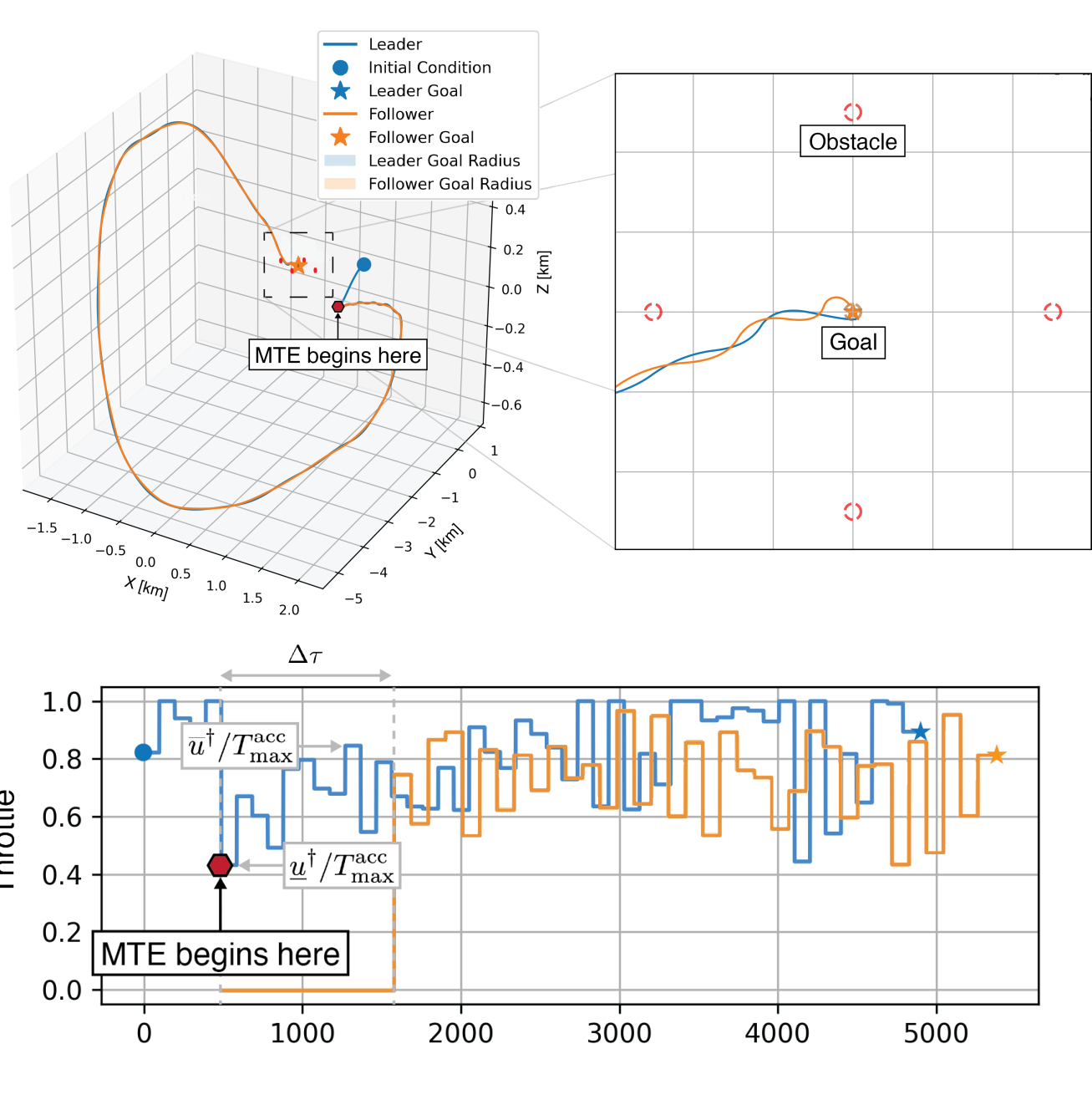
Circular
Eccentric
Phase I



Phase II



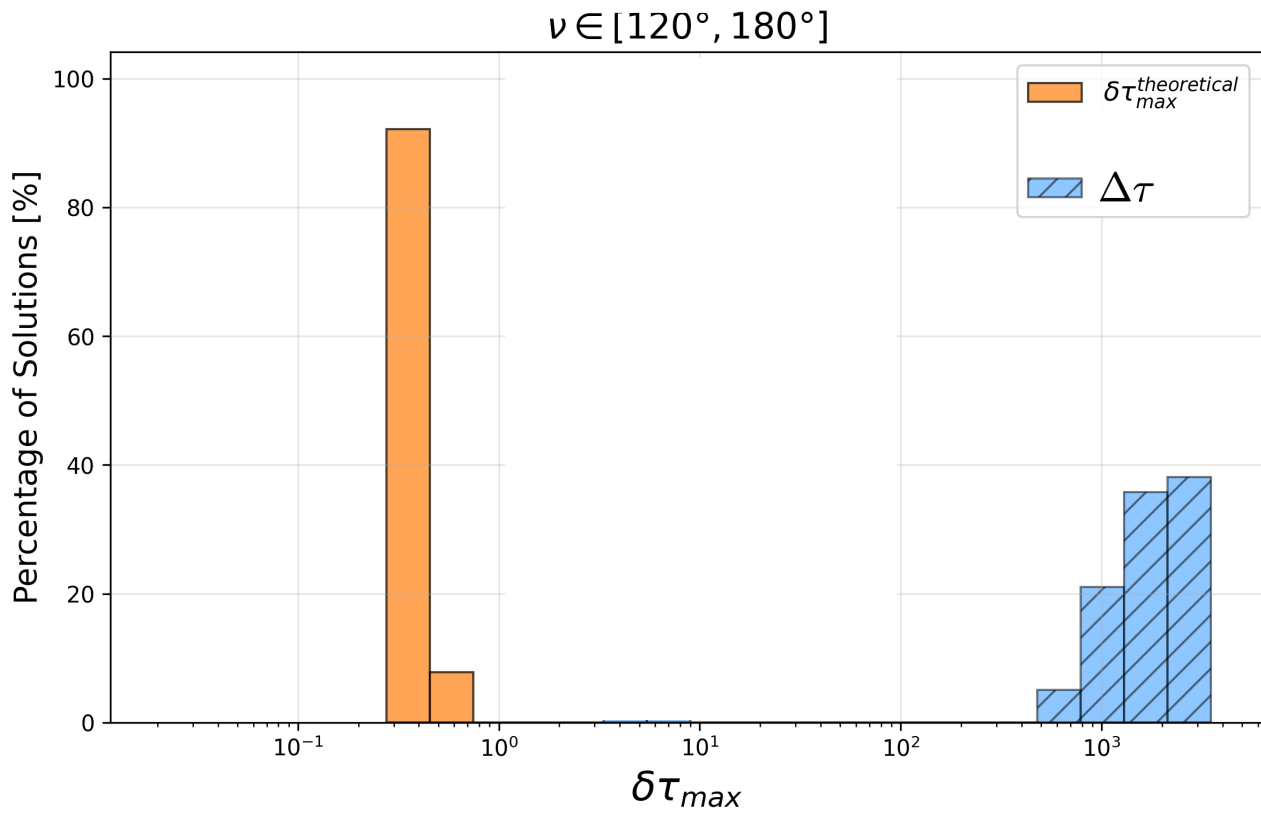
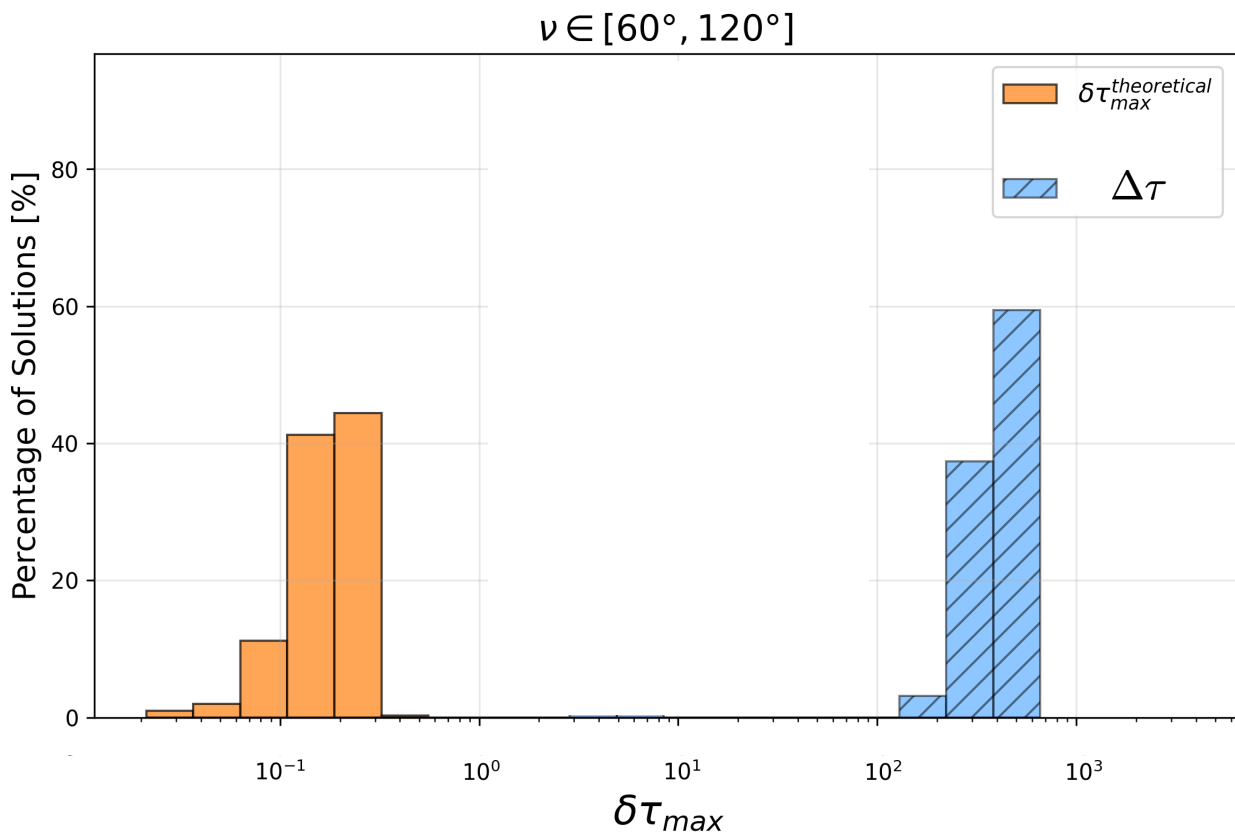
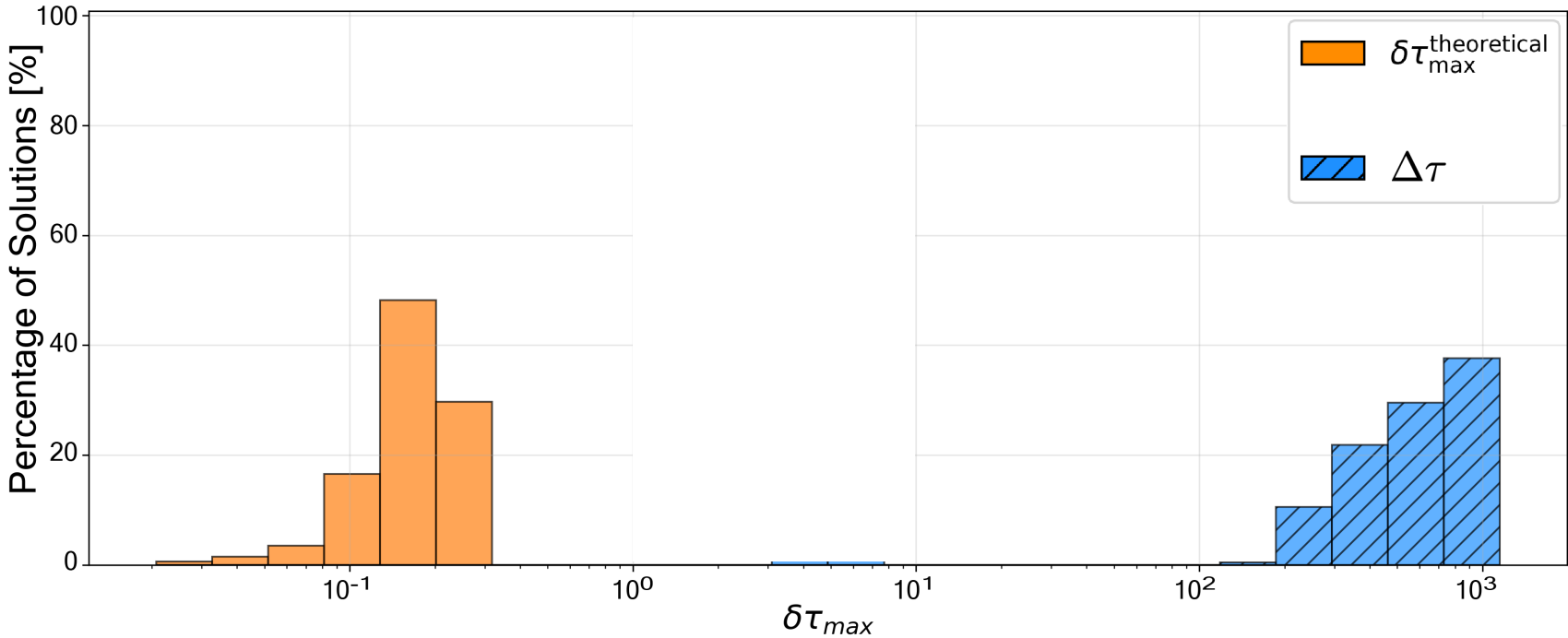
Feasible solutions may fail the Phase I certification, only if they pass the Phase II certification.



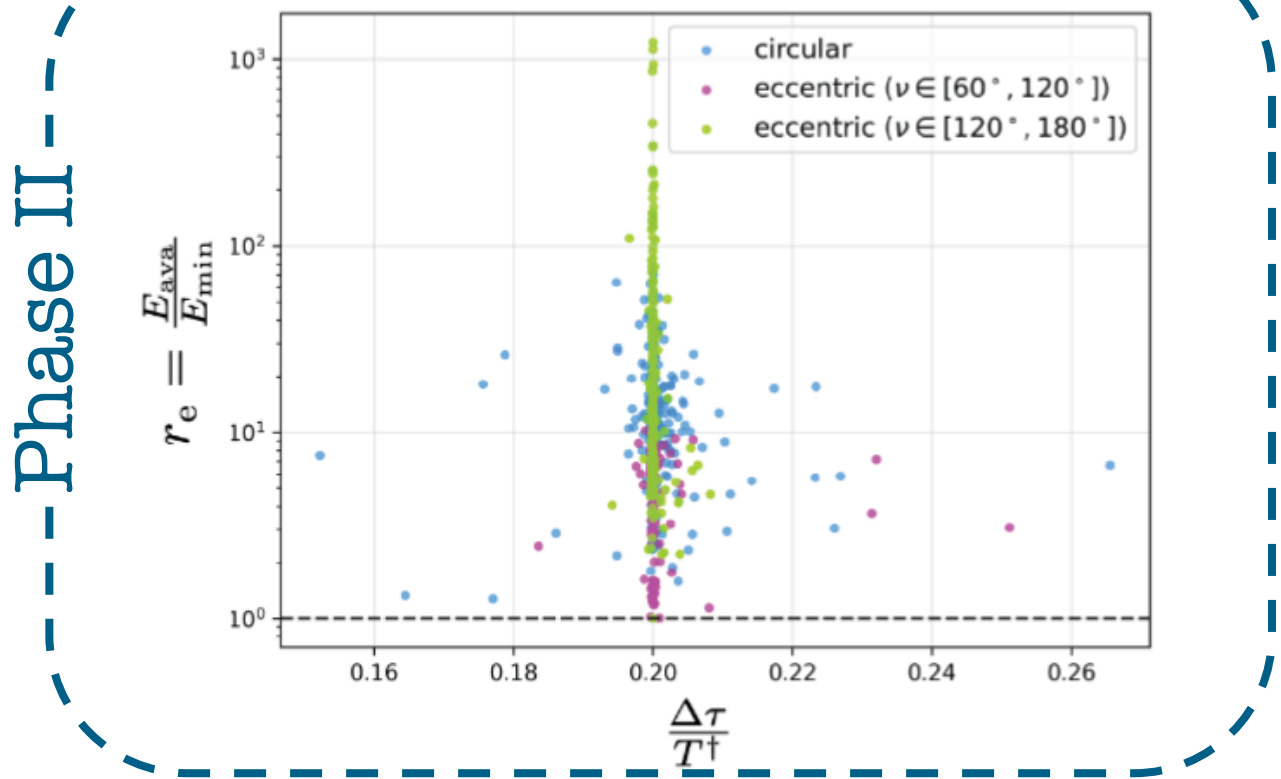
Sinha and Beeson (JGCD, 2025)

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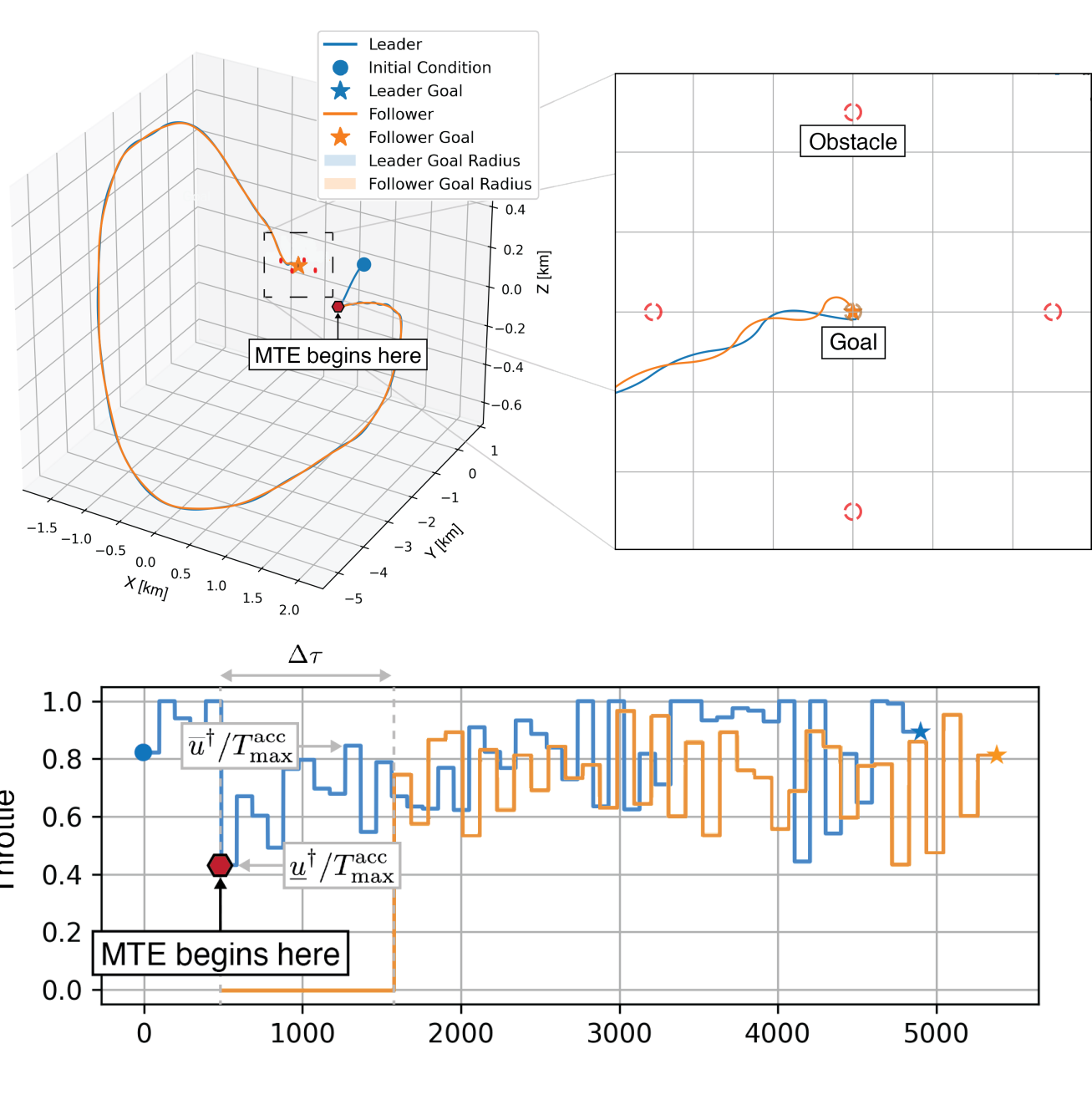
Circular
Eccentric



Phase II certification passed: $r_e > 1$



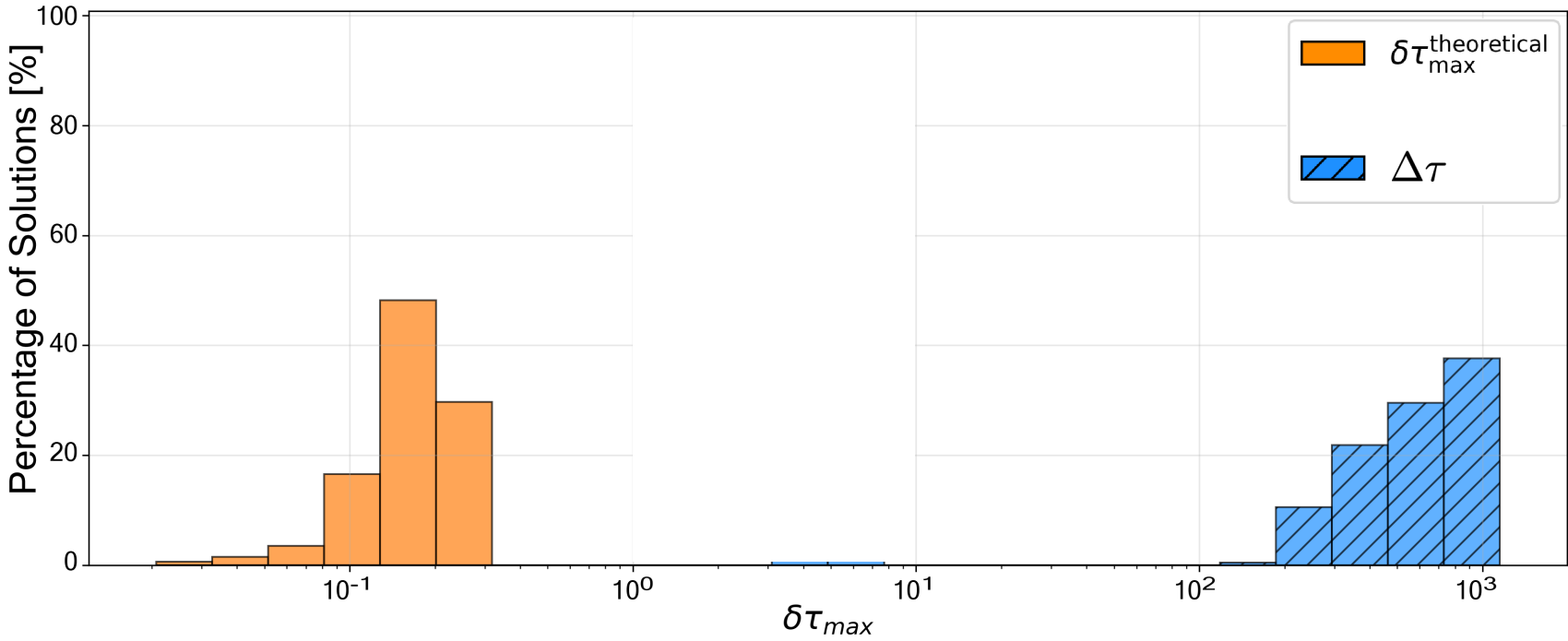
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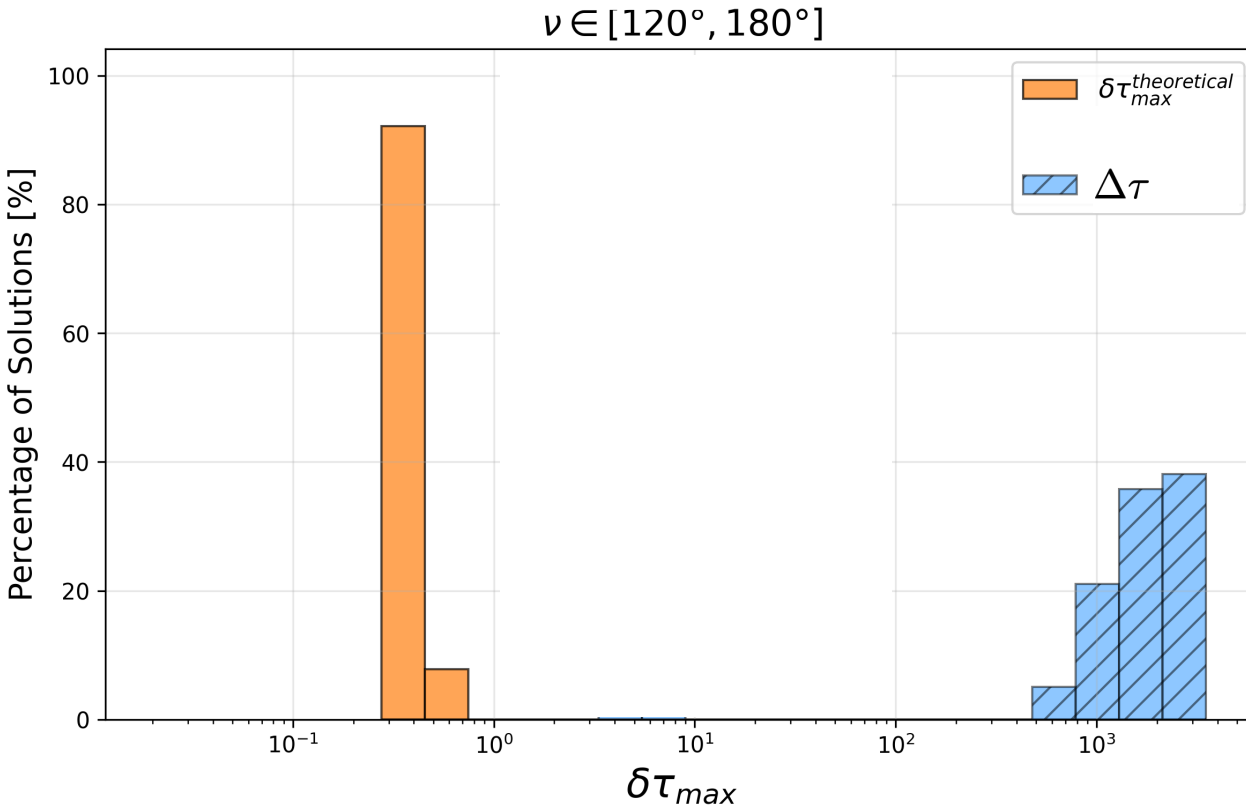
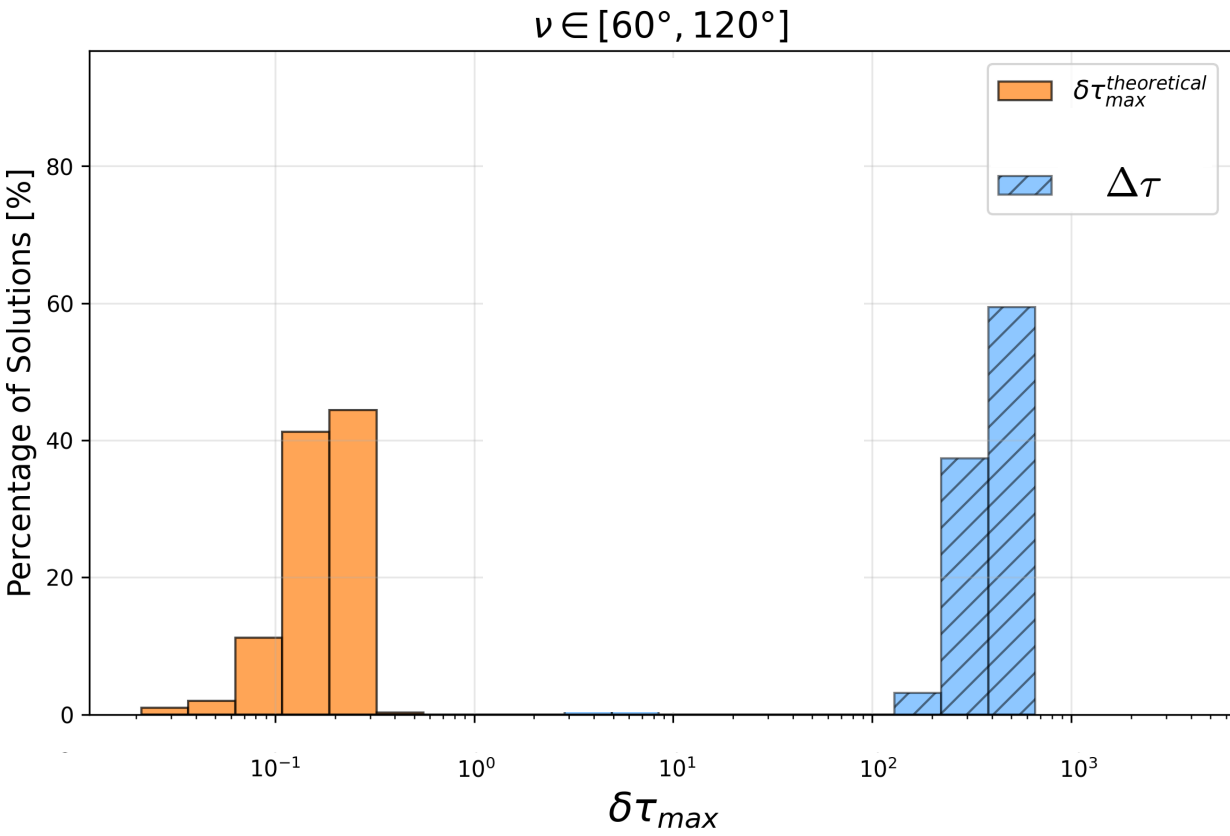
Sinha and Beeson (JGCD, 2025)

Phase I certification failed: $\delta\tau_{\max}^{\text{theoretical}} \ll \Delta\tau$

Circular

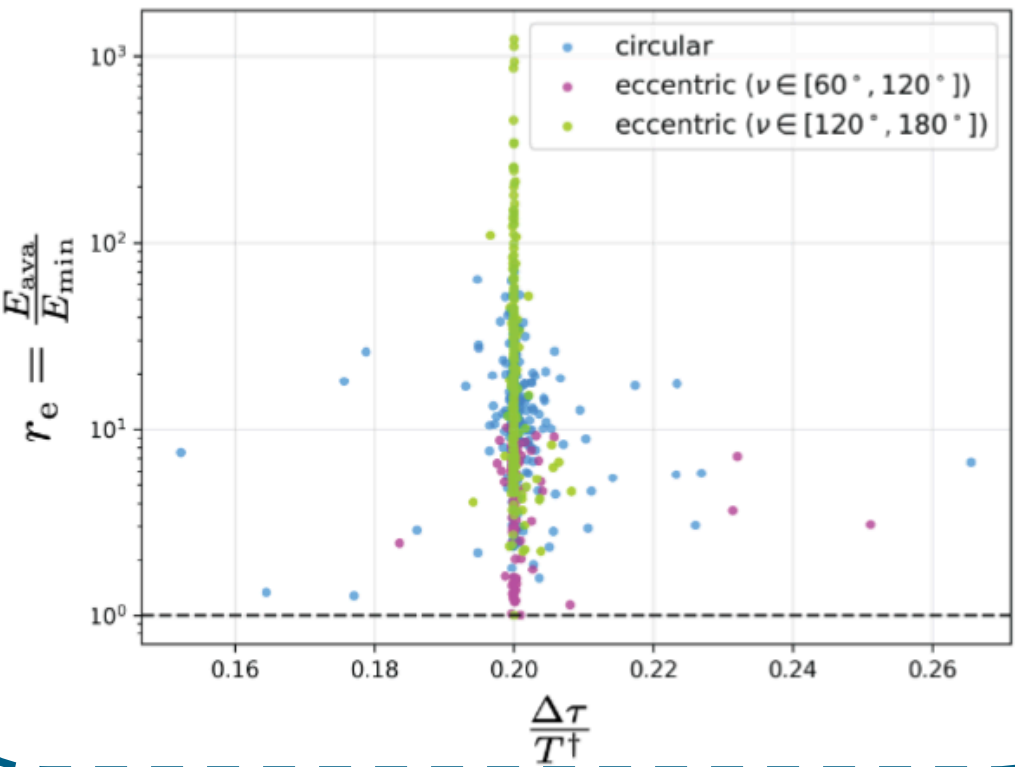


Eccentric



Phase II certification passed: $r_e > 1$

Phase II



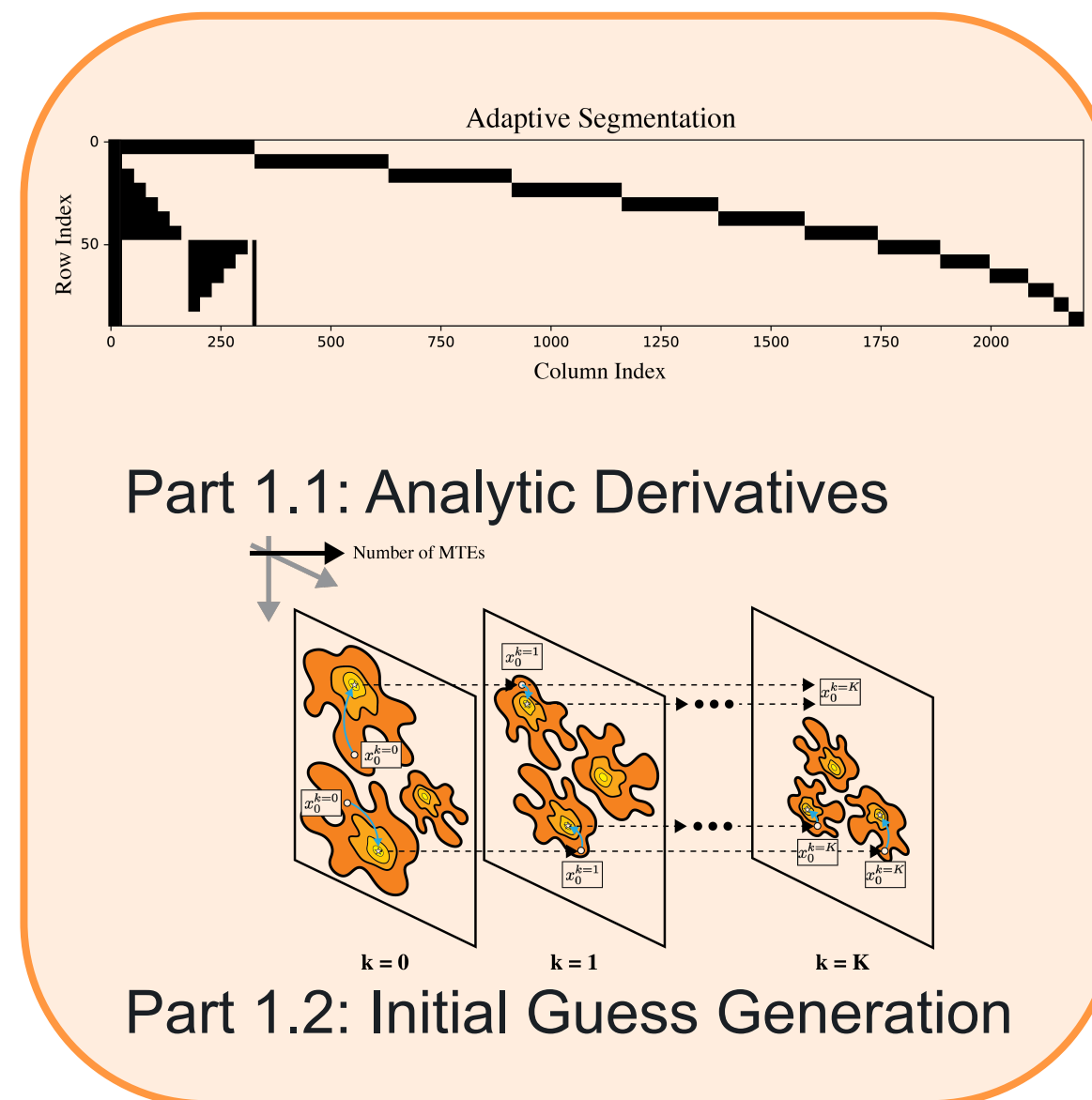
$r_e > 1$ explains why recovery can succeed even when $\Delta\tau > \delta\tau_{\max}$



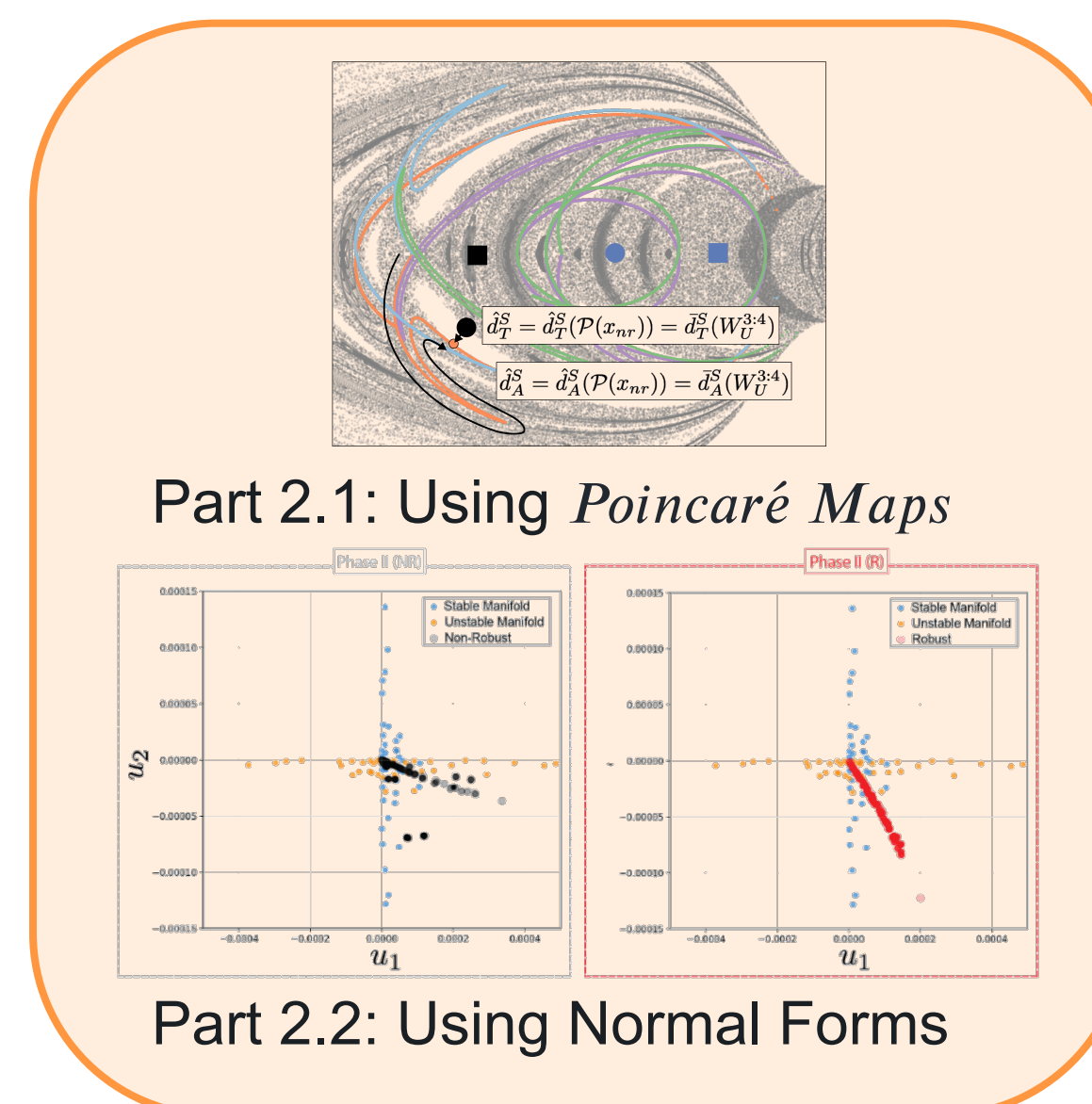
Concluding Remarks

This dissertation develops a framework for robust low-thrust trajectory design under missed thrust events...

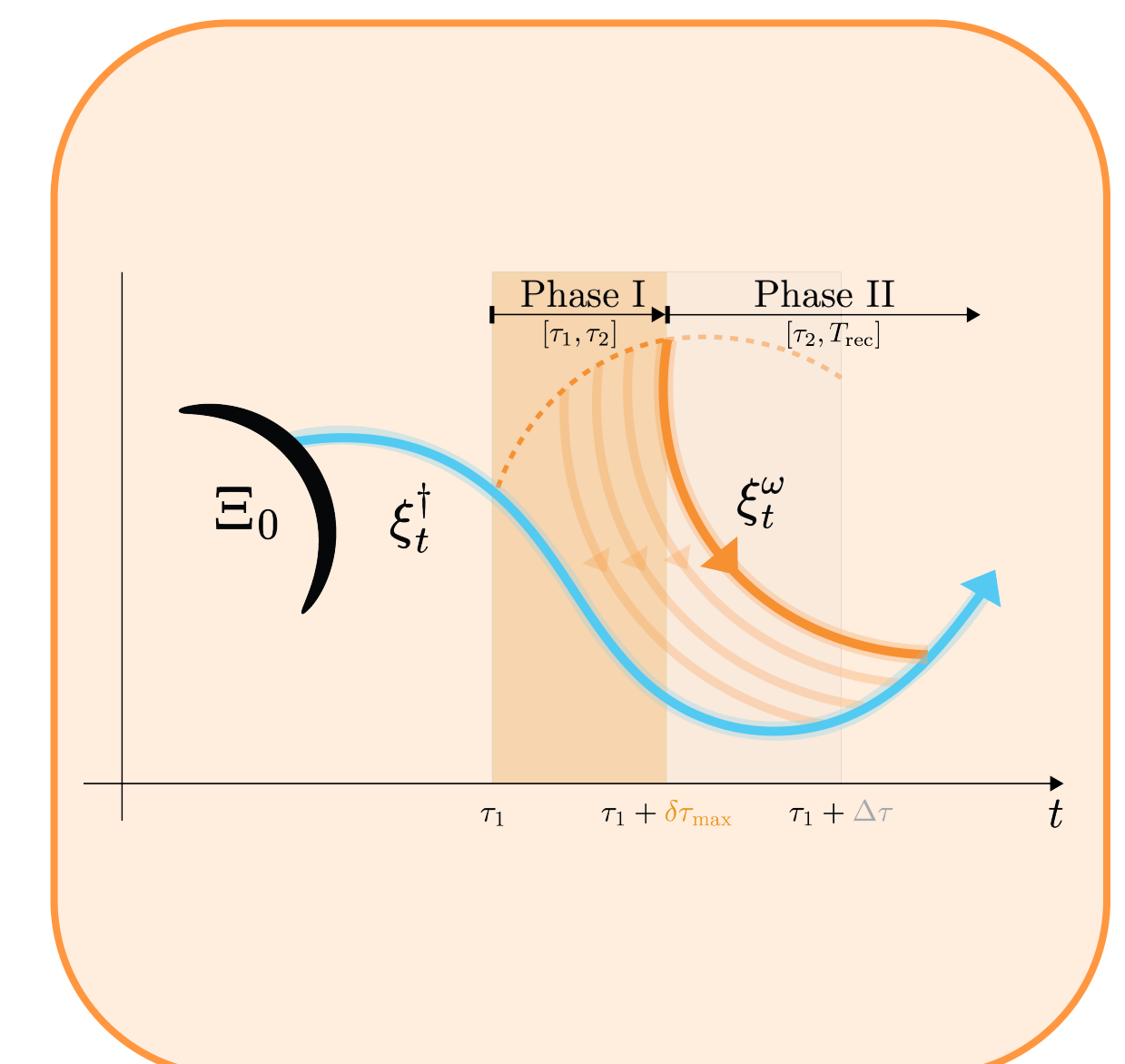
Part 1: Scalable Computational Methods



Part 2: Dynamical Systems Analysis



Part 3: Bilevel Optimal Control Framework

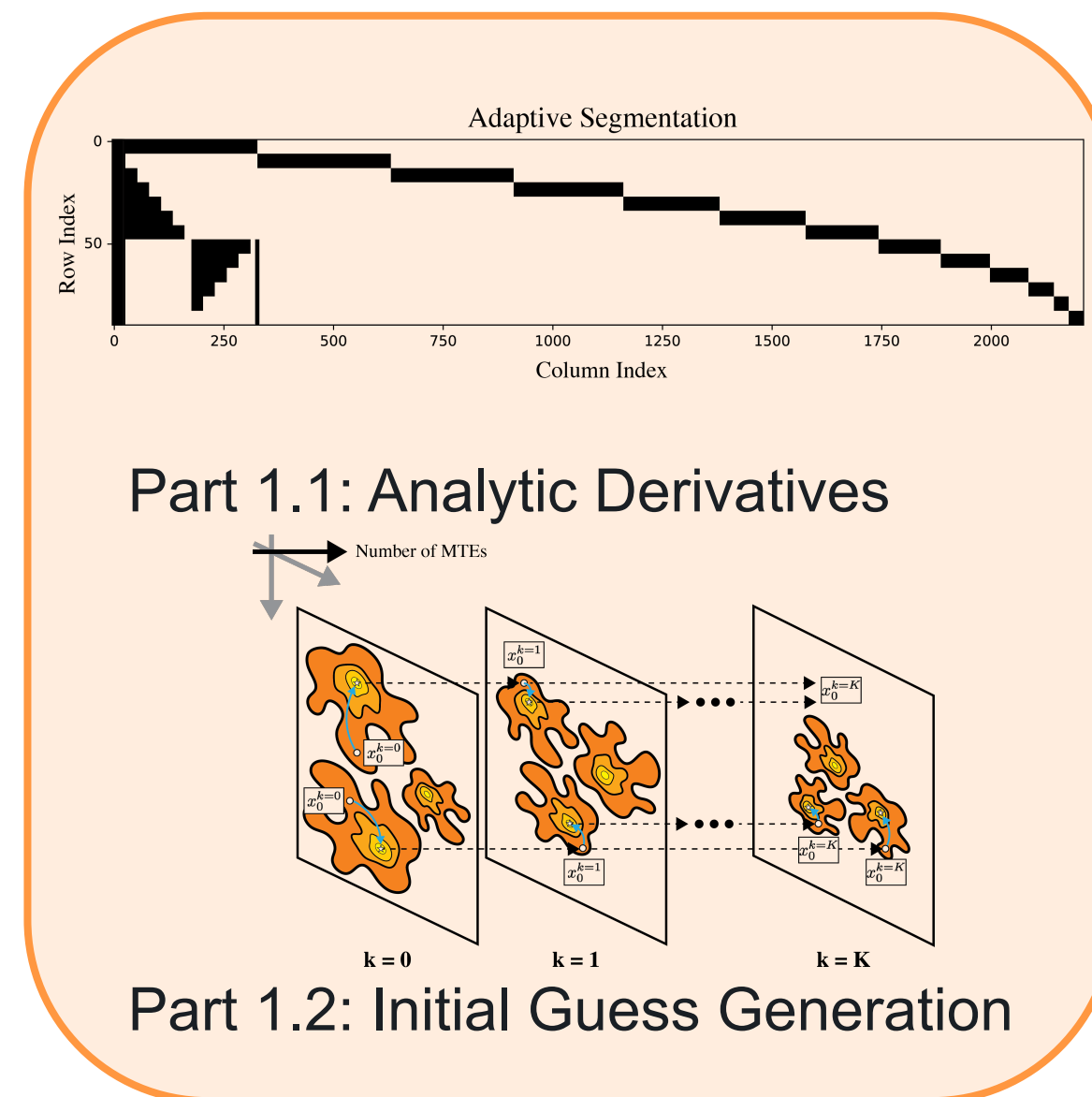


Part 0: Missed Thrust Design Problem Formulation

This dissertation develops a framework for robust low-thrust trajectory design under missed thrust events...

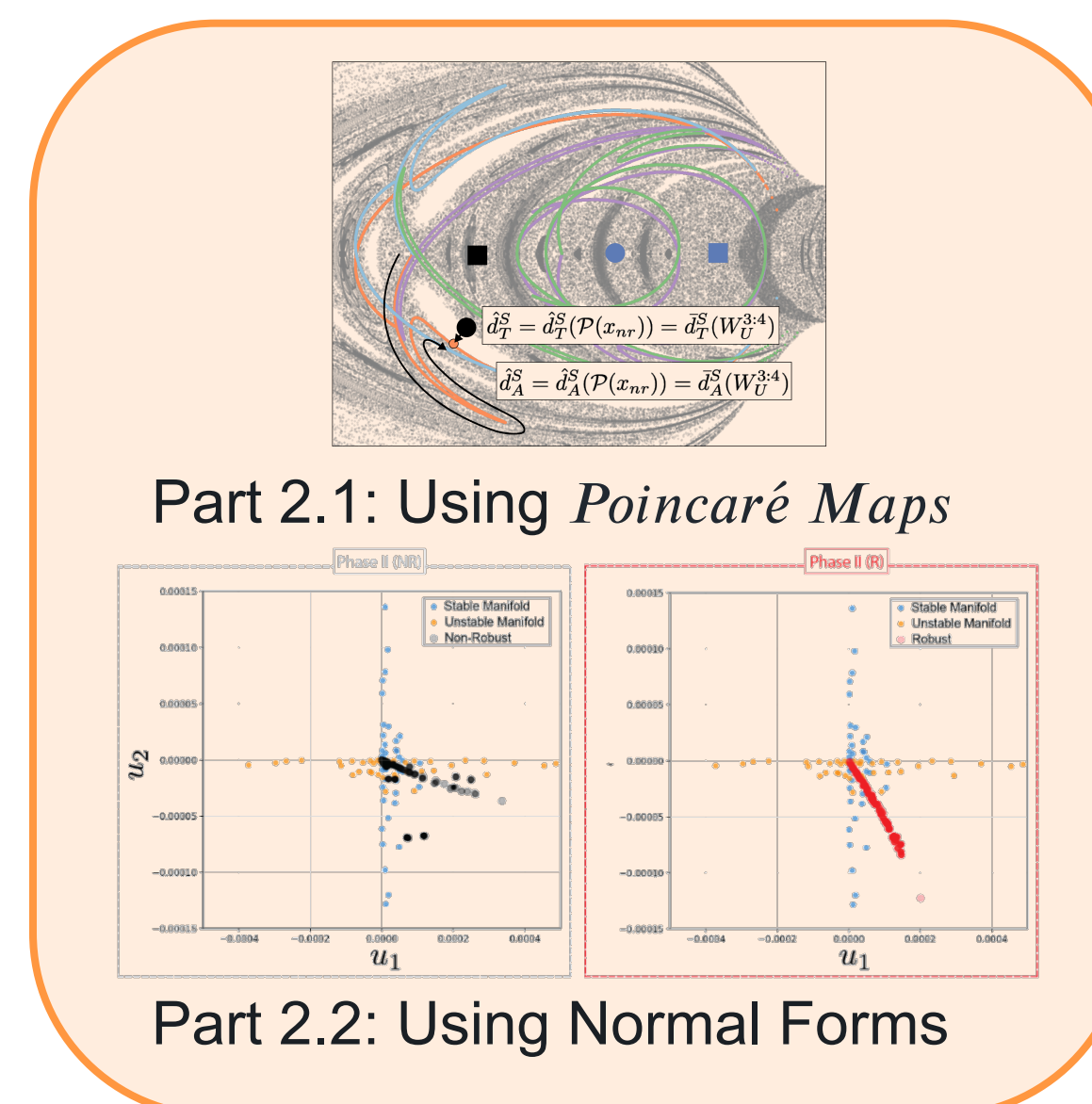
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Scalable Computational Methods



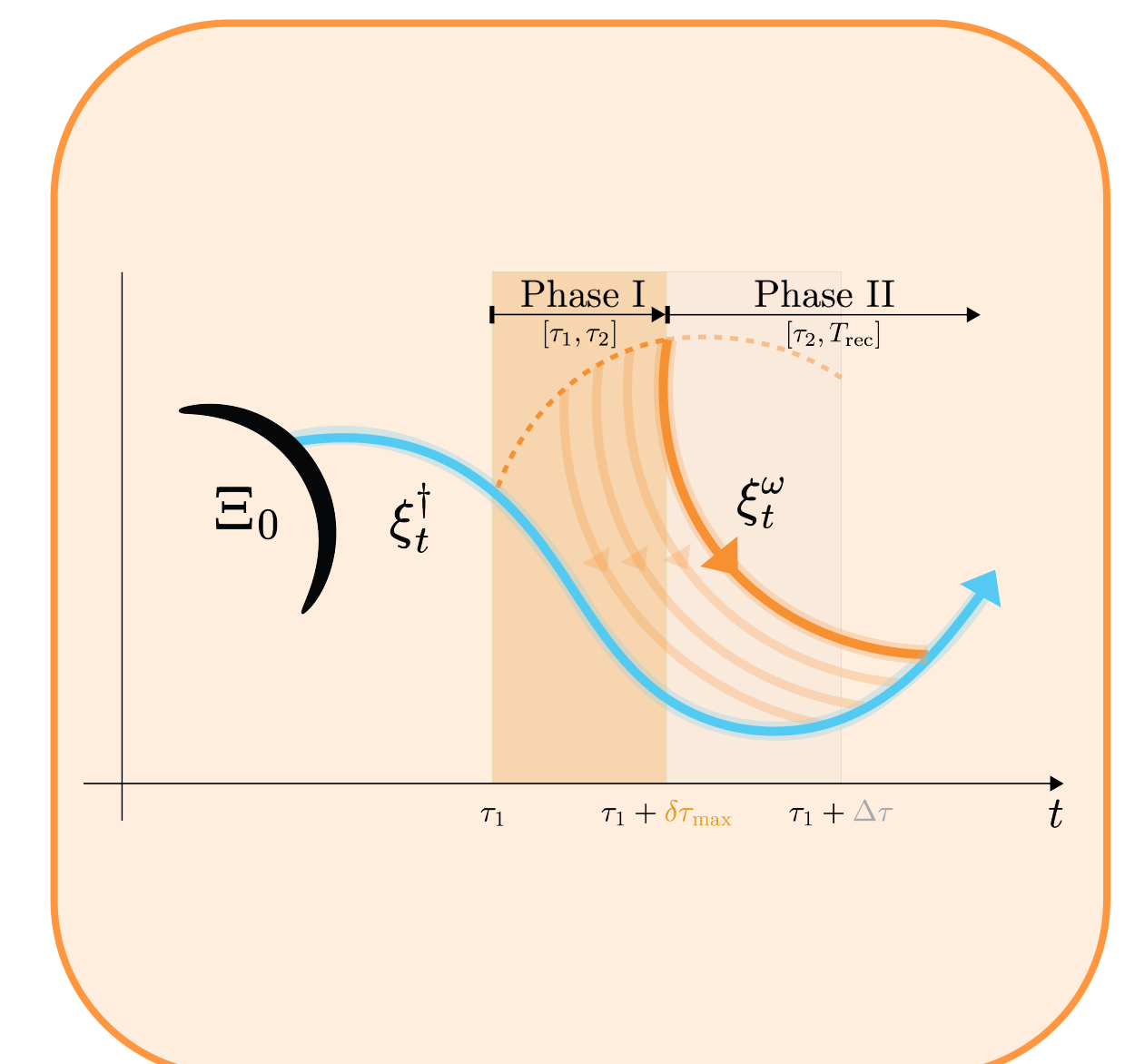
Part 2:

Dynamical Systems Analysis



Part 3:

Bilevel Optimal Control Framework



Formulation as a continuous time robust optimal control problem, coupling the nominal solution with recovery solutions under discrete missed thrust scenarios

This dissertation develops a framework for robust low-thrust trajectory design under missed thrust events...

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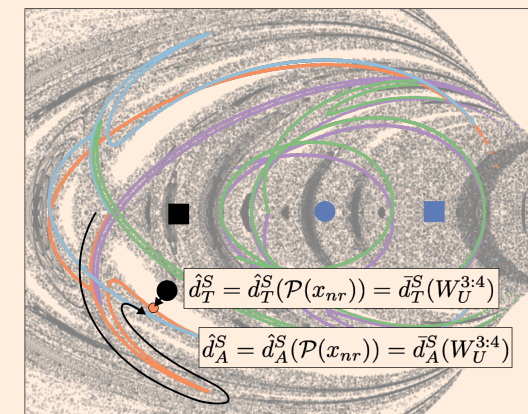
Scalable Computational Methods

Analytic derivatives are derived for the missed thrust design problem, along with a conditional global search strategy which collectively improve

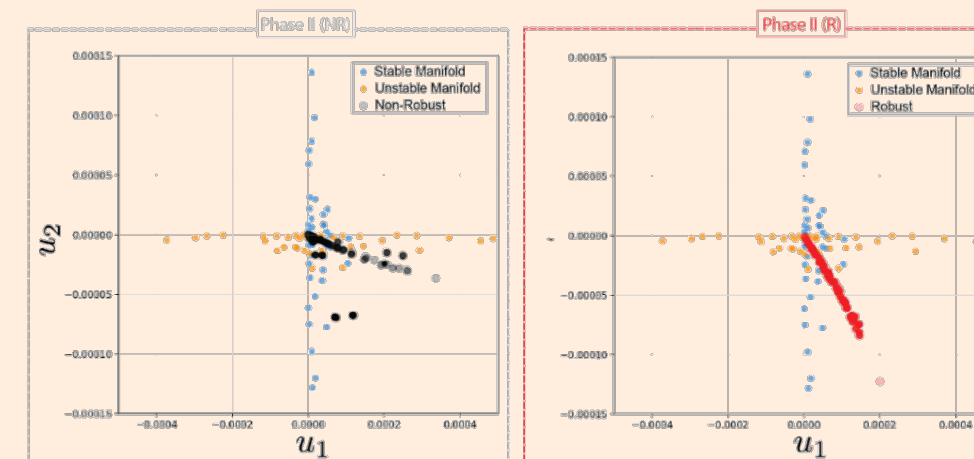
- Convergence rates
- Solution diversity
- Runtime

Part 2:

Dynamical Systems Analysis



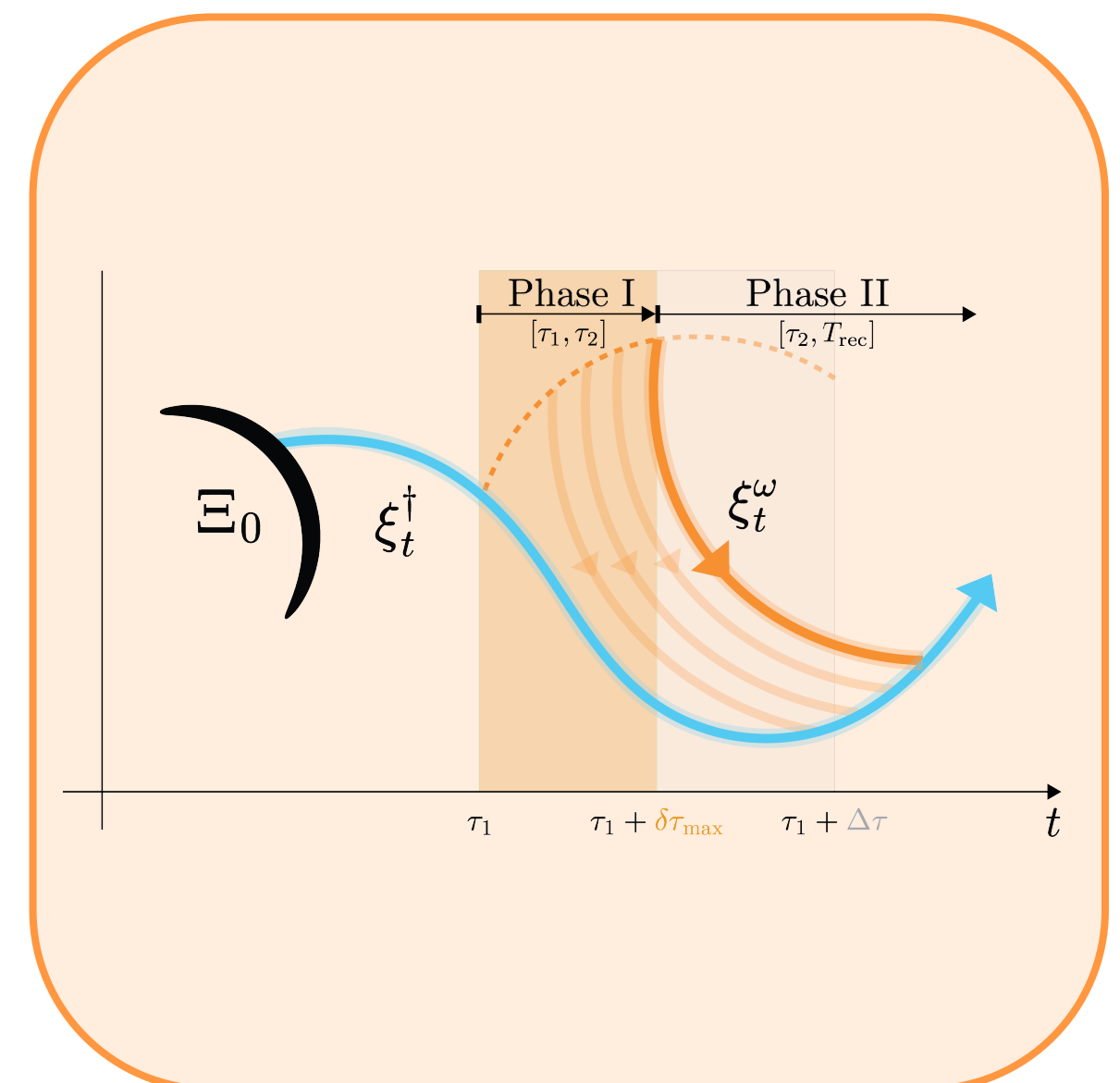
Part 2.1: Using *Poincaré Maps*



Part 2.2: Using Normal Forms

Part 3:

Bilevel Optimal Control Framework



Formulation as a continuous time robust optimal control problem, coupling the nominal solution with recovery solutions under discrete missed thrust scenarios

This dissertation develops a framework for robust low-thrust trajectory design under missed thrust events...

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- Convergence rates
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- Runtime

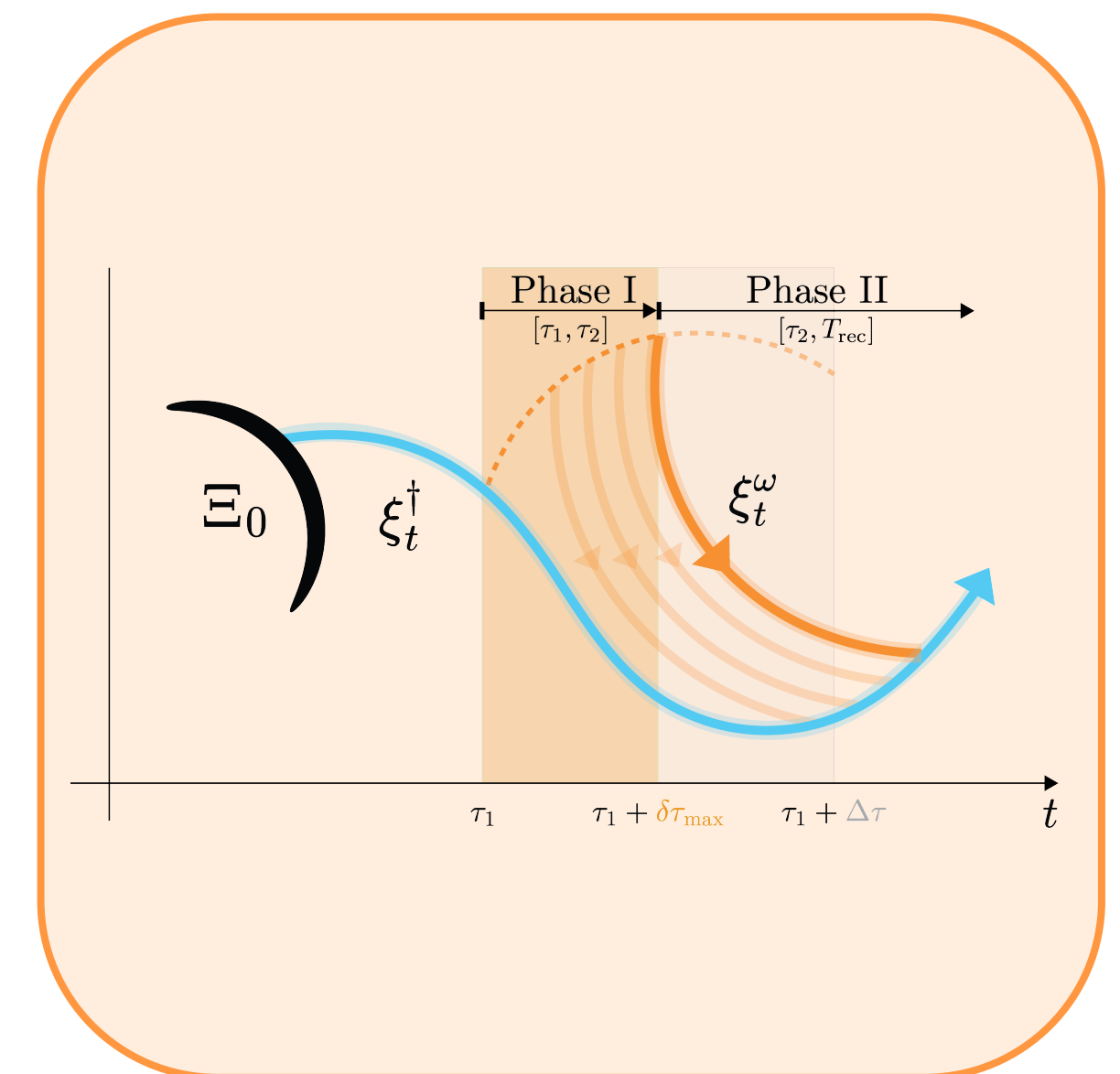
Part 2:

Dynamical Systems Analysis

Robust solutions must stay in close proximity to the invariant manifolds throughout the transfer to achieve **optimality**, sometimes more closely than the non-robust solutions.

Part 3:

Bilevel Optimal Control Framework



Formulation as a continuous time robust optimal control problem, coupling **the nominal solution with recovery solutions** under discrete missed thrust scenarios

This dissertation develops a framework for robust low-thrust trajectory design under missed thrust events...

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Robust solutions must stay in close proximity to the invariant manifolds throughout the transfer to achieve optimality, sometimes more closely than the non-robust solutions.

Part 3:

Bilevel Optimal Control Framework

By posing the missed thrust design problem as bilevel optimal control problem, we can obtain theoretical bounds on the maximum missed thrust duration as well as post outage recoverability conditions.

Formulation as a continuous time robust optimal control problem, coupling the nominal solution with recovery solutions under discrete missed thrust scenarios

**This
dissertation
opens several
paths toward
a broader
robust
trajectory
design
pipeline...**

Richer Uncertainty Models

- Multiple MTEs
- Stochastic models
- Additional uncertainties

Deeper integration of dynamical structures

- Embedding dynamical structure information
- Multi-chart atlas
- Efficient computation of invariant structures

More Scalable Algorithms

- Data driven frameworks
- Continuation methods

Stronger robustness certification

- LL generalization beyond LQR
- Segment-wise bounds

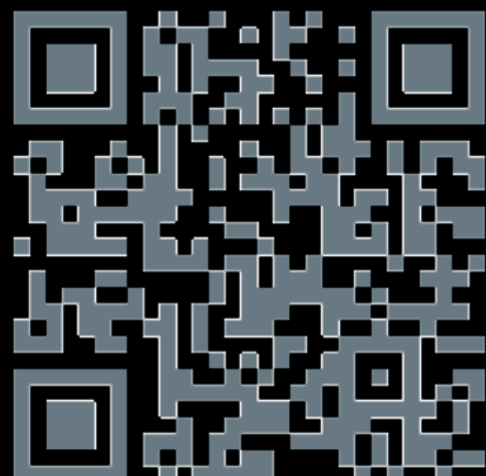
This work has culminated in...

Personal Website



amlan-sinha.github.io

Group Website



beeson.princeton.edu

3

Journal Papers
as first author

- 2 published in the *Journal of Guidance, Control, and Dynamics*;
- 1 in review with the *Journal of Guidance, Control, and Dynamics*;

6

Conference Papers
as first author

- 2 at 2023 AAS/AIAA Astrodynamics Specialist Conference
- 2 at 2024 AAS/AIAA Astrodynamics Specialist Conference
- 1 at 2025 AIAA/AAS Space Flight Mechanics Meeting
- 1 at 2025 AAS/AIAA Astrodynamics Specialist Conference

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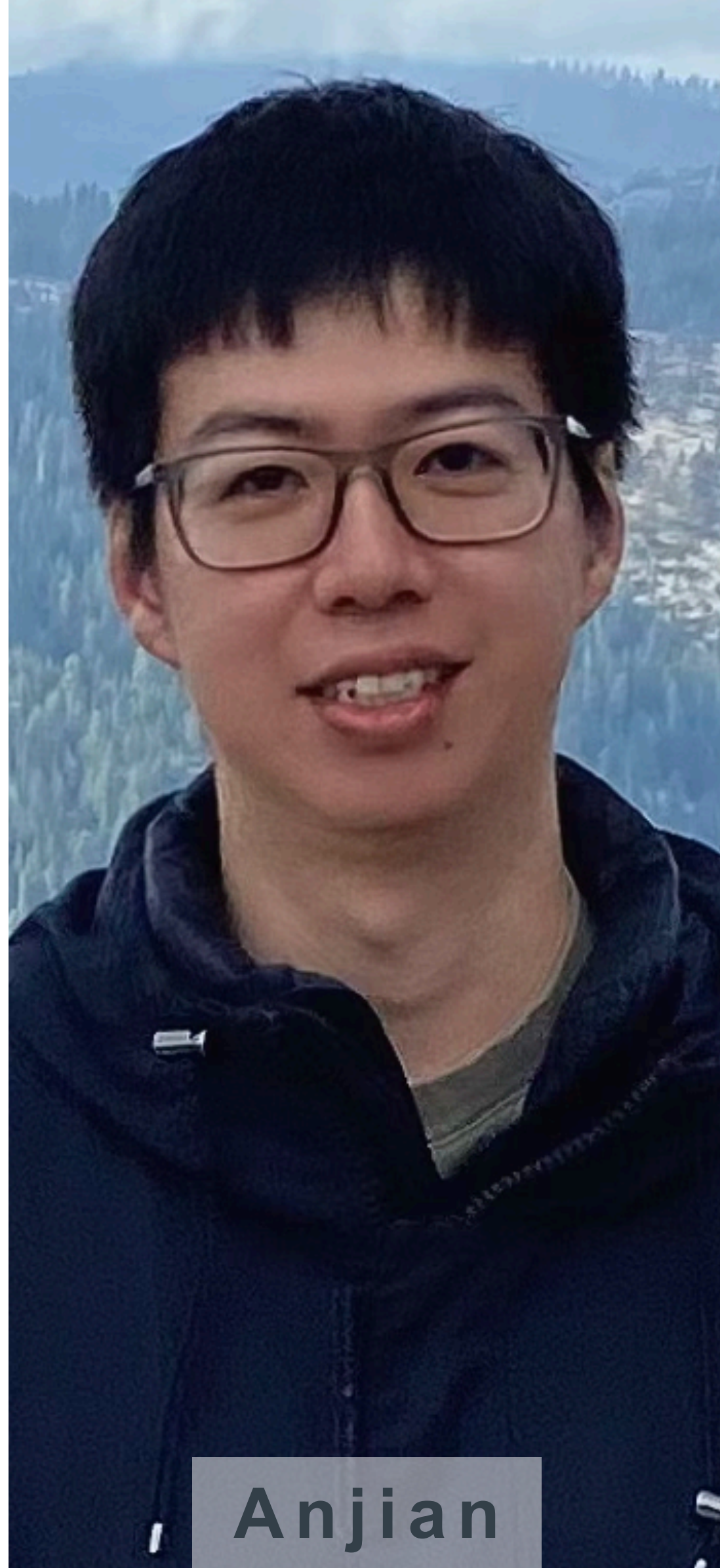
Collaborative Efforts

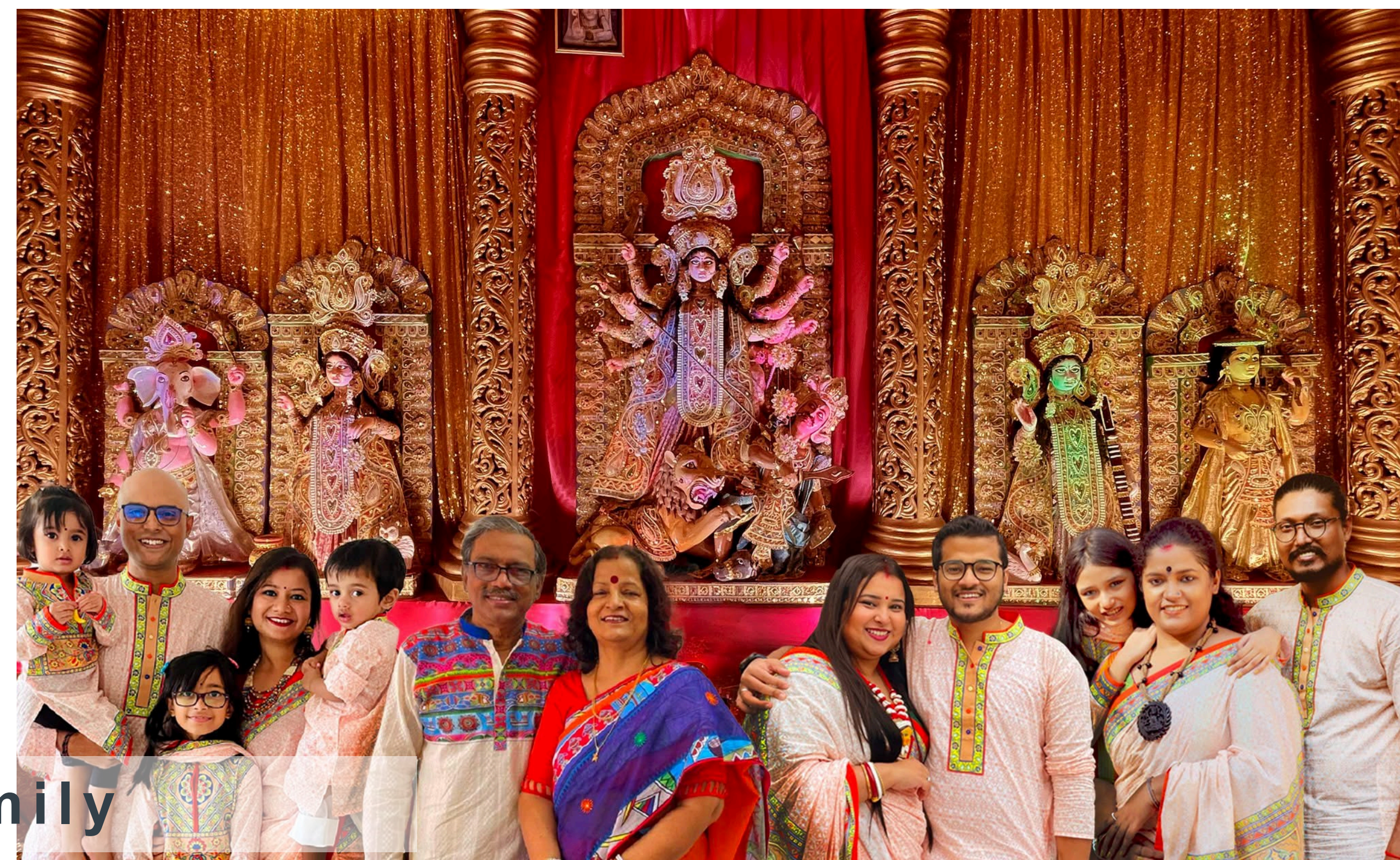
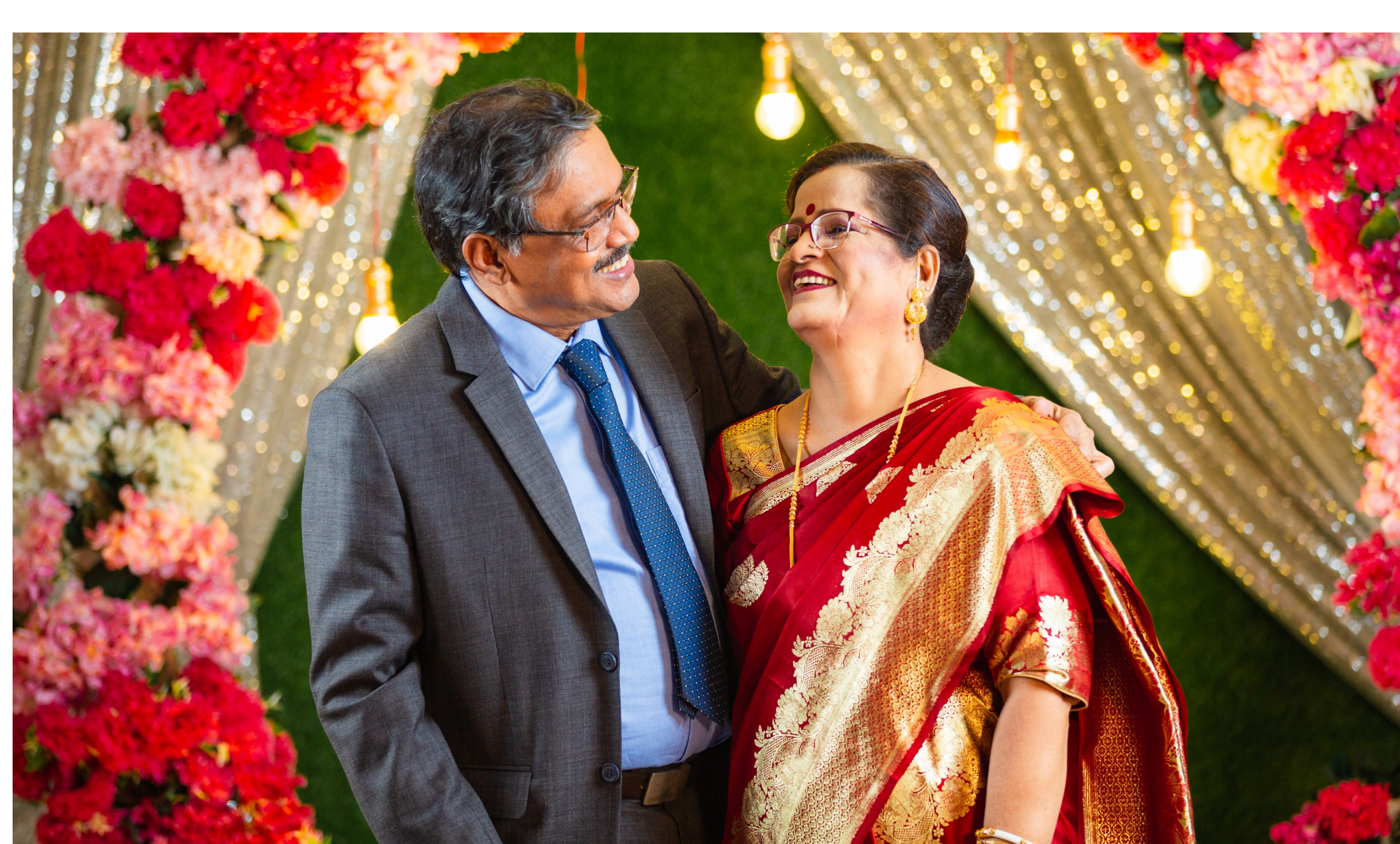
- 2 journal publications as second author (1 published, 1 in review)
- 10 conference publications as co-author



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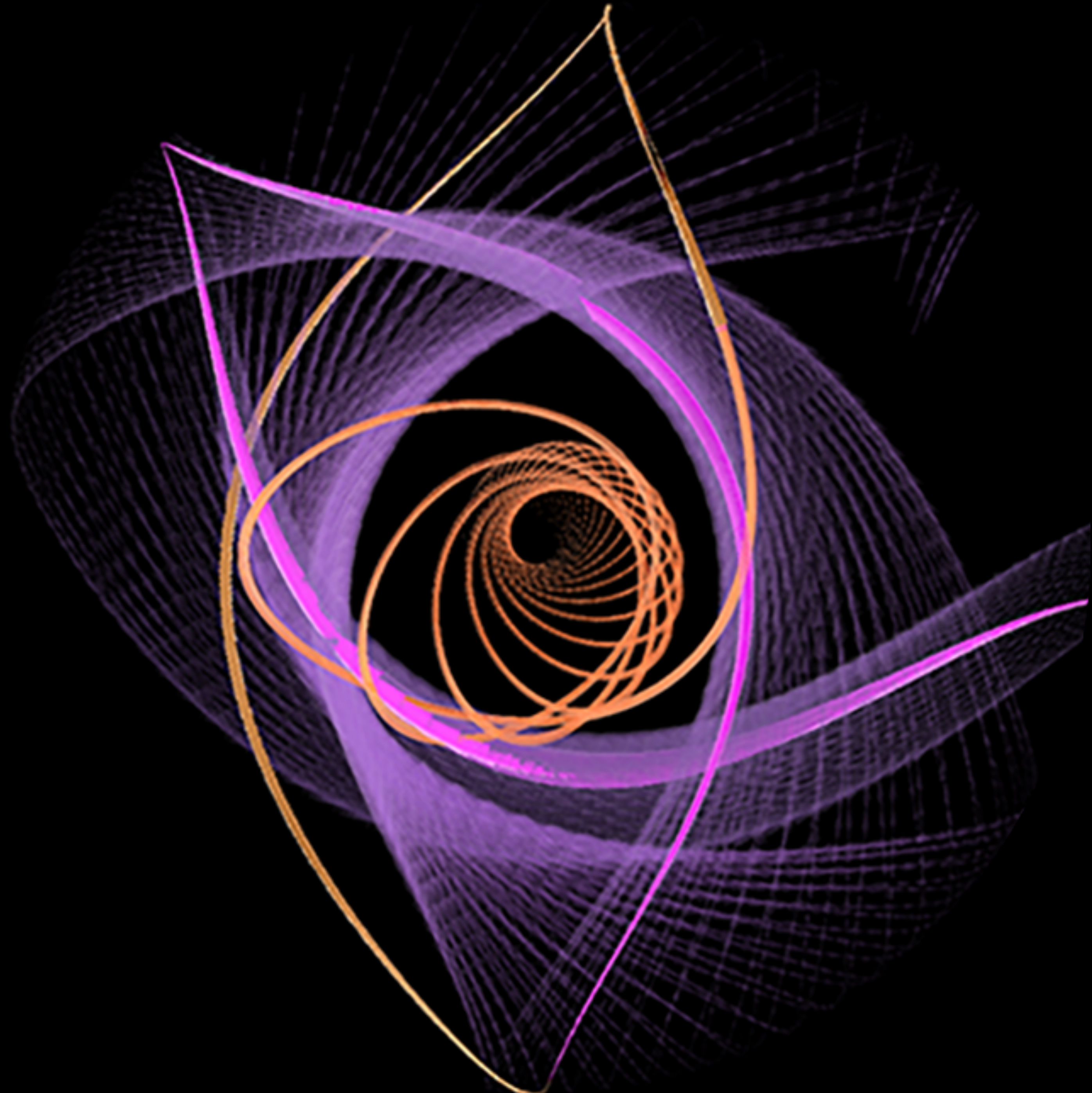


Saiti Srabonti Halder, Ph.D.



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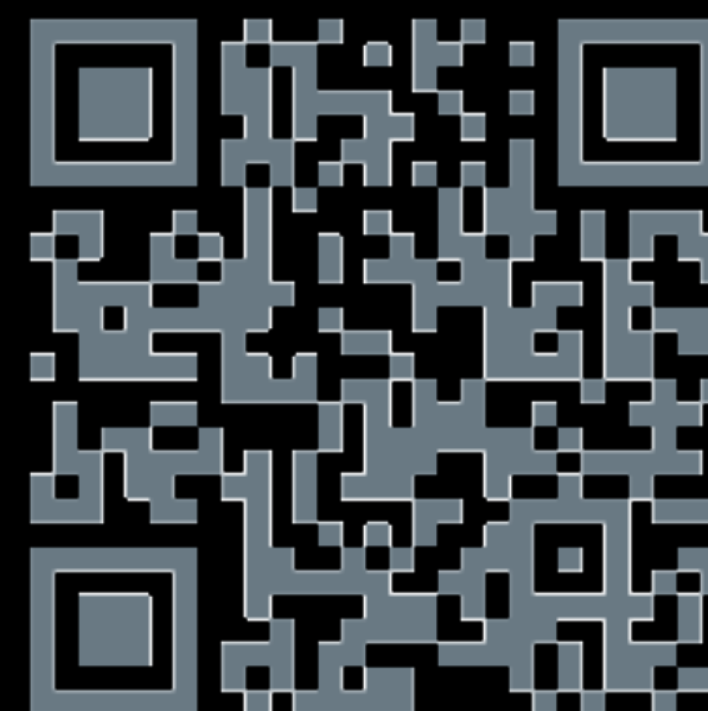


Personal Website



amlan-sinha.github.io

Group Website



beeson.princeton.edu

Questions?